Selection of One Plan Suspension System with Special Type Double Sampling Plan Through Minimum Sum of Risks

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Abstract

Acceptance sampling plans are the practical tools for quality assurance applications involving product quality control. Acceptance sampling systems are advocated when small sample size are necessary or desirable towards costlier testing for product quality. When the lots in a series are accepted without any inspection, then sampling inspection is carried for immediately preceding lots, when meets the acceptance criteria towards decision making. Whenever a sampling inspection is considered, the lot is either accepted or rejected along with associated producer and consumer’s risk.

This paper presents a new procedure for designing one plan suspension system involving minimum sum of risks. Procedures and necessary tables are provided for the selection of sampling system through minimum angle criteria involving producer and consumer quality levels.

Keywords:
One plan suspension system, Minimum sum of risks, Minimum Angle Method, Accepting quality level, Limiting quality level.

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Introduction

Cone and Dodge (1962) have first shown that the effectiveness of a small sample lot-by-lot sampling system can be greatly improved by using cumulative results as a basis for suspending inspection. It requires the producer to correct what is wrong and submit satisfactory written evidence of action taken before inspection is resumed. The small sample is considered due to small quantity of production or costly / destructive nature of sample.

Troxell (1972) has applied this suspension principle to acceptance sampling system incorporating a suspension rule to suspend inspection on the basis of unfavorable lot history, when small sampling plans are necessary or desirable. Here suspension rule is seen to be a stopping time random variable and a suspension system is a rule used with a single sampling plan or a pair of normal and tightened sampling plans. When single plan is used with a suspension rule it is called One Plan (OP) suspension system.

In this paper a new concept has been presented, what is the behaviour of submitted lots before suspension occurs if it follows Poisson distribution. Attention is centered primarily on small sample single sampling plan with acceptance number c=0 and c=1. Lilly Christina (1995) has studied the design and analysis of suspension system.

A suspension rule, which is designated as (j, k), 2 ≤ j ≤ k, is a rule for suspending inspection based on finding j lot rejections in k or less lots. Specifically, an account is kept of lot dispositions from the present lot to a fixed number of k-1 previous consecutive lots. If at any time the present lot increases the total number of lot rejections observed over the fixed span of length k to some predetermined integer j, inspection is suspended; a run of j out of k or less lots is said to have occurred. Given j and k, at least j lots must be inspected before a decision is possible upon the beginning of a new process or from the time of the last suspension. Upon restart of inspection after suspension, history starts a new in that all previous dispositions are ignored. The rule then determines uniquely at every lot whether to continue or suspend inspection.

The phrase “lot disposition” always refers to either lot acceptance (A) or lot rejection (R), while the term ‘lot history’ refers to a sequence of lot dispositions e.g.(AARARA….). A one plan suspension system is a combination of a suspension rule and a single lot-by-lot sampling plans. Under OP suspension system, a lot-by-lot sampling plan is used in the usual
way to decide whether individual lots shall be accepted or rejected. The sampling inspection
procedures being treated here is one involving the sampling of a continuous process with
samples taken from each lot or partition of the product. The conditions for application are
given below:

**Conditions for Application**

1. Production is reasonably steady. So that results on current and proceeding lots are
   broadly indicative of a continuous process.
2. Samples are taken from lot substantially in the order of production so that observed
   variations in quality of product reflect process performance.
3. Inspection is performed close to the production source so that inspection information
   can be made available promptly.
4. Inspection is by attributes, with quality measured in terms of fraction defective p
5. A single sample of size n or double or multiple samples of equal size n is taken from
   each sampled lot.

**Operating Procedure**

1. For the product under consideration establish a reference quality level (RQL).
   This RQL termed as np represents the desired quality at delivery considering
   the needs of service and cost of production.
2. Consider the established RQL, select a suspension system.
3. Apply the suspension rule to the first, second…kth lot, then to each successive
   group of k lots.
4. If any lot is rejected, declare the lot nonconforming and dispose it in
   accordance with standard procedures.
5. If for any lot, the suspension rule occurs, declare the current lot
   nonconforming and also declare the process nonconforming.
6. When the process is judged nonconforming:
   a. Notify the submitting agency that no additional lots may be submitted
      for inspection until that agency has furnished evidence, satisfactory to
      the inspection agency that action has been taken to assure the
      submission of satisfactory material.
b. Dispose the current nonconforming lot in accordance with standard procedures.

c. When satisfactory evidence of corrective action is furnished, start inspection again with the next succeeding lot and with this lot begin accumulation.

d. If it becomes necessary to refuse lot submissions a second time, so advice an appropriate higher authority and notify the submitting agency that further submissions will be refused until evidence satisfactory to the higher authority has been approved.

### Average Run Length

According to Troxell (1972) the expected time to suspension or average run length of the suspension rule \((j, k)\) designated as \(ARL(j, k)\) can be calculated as follows:

First, the expected number of lot rejections until suspension is calculated. Since the rejections are interspersed with lot acceptances, the second step is to find the total expected number of lots inspected, including the rejected lot, between successive lot rejections the \(ARL\) equals the sum of the total number of lots inspected until suspension.

\[
ARL(j, k) = \text{Total number of inspected lots between two rejections} \times \frac{\text{Expected number of rejections until suspension}}{1 - P_A}
\]

Using this fact, for \(j=2\), the expression is given by a single term and for \(j=3\), the result is best expressed in the form of a continued fraction, which is found by solving for the stationary distribution of a particular Markov chain. For higher rules, a discussion is given indicating the method of solving for the expected number of rejections until suspension.

Troxell (1972) has derived the following results:

i) ARL for the rule \((j, j)\), \(j \geq 2\) is

\[
ARL(j, j) = \frac{1 - (1 - P_A)^j}{P_A (1 - P_A)^j}
\]

ii) ARL for the rule \((j, \infty)\) is

\[
ARL(j, \infty) = \frac{j}{1 - P_A}
\]

which is the waiting time for the \(j^{th}\) occurrence of a lot rejection, or the mean of the negative binomial distribution with parameter \(j\).
iii) ARL for the rule \((2, k)\) is

\[
\text{ARL}(2, k) = \frac{(2 - P_a^{-k})}{(1 - P_a)(1 - P_a^{-k})}, \quad k \geq 2
\]

For any \(k\) such that \(j < k < \infty\) and \(0 < P_a < 1\)

\[
\text{ARL}(j, j) > \text{ARL}(j, k) > \text{ARL}(j, \infty)
\]

So that the rules \((j, j)\) and \((j, \infty)\) respectively are upper and lower bounds for all rules in the class \((j, k)\). Troxell (1980) has given the new procedure for one plan suspension system using single sampling plan with \(c=0\). Further tables are provided for solving ARL equations in terms of Probability of acceptance \((P_a)\).

In the literature, this paper provides a new procedure for designing one plan suspension system with special type double sampling plan as reference plan involving through Minimum Sum of Risks. Tables are developed for the selection of plan parameters for the sampling plan. Illustrations are also provided for easy selection of plan parameters.

Golub (1953) has developed a method for designing single sampling plan when the sample size is fixed and given an expression for \(c\) such that the sum of producer’s risk \((\alpha)\) and consumer risk \((\beta)\) is minimum. Soundararajan (1981) has extended the Golub’s approach to single sampling plan under the conditions of Poisson model for the OC curve. Vijayaraghavan (1989) has explained on designing Multiple Deferred State Sampling plans involving Minimum Sum of Risks.

Suresh and Saminathan (2007) have given a procedure to define multiple repetitive group sampling plans indexed with MAPD and MAAOQ. Suresh and Jayalakshmi (2007) have suggested new procedures on quick switching system with STDS using specified quality levels. Suresh and Kaviyarasu (2008) have explained the desirability of developing quick switching system indexed by the quality levels.

**Selection procedure**

**Selection of minimum risks one plan suspension system with Special Type Double Sampling (STDS) Plan**

Table (1) is used to select a minimum risks one plan suspension system with Special Type Double Sampling (STDS) Plan as reference plan for given \(p_1\) and \(p_2\). For the system of table, the producer’s and the consumer’s risks will be almost 10% each against the fixed values of the operating ratio \(p_2/p_1\). Tables give the parameter \(k, \phi\) and the associated
producer’s and consumer’s risks in the body of the table against the product of the sample size and the acceptable quality level (np₁).

The following procedure is used for selecting plan for given p₁, p₂, α and β.

1. Compute the operating ratio p₂/p₁.
2. With the computed value of p₂/p₁ enter table in the row headed by p₂/p₁, which is equal to or just smaller than the computed ratio.
3. For determining the parameter k and φ, one proceeds from left to right in the row identified in step 2 such that the tabulated producer’s and the consumer’s risks are equal to or just less than the desired values.
4. The sample size n is obtained as n = np₁/p₁ values are given in the column heading to the parameter k, φ identified in step 3.

For example, for given p₁ = 0.01 and p₂ = 0.06 with α = 0.05 and β = 0.10, from table one finds the plan suspension system with as reference plan involving minimum sum of risks as follows:

1. p₂/p₁ = 30
2. Tabulated p₂/p₁ = 30
3. corresponding to k = 25 and φ = 1.15 given in the body of the table () one obtain α = 0.004 and β = 0.005 against the desired α = 0.05 and β = 0.10
4. n = np₁/p₁ = 0.25/0.01 = 25.

**Construction of Tables**

The probability of accepting a lot given the proportion non-conforming under one plan suspension system with Special Type Double Sampling (STDS) plan as reference plan is given as

\[
P_a(2,k) = \frac{1 + P_a - P_a^k}{2 - P_a^{k-1}}
\]

Here Pₐ is the probability of acceptance for Special Type Double Sampling (STDS) plan. It is well known that for a series of lots from a process, the binomial model for the OC curve will be exact in the case of fraction non-conforming. It can be satisfactorily approximated with the Poisson model where p is small, n is large, and np < 5 when the quality is measured in terms of non-conformities, the Poisson model is the appropriate one. Under Poisson assumption, the expression for Pₐ under Special Type Double Sampling (STDS) plan,
\[ P_a = e^{-np} (1 + \phi np) \]  

(1)

Which is,

\[ P_a(2, k) = \frac{1 + (e^{-np} (1 + \phi np)) - (e^{-np} (1 + \phi np))^k}{2 - (e^{-np} (1 + \phi np))^{k-1}} \]  

(2)

For fixed \( np_1 \) the value of \( np_2 \) is calculated from equation \( np_2 = (p_2/p_1) (np_1) \) and is used in equation \( \alpha + \beta = [1 - P_a (p_1)] + P_a (p_2) \). The parameter \( k \) and \( \phi \) corresponding to the minimum \( [1 - P_a (p_1)] + P_a (p_2) \) are obtained by searching for \( k = 2(1)30, \phi = 0(0.05)1.40 \) with the help of a computer program. The following tables (1) gives the producer and consumer’s risks are obtained corresponding \( k \) and \( \phi \)

### Conclusion

Acceptance sampling is the techniques which deals with the procedure in which decision either accept or reject lots or processes which are based on the examination of samples. This paper relates to the new procedure for the selection of One Plan Suspension System with Special Type double Sampling Plan (STDS) as reference plan using Minimum sum of risks.

In acceptance sampling the producer and consumer plays a dominant role and hence one allows certain level of risk for producer and consumer, namely \( \alpha = 0.05, \beta = 0.10 \). In practice it is desirable to design any sampling plan with the associated quality levels which concern to producer and consumer.

The result presented in this paper are mainly related with new procedure and necessary tables for selection of sampling system through minimum sum of risks involving producer’s and consumer’s quality levels.

The emphasis in the present work is that the selection of sampling system with this procedure is more advantages to the producer and consumer. Tables are provided here which are tailor made, handy and ready- made uses to the industrial shop-floor condition. These tables are useful for both producer and consumer for obtaining good quality products with less cost for inspection.
REFERENCES


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