ON EFFICIENT ESTIMATION FOR THE LOG-NORMAL MODEL MEAN

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ABSTRACT

It is not uncommon to have situations where highly skewed data appear in research investigations. In many instances, transformations of such data are advocated, for the primary purpose of establishing a normal distribution required for parametric statistical analysis. Such a transformation retains its advantage in the context of 'meta-analysis' (Joseph et. al. (2000)), as well as in the context of 'Bayesian' Case-Studies (Stevens et. al. (2003)). The logarithmic transformation is a commonly-employed method within the decision sciences used to establish a normal distribution of skewed data. Patterson (1966) discussed and defined the challenges involved in the estimation of the population mean following the transformation of sample data. The same is equally true for the logarithmic transformations as well. The purpose of this paper is to address the logarithmic transformation and to ultimately provide the most efficient estimation of the lognormal mean. This has been achieved by using the sample information from the resultant normal distribution, in order to ultimately estimate the mean of the non-transformed population. The aspects of the gains in efficiency of the proposed 'Optimal Mean Estimator' are numerically illustrated through a simulation study, comparing it with the minimum "Risk/RMSE (Relative Mean Square Error)" estimator of log-normal mean recently studied by Shen et. al. (2006).

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1. INTRODUCTION

This paper addresses the transformation problem, and looks for the 'most efficient' mean estimation for the Log-normal model. This has been achieved by using the sample information from the resultant normal distribution (after the logarithmic transformation) more fully in the context of estimation of the population mean of the original (lognormal) distribution before the transformation. Suppose Y is a random variable which has a log-normal distribution with mean E(Y) = ς. Then log (Y) will be normally distributed with mean, say, μ and variance, say, σ^2.

Let us say that Y ~ LN (μ, σ^2) with mean ς. Consequently, the three parameters have the following relationship:

$$\varsigma = \exp(\mu + \sigma^2/2) \quad \ldots (1.1)$$

Consider a random sample $y_1, y_2, \ldots, y_n$ that is i. i. d. LN (μ, σ^2) with mean ς. Then $x_i = \log(y_i)$ is i. i. d. N (μ, σ^2); for i = 1, …, n.

Let us define the following expressions:

$$\bar{y} = \sum_{i=1}^{i=n} y_i / n, \ \bar{x} = \sum_{i=1}^{i=n} x_i / n, \text{ and } s^2 = \sum_{i=1}^{i=n} (x_i - \bar{x})^2 / (n-1) \quad \ldots (1.2)$$

We know that $\bar{x}$ and s^2 are the ML (Maximum Likelihood) estimators for μ and σ^2 respectively. The simple plug-in principle, in view of (1.1), leads to the usual ML estimator for ς:

$$\exp(\bar{x} + s^2/2) \sim \text{UER, say.} \quad \ldots (1.3)$$

Shen et. al. (2006) have proposed a new estimator by considering its Relative Mean Square Error (RMSE). For example the RMSE of 'UE' in (1.3) above will be:

$$\text{RMSE (UER)} = E\left[(\text{UER} - \varsigma)/\varsigma\right]^2 \quad \ldots (1.4)$$

Shen et. al. (2006) use the following class of estimators:

$$\delta_c = \exp(\bar{x} + c.s^2/2), \ c = 1/(n+d); \ d > -n. \quad \ldots (1.5)$$

By minimizing the RMSE of the estimators in their class in (1.5) above up to the terms of order $1/n^2$, they found the optimal value of the'd'. They found their optimal estimator in the class when:

$c = 1/ (n + 4 + 3. \sigma^2/2)$ … (1.6)

Using the unbiased estimate 's^2 'of 'σ^2 'in (1.6), they found their 'Minimum Risk/RMSE (Relative Mean Square Error) estimator of ς:

$\exp \{\bar{x} + (n-1).s^2/ (2. (n + 4) + 3. s^2)\}$ = ER2006, say. … (1.7)

The RMSE of 'ER2006' in (1.7) above will be:

RMSE (ER2006) = E [(ER2006 - ς)/ ς]^2 … (1.8)

2. PRELIMINARY RESULTS

It is well-known that the sample mean $\bar{x}$, and the sample variance s^2 are independently distributed for a random sample from a Normal population with mean μ and standard deviation σ. While the original variable (on which we have the original data), say y is not normally distributed, the transformed variable x = log(y) is N (μ, σ^2). The population mean ς of y is a function of μ and σ^2, as noted in (1.1). As such, we could use the sample mean, and the sample variance:

$$\bar{x} = \sum_{i=1}^{i=n} x_i \text{ /n \& } s^2 = \sum_{i=1}^{i=n} (x_i - \bar{x})^2 /(n-1)$$

Where 'n' is the size of the random sample drawn from a normal population: x ~ N (μ, σ^2). Doing so leads to the usual estimator of μ as exp (UER), as in (1.3), wherein UER $= \bar{x} + s^2/2$. It is a very significant fact to note that, in view of the celebrated Rao- Blackwell theorem, any function of the sample mean/sample variance will be UMVUE (Uniformly Minimum Variance Unbiased Estimate) or UMMSE (Uniformly Minimum Mean Squared Error Estimator) of the relevant function of Population mean (μ)/population variance (σ^2), depending on the fact that the corresponding estimator at hand is simply an unbiased/ minimum mean squared estimator (mmse) of that parametric function of the population, which we are interested in the estimation of.

Hence, the estimator 'URE' is UMVU estimator of ln ς (the population mean of the original lognormal distribution).

We know that $(n-1)*s^2/\sigma^2 \sim \chi^2$ (Chi-Square) distribution on 'n-1' degrees of freedom (d. f.).

Using that sampling distribution, it could well be seen that the aforesaid estimator of the variance of T is not unbiased. In fact, one could easily check that:

$E(s^2) = \sigma^2$. So that s^2 is an unbiased estimate of σ^2, whereas $E(s^4)$ is not equal to σ^4.

On the other hand, we could also easily check that:

$E(cn1 * s^4) = \sigma^4$, wherein $cn1 = (n-1)/(n+1)$.

Therefore, $cn1 * s^4$ is an unbiased estimate of σ^4.

However, it could easily be verified that the MMSE (Minimum Mean square Error) Estimator of σ^2 is as below:

$MMSE(\sigma^2) = cn1*s^2$; wherein $cn1 = (n-1)/(n+1)$ … (2.1)

3. PREPARATORY IMPROVEMENT

It would be very common to encounter situations in which the sample estimate of the coefficient of variation of the sample mean (a more stable random variable than the original variable in the study 'X') will not be that large.

In such situations, we might prefer to have 'C.V. ($\bar{x}$) < 1.0 or even much less than 1.0'. Hence for such cases, which are quite frequent, we propose an alternative estimator, say $t^{\otimes}$, as below:

$$t^{\otimes} = \bar{x} + \bar{x} / [n.(\bar{x})^2/s^2 - 1] \quad \text{… (3.1)}$$

We note that the relative efficiency of $t^{\otimes}$ with respect to (wrt) the usual estimator $\bar{x}$ (In %) would be:

$$100/\eta = 100\frac{E(\bar{x}-\mu)^2}{E(t^{\otimes}-\mu)^2} \quad \text{… (3.2)}$$

It suffices for us to find an unbiased estimator of the 'Efficiency Ratio' η, as a function of ($\bar{x}$, s^2) alone; since ($\bar{x}$, s^2) is jointly a complete sufficient statistic for (μ, σ^2).The so-determined unbiased estimator, being a function of a "Complete Sufficient Statistic", would be UMVUE.

Now, we note that: $\eta = \frac{E(t^{\otimes} - \mu)^2}{E(\bar{x} - \mu)^2}$

$$= \left(\frac{n}{\sigma^2}\right) \mathbf{E} \left[(\bar{x} - \mu) + \bar{x} / \{n.(\bar{x})^2 / s^2 - 1\}\right]^2$$

= 1 + 2A + B; **… (3.3)**

Wherein: →

$$\mathbf{A} = \left(\frac{n}{\sigma^2}\right) \mathbf{E} \left[\bar{x}.(\bar{x} - \mu).\{n.(\bar{x})^2 / s^2 - 1\}^{-1}\right]$$

$$= \left(\frac{n}{\sigma^2}\right) \mathbf{E}_{s^2} . \mathbf{E}_{\bar{x}} . \left[\bar{x}.(\bar{x} - \mu).\{n.(\bar{x})^2 / s^2 - 1\}^{-1}\right] = \left(\frac{c.n}{\sigma^2}\right) \mathbf{E}_{s^2} .$$

$$\left[\int_{-\infty}^{+\infty} \bar{x}.\{n.(\bar{x})^2 / s^2 - 1\}^{-1}.(\bar{x} - \mu).\exp\left(\frac{-n(\bar{x} - \mu)^2}{2\sigma^2}\right) d\bar{x}\right] \text{ With } c = \left[\frac{n}{2\pi\sigma^2}\right]^{1/2}.$$

$$= (-c)\, \mathbf{E}_{s^2} . \left[\int_{-\infty}^{+\infty} \bar{x}.\{n.(\bar{x})^2 / s^2 - 1\}^{-1}.(d / d\bar{x}).\{\exp\left(\frac{-n(\bar{x} - \mu)^2}{2\sigma^2}\right)\} d\bar{x}\right].$$

Now, using the technique of integration by parts, we have the following proceedings:

A = E$_{s^2}$ **. E**$_{\bar{x}}$ $[a]$ **= E[a]; Wherein, a =** $-[\{n.(\bar{x})^2 / s^2 + 1\}.\{n.(\bar{x})^2 / s^2 - 1\}^{-2}]$

= - [(u +1). (u -1)$^{-2}$]. **… (3.4)**

And, $\{n.(\bar{x})^2 / (s^2)\} = u$ **… (3.5)**

Also; B = $\left(\frac{n}{\sigma^2}\right) \mathbf{E} \left[(\bar{x})^2.\{n.(\bar{x})^2 / s^2 - 1\}^{-2}\right]$

Similarly, we now take note of the well-known fact that independently of $\bar{x}$ **, (n-1)** s^2/σ^2 **has a** χ^2**-distribution with (n-1) 'degrees of freedom' ~ d. f. Therefore, we have: →**

$$\mathbf{B} = \left(\frac{n}{\sigma^2}\right) \mathbf{E}_{s^2} \mathbf{.}\ \mathbf{E}_{\bar{x}} \mathbf{.} \left[(\bar{x})^2.\{n.(\bar{x})^2/s^2-1\}^{-2}\right]$$

$$=(c^*/\sigma^2).\ \mathbf{E}_{\bar{x}}.\ [\int_0^{+\infty}\{[(\bar{x})^2/s^2].(n.(\bar{x})^2/s^2-1)^{-2}\}.((s^2)^{(n-1)/2})\circ\exp\{-(n-1).s^2/2.\sigma^2\}.ds^2] \quad \text{… (3.6)}$$

Where, $C^* = \{(n-1)/2.\sigma^2\}^{(n-1)/2}.\{1/(\Gamma((n-1)/2)\}.$

Now, because we have:

$$(d/ds^2).[(s^2)^{(n-1)/2}.\exp\{-(n-1).s^2/2\sigma^2\}]$$
$$=\{(n-1)/2\sigma^2\}.[\sigma^2.(s^2)^{(n-3)/2}.\exp\{-(n-1).s^2/2\sigma^2\}-(s^2)^{(n-1)/2}.\exp\{-(n-1).s^2/2\sigma^2\}]. \quad \text{… (3.7)}$$

Using (3.7) in (3.6), we could lead ourselves to the following:

$$\mathbf{B} = (c/n)^*.\ \mathbf{E}_{\bar{x}}\ [\int_0^{+\infty}\{[n.(\bar{x})^2/s^2].(n.(\bar{x})^2/s^2-1)^{-2}\}.((s^2)^{(n-3)/2})\circ\exp\{-(n-1).s^2/2.\sigma^2\}.ds^2]-$$

$$-\ \{2c^*/[n.(n-1)]\}.\ \bar{x}\ \{\int_0^{+\infty}[n.(\bar{x})^2/s^2].(n.(\bar{x})^2/s^2-1)^{-2}.\ (d/ds^2).[(s^2)^{(n-1)/2}.\exp\{-(n-1).s^2/2\sigma^2\}\circ ds^2\}].$$

Again resorting to the integration by parts, we obtain:

$$\mathbf{B} = \mathbf{E}_{s^2}\ \square\ \mathbf{E}_{\bar{x}}\ \square[\{n.(\bar{x})^2/(s^2)\}.(n.(\bar{x})^2/s^2-1)^{-2}]\ -\ \{2c^*/[n.(n-1)]\}$$

$$\square\mathbf{E}_{\bar{x}}\ \{\int_0^{+\infty}[n.(\bar{x})^2/(s^2)]].[n.(\bar{x})^2/s^2-1]^{-2}.\ (d/ds^2).[(s^2)^{(n-1)/2}.\exp\{-(n-1).s^2/2\sigma^2\}\circ ds^2\}]$$

$$\mathbf{B} = \mathbf{E}\ [\{n.(\bar{x})^2/(s^2)\}.(n.(\bar{x})^2/s^2-1)^{-2}]-\{2/(n-1)\}.\mathbf{E}[n.(\bar{x})^2/s^2.\{n.(\bar{x})^2/s^2+1\}.\{n.(\bar{x})^2/s^2-1\}^{-3}]$$

$$= \mathbf{E}\ ([\{n.(\bar{x})^2/(s^2)\}.(n.(\bar{x})^2/s^2-1)^{-2}\ -$$
$$\{2/(n-1)\}.\ [n.(\bar{x})^2/s^2.\{n.(\bar{x})^2/s^2+1\}.\{n.(\bar{x})^2/s^2-1\}^{-3}])$$

Or B = E (b), where: →

$$b = u.\ (u-1)^{-2} - \{2/(n-1)\}.\ u.\ (u+1)\ (u-1)^{-3} = u.\ (u-1)^{-3}.\ [(n-3) - u.\ (n+1)]/(n-1) \quad \text{… (3.8)}$$

Wherein, as mentioned earlier, we set $\{n.(\bar{x})^2/(s^2)\} = u$ **… (3.9)**

Thus, the 'UMVUE' of the relative efficiency of $t^{\otimes}$ with respect to (w.r.t.) the usual estimator $\bar{x}$ would be that of:1 + 2A +B [i.e., 1 + 2a + b].

Which is, as per (3.4), and (3.7)

$= 1 - 2. [(u +1). (u -1)^{-2}] + u. (u -1)^{-3.} [(n-3) – u. (n+1)]/ (n-1)$

$= 1 – [(u – 1)^{-3}. \{(n – 3). u^2 + (n – 3).u – 2. (n – 1)\}/ (n – 1)]$ … (3.10)

Now, $100/\eta = 100 \dfrac{E(\bar{x}-\mu)^2}{E(t^{\otimes}-\mu)^2} > 100\%$ **… (3.11)**

If $0 \prec \hat{\eta} \prec 1$ **; Or If $0 < 1 + 2a + b < 1$; or If $0 < \{(n – 3). u^2 + (n – 3).u – 2. (n – 1)\}$ {As per (3.10), as u > 1 for all** $(\bar{x})^2 \succ s^2/n$ **[As "C.V." of $\bar{x}$ is < 1], as per (3.9); or If u > (n + 1)/ (n – 3); which is so for all "C.V." of $\bar{x}$, in practice!**

4. THE IMPROVED LOGNORMAL MEAN ESTIMATOR

As noted in the introduction section, our aim is that of improving the estimator proposed by Shen et. al. (2006) in (1.7) as below:

$\exp \{\bar{x} + (n-1).s^2/(2. (n + 4) + 3. s^2)\}$ = ER2006. … (4.1)

Using the improvement achieved through $t^{\otimes}$ in the estimation of μ we use that in (4.1) in place of $\bar{x}$

$t^{\otimes} = \bar{x} + s^2 / (n\bar{x}) \qquad = \bar{x}. (1+v)$ … (4.2)

Also, we use the "MMSE" in (2.1), in (4.1) above, rather than s^2 therein: MMSE (σ^2) = cn1*s^2

Thus, our proposed efficient estimator say, 'ER2007I' &'ER2007II', using (4.2) and then cn1*s^2 also in (4.1) are respectively, as follows:

$\exp \{t^{\otimes} + (n-1).s^2/ (2. (n + 4) + 3.s^2)\}$ = ER2007I. … (4.3)

&

$\exp \{t^{\otimes} + (n-1).cn1.s^2/ (2. (n + 4) + 3. cn1.s^2)\}$ = ER2007II. … (4.4)

Wherein, $t^{\otimes} = \bar{x} + \bar{x} / [n. (\bar{x})^2 /s^2 – 1]$ defined in (3.1).

Now, in order to compare the two estimators, namely ER2006 vis-à-vis its improvement proposed by us ER2007, that is:→

Shen et. al. (2006) in (4.1), vis-à-vis its improvement as in (4.3), we define their 'Relative Efficiencies w.r.t. "usual ML estimator", namely UER in (1.3) as follows:→

ReffOER2006 = RMSE (UER)*100/RMSE (ER2006) % --- (4.5)

ReffOER2007I = RMSE (UER)*100/RMSE (ER2007I) % --- (4.6)

&

ReffOER2007II = RMSE (UER)*100/RMSE (ER2007II) % --- (4.7)

Wherein,

RMSE (UER) = E [(UER - ς)/ ς]^2 as in (1.4), --- (4.8)

&

RMSE (ER2006) = E [(ER2006 - ς)/ ς]^2 as in (1.8), --- (4.9)

And similarly,

RMSE (ER2007I) = E [(ER2007I - ς)/ ς]^2 --- (4.10)

&

RMSE (ER2007II) = E [(ER2007II - ς)/ ς]^2 --- (4.11)

5. SIMULATION AND COMPARISONS

The simulation conducted for the proposed point estimator considered various sample sizes n: 11, 21, 31, 41, 51, 71, 101, & 151 and population mean values for μ: 0.5000, 0.5500, 0.6000, 0.6500, 0.7000, 0.7500, & 0.8000 for a fixed variance of σ^2 = 0.25 (i.e., σ = 0.50, as well !). This simulation considered 11,000 iterations of the relevant sample size, drawing randomly from a population of $N(\mu, \sigma^2)$. The actual *RMSEs* of the estimators (usual ML estimator UER, and the improved, and the three estimators: ER 2006, ER 2007I & 2007II) respectively, were then calculated per the expressions in (4.8), (4.9), (4.10), and (4.8) as an average of "the actual relative mean square errors on each one of these 11,000 iterations" to validate the findings. Thence "Relative Efficiencies" of the improved, and the proposed estimators: ER 2006, ER 2007I & ER 2007II with respect to the usual ML estimator UER were calculated per the expressions in (4.5), (4.6) & (4.7), respectively. The resulting values of these relative efficiencies appear in Table 4.1, in the APPENDIX.

6. CONCLUSION

The need for an accurate determination of the point estimate for a lognormal mean is great within the context of clinical research and other scientific disciplines. For numerous, and often well-established reasons, the transformation of data is undertaken to reach the assumptions required for parametric statistical inference. However, standard methods, which capture only a sample's mean and variance, do not necessarily yield the most efficient estimator. By utilizing more complete information available within the sample coefficient of variation: " $\frac{s}{\sqrt{n}.\bar{x}}$ **", this paper presented, and tested, more efficient point estimators for the lognormal mean. The results from the simulation which incorporated various population means and sample sizes indicated that a relative improvement in efficiency was observed, particularly as a function of increasing population CV {i.e. (σ / μ) } and decreasing sample size. It might be in place here to note that in most of the applications (in the context of) biomedical research in general, and in the context of 'Pharmacokinetic data', in particular, the sample size is rather small, always.**

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APPENDIX

Relative Efficiencies of Er2007I & Er2007II Vis-à-Vis that of Shen et. al (2006)

TABLE 4.1:

Relative Efficiencies ~ REff's Of Er 2006, Er 2007I & Er 2007II with Respect To Usual Estimator UER [%]↓[11,000 iterations/samples with σ = 1.00]→

n=11							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**112.092**	**113.836**	**114.865**	**115.021**	**113.545**	**114.075**	**114.381**
REffEr2007I	**114.374**	**115.104**	**115.873**	**116.002**	**115.611**	**116.3621**	**115.731**
REffEr2007II	**120.825**	**121.771**	**121.250**	**121.661**	**120.574**	**119.271**	**119.161**
n=21							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**107.299**	**107.889**	**106.820**	**107.684**	**108.008**	**108.267**	**107.867**
REffEr2007I	**111.158**	**110.874**	**110.494**	**110.061**	**109.171**	**109.188**	**109.029**
REffEr2007II	**112.216**	**114.544**	**113.628**	**113.030**	**111.455**	**111.543**	**111.283**
n=31							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**104.675**	**105.389**	**106.196**	**105.139**	**105.243**	**105.149**	**105.376**
REffEr2007I	**108.363**	**107.747**	**107.506**	**106.793**	**106.524**	**106.330**	**106.215**
REffEr2007II	**110.812**	**110.097**	**109.887**	**108.552**	**108.145**	**107.758**	**107.597**
n=41							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**104.363**	**104.090**	**103.924**	**104.268**	**104.620**	**104.165**	**104.720**
REffEr2007I	**106.387**	**105.850**	**105.464**	**105.302**	**105.261**	**104.897**	**105.080**
REffEr2007II	**108.425**	**107.547**	**106.895**	**106.702**	**106.666**	**106.026**	**106.313**
n=51							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**102.972**	**103.339**	**103.794**	**103.553**	**103.625**	**102.770**	**103.056**
REffEr2007I	**105.054**	**104.729**	**104.584**	**104.337**	**104.208**	**103.752**	**103.742**
REffEr2007II	**106.431**	**106.068**	**105.938**	**105.477**	**105.273**	**104.408**	**104.439**
n=71							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**102.384**	**102.477**	**102.342**	**102.486**	**102.160**	**102.526**	**101.811**
REffEr2007I	**103.642**	**103.423**	**103.211**	**103.101**	**102.876**	**102.953**	**102.538**
REffEr2007II	**104.687**	**104.377**	**104.011**	**103.867**	**103.450**	**103.600**	**102.869**
n=101							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**101.768**	**101.663**	**101.505**	**101.940**	**101.743**	**102.318**	**101.854**
REffEr2007I	**102.586**	**102.386**	**102.200**	**102.279**	**102.137**	**102.352**	**102.073**
REffEr2007II	**103.337**	**103.008**	**102.693**	**102.872**	**102.603**	**102.994**	**102.509**
n=151							
	mu=0.50	**mu=0.55**	**mu=0.60**	**mu=0.65**	**mu=0.70**	**mu=0.75**	**mu=0.80**
REffEr2006I	**100.922**	**101.720**	**101.313**	**100.843**	**101.329**	**100.751**	**101.089**
REffEr2007I	**101.673**	**101.780**	**101.585**	**101.340**	**101.503**	**101.185**	**101.327**
REffEr2007II	**102.067**	**102.412**	**102.019**	**101.560**	**101.869**	**101.312**	**101.551**