Modified Goodness-of-Fit Tests for Exponentiated Gamma Distribution with Unknown Shape Parameter

A. I. Shawky[1*] and R. A. Bakoban[2]

[1]King Abdulaziz University, Girls College of Education, P.O. Box 32691, Jeddah 21438, Saudi Arabia.
[2]King Abdulaziz University, Girls College of Education, Department of mathematics, P.O. Box 4269, Jeddah 21491, Saudi Arabia.

Abstract

This paper gives tables of critical values of the modified Kolmogorov-Smirnov, Anderson-Darling and Cramer-von Mises goodness-of-fit tests for the exponentiated gamma distribution with unknown shape parameter based on complete and type II censored samples. The powers of these tests are given for a number of alternative distributions.

Key words: Kolmogorov-Smirnov; Anderson-Darling; Cramer-von Mises; Empirical distribution function; Monte Carlo simulation; Exponentiated gamma distribution.

1. Introduction

One of the important families of distributions in lifetime tests is the exponentiated gamma (*EG*) distribution. The cumulative distribution function (c.d.f.) of *EG* distribution with shape parameter θ is given by

$$F(x;\theta) = [1 - e^{-x}(x+1)]^{\theta}, \qquad x > 0, \; \theta > 0, \qquad (1.1)$$

and the probability density function (p.d.f.) is given by

$$f(x;\theta) = \theta\, x\, e^{-x}[1 - e^{-x}(x+1)]^{\theta-1}, \qquad x > 0, \; \theta > 0. \quad (1.2)$$

When the shape parameter $\theta = 1$ in both (1.1) and (1.2) give the c.d.f. and p.d.f. of gamma distribution with shape parameter $\alpha = 2$ and scale parameter $\beta = 1$, *i.e.*, $G(2,1)$. For more details about this distribution, see Shawky and Bakoban (2008 a, b, 2009 a, b, c).

It is often to interest to determine whether a set of data can be considered to come from a population governed by certain family. One class of goodness-of-fit tests that can be used consists of tests based on the distance between the empirical distribution function and the hypothesized distribution function. Three of the better known tests in this class are Kolmogorov-Smirnov (KS), Anderson-Darling (AD) and Cramer-von Mises (CvM) tests. These tests are valid when there are no unknown parameters in the hypothesized distribution. These tests become extremely conservative, however, if they are used in case where unknown parameters must be estimated from the sample data.

[*] Permanent address: Fac. of Eng. at Shoubra, P.O. Box 1206, El Maadi 11728, Cairo, Egypt.

Several papers exist in the literature that address the problems of finding critical values for these tests when unknown parameters must be estimated. Lilliefors (1967, 1969) has used Monte Carlo methods to construct tables for the Kolmogorov-Smirnov test when the mean and variance of a normal distribution are estimated and when the mean of an exponential distribution is estimated. Subsequent papers by Stephens (1970, 1974), Durbin (1975), and Green and Hegazy (1976) have extended the work on goodness-of-fit testing for normal and exponential distribution with unknown parameters. Also, Chandra *et al*. (1981) studied the Kolmogorov statistics for tests of fit for the extreme-value and Weibull distributions. Goodness-of-fit tests for the Weibull distribution were discussed by Littell *et al*. (1979), Bush *et al*. (1983), Aho *et al*. (1985) and Murthy *et al*. (2004). Goodness-of-fit tests have also been developed for gamma distributions with unknown location and scale parameters, but with known shape parameter, paper by Woodruff *et al*. (1984) has addressed this problem. Moreover, Yen and Moore (1988) have discussed the Laplace distribution. Also, Mudholkar and Srivastava (1993) and Mudholkar *et al*. (1995) have studied the goodness-of-fit of the null hypothesis for the shape parameter (θ=1) within the exponentiated Weibull model. Further, the exponential distribution were discussed by Balakrishnan and Basu (1995). Also, Hassan (2005) was studied Goodness-of-fit for the generalized exponential distribution.

In this paper, we use Monte Carlo techniques to obtain critical values for modified KS, AD, and CvM tests for exponentiated gamma distribution with unknown shape parameter for complete and censored samples. Tables of critical values are obtained for sample sizes 10(5)30(10)50 and 100. Also, we obtain tables of critical values for censoring sizes 8(4)24(8)40 and 80. In section 2, we derive the maximum likelihood estimator (MLE) of the shape parameter in both cases complete and censored samples. The generation of these tables is described more fully in section 3. Section 4 describes a power investigation of the modified tests.

2. Maximum Likelihood Estimation

Suppose a Type-II censored sample $\underline{X} = (X_{1:n}, X_{2:n}, \ldots, X_{r:n})$ where $X_{i:n}$ is the i^{th} order statistics. This sample are obtained and recorded from *EG* distribution with c.d.f. and p.d.f. given, respectively, by (1.1) and (1.2). The likelihood function (LF) in this case can be written as:

$$\ell(x;\theta) \propto \theta^r e^{-T} (1 - V^{\theta})^{n-r}, \tag{2.1}$$

where

$$T = \sum_{i=1}^{r} [x_{i:n} - \ln x_{i:n} - (\theta - 1) \ln u_{i:n}],$$

$$u_{i:n} = 1 - e^{-x_{i:n}}(1 + x_{i:n}), \qquad \text{and} \qquad V = 1 - e^{-x_{r:n}}(1 + x_{r:n}).$$

The logarithm of the LF is given by

$$L = \ln \ell(x;\theta) \propto r \ln \theta - T + (n-r)\ln(1 - V^{\theta}). \tag{2.2}$$

The MLE of θ, denoted by $\hat{\theta}_{MLE}$, is given by

$$\hat{\theta}_{MLE} = r/[(n-r)(V^{-\hat{\theta}_{MLE}} - 1)^{-1} \ln V - \sum_{i=1}^{r} \ln u_{i:n}]. \tag{2.3}$$

This equation is in implicit form, so it may be solved by using numerical iteration such as Newton-Raphson method by using Mathematica 4.0.

In the sense of complete sample (that is, $r = n$), the estimator $\hat{\theta}_{MLE}$ in (2.3) reduces to

$$\hat{\theta}_{MLE} = \frac{-n}{\sum_{i=1}^{n} \ln[1 - e^{-x_{i:n}}(1 + x_{i:n})]}. \tag{2.4}$$

Now note that, if X_i's are independent and identically distributed *EG*, then $-\theta \sum_{i=1}^{n} \ln[1 - e^{-x_{i:n}}(x_{i:n} + 1)]$ follows $G(n,1)$. Therefore, for $n > 2$,

$$E(\hat{\theta}_{MLE}) = \frac{n}{n-1}\theta \qquad \text{and} \qquad Var(\hat{\theta}_{MLE}) = \frac{n^2}{(n-1)^2 (n-2)}\theta^2.$$

Using (2.4), an unbiased estimate of θ can be obtained as

$$\hat{\theta}_{UBE} = \frac{n-1}{n}\hat{\theta}_{MLE} = -\frac{n-1}{\sum_{i=1}^{n} \ln[1 - e^{-x_{i:n}}(x_{i:n} + 1)]}, \tag{2.5}$$

where

$$Var(\hat{\theta}_{UBE}) = \frac{\theta^2}{n-2}.$$

3. Goodness-of-Fit Tests

Consider a random variable X with distribution function $F(x)$. The main problem discussed here is that of testing hypotheses about $F(x)$ of the form (see, Lawless (1982) and D'Agostino and Stephens (1986))

$$H_\circ : F(x) = F_\circ(x) \tag{3.1}$$

where $F_\circ(x)$ is a specified family of models. Usually $F_\circ(x)$ will involve unknown parameters. Tests of $H_\circ$ are frequently referred to as goodness of fit tests. We consider here the classical goodness of fit tests based on the empirical distribution function (EDF). The EDF is a step function with jumps at the order statistics $x_{1:n} < x_{2:n} < \dots < x_{n:n}$, calculated from the sample, which estimates the population distribution function. The EDF is given by

$$F_n(x) = \frac{\text{number of } x_i\text{'s} \le x}{n}, \qquad -\infty < x < \infty,$$

where $x_1, x_2, \dots, x_n$ a random sample from the distribution for X.

EDF statistics are measures of the discrepancy between the EDF and a given distribution function, and are used for testing the fit of the sample to the distribution; this may contain parameters which must be estimated from the sample. The following goodness-of-fit test statistics were reviewed by D'Agostino and Stephens (1986) for complete and censored samples.

3.1 Modified EDF for complete samples and unknown parameters

Suppose a given random sample of size n from *EG* distribution with c.d.f. given in (1.1) and Let $X_{1:n} < X_{2:n} < \dots < X_{n:n}$ be the order statistics, then

1- The modified Kolmogorov-Smirnov statistics

$$D_n^+ = \max_{1 \le i \le n} \{\frac{i}{n} - F_\circ(x_{i:n}; \hat{\theta})\};$$

$$D_n^- = \max_{1 \le i \le n} \{F_\circ(x_{i:n}; \hat{\theta}) - \frac{i-1}{n}\};$$

$$D_n = \max(D_n^+, D_n^-). \tag{3.1}$$

2- The modified Cramer-von Mises statistics

$$W_n^2 = \sum_{i=1}^{n} [F_\circ(x_{i:n}; \hat{\theta}) - \frac{2i-1}{2n}]^2 + \frac{1}{12n}. \tag{3.2}$$

3- The modified Anderson-Darling statistics

$$A_n^2 = -n - \sum_{i=1}^{n} \frac{2i-1}{n}\{\log[F_\circ(x_{i:n}; \hat{\theta})] + \log[1 - F_\circ(x_{n+1-i:n}; \hat{\theta})]\}. \tag{3.3}$$

For each of the three test procedures (KS, CvM, AD), each sample size n=10(5)30(10)50 and 100, the random sample $x_1, \dots, x_n$ were generated from *EG* distribution. This random sample was used to estimate the shape parameter by (2.5). The resulting unbiased estimator of the shape parameter ($\hat{\theta}_{UBE}$) were then used to determine $F_\circ(x)$, the hypothesized c.d.f.. The appropriate test statistic was calculated for the given values of n. The quantiles (80%, 85%, 90%, 95%, 99%) were found from 2000 samples a total of 20 times, thus establishing the critical value for that particular test and sample size. The critical values reported in Table 1 are the means of 20 individual quantiles. Table 1 contains critical values for modified KS, CvM and AD tests.

Table 1: Critical values for modified KS, CvM and AD tests for complete samples.

n	Statistic	Critical values				
		α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	KS	0.257313	0.272003	0.290829	0.320447	0.376765
	CvM	0.124996	0.141909	0.166837	0.210630	0.320901
	AD	0.773337	0.870689	1.015320	1.270230	1.966800
15	KS	0.213610	0.225524	0.241287	0.266253	0.315070
	CvM	0.126520	0.143726	0.168938	0.215254	0.324454
	AD	0.782284	0.882611	1.024250	1.273190	1.985650
20	KS	0.187531	0.197432	0.211196	0.231895	0.274494
	CvM	0.128163	0.145783	0.171950	0.218875	0.332291
	AD	0.787979	0.886478	1.033510	1.299020	1.976240
25	KS	0.169251	0.178378	0.190722	0.210173	0.248437
	CvM	0.129745	0.147302	0.172935	0.217770	0.332140
	AD	0.799034	0.899678	1.040440	1.307280	2.005870
30	KS	0.155112	0.163930	0.175219	0.192363	0.227527
	CvM	0.127277	0.145095	0.170753	0.217313	0.334190
	AD	0.800541	0.905146	1.050150	1.316750	1.996590
40	KS	0.134694	0.142135	0.151953	0.167747	0.200837
	CvM	0.128166	0.146251	0.172045	0.219229	0.337866
	AD	0.798966	0.895852	1.037410	1.297300	2.049810
50	KS	0.120986	0.127761	0.136607	0.150532	0.179287
	CvM	0.127631	0.145201	0.169329	0.213800	0.336666
	AD	0.796180	0.891604	1.032520	1.289790	2.044470
100	KS	0.086273	0.090984	0.097436	0.107544	0.129204
	CvM	0.126963	0.144374	0.169050	0.215722	0.343240
	AD	0.794364	0.894208	1.036680	1.297190	2.027520

To use this table one estimates the shape parameter by using (2.5) and inserts the estimator in (1.1) to obtain a value for $F_{\circ}(x)$. The value of $F_{\circ}(x)$ is used in calculating D_n, W_n^2 and A_n^2 respectively by equations (3.1), (3.2) and (3.3).

If the calculated value of the test statistics from a random sample exceeds the tabulated value, the null hypothesis of an *EG* distribution is rejected at the chosen significance level.

Table 1 shows that all critical values increase as the significant level (α) decreases. Also, the critical values for KS test statistics decrease as n increases. Moreover, the critical values for AD and CvM test statistics increase usually as n increases. However, we note from Table 1 that for the specific sample size the critical values for AD test statistics is the biggest for all significant levels. Also, for the specific sample size the critical values for KS test statistics is bigger than the critical values for CvM test statistics when n=10 for all α and for n=15 and 20 for all α except α=0.01. Then as n increases and α decreases the critical values for CvM test statistics become grater than the critical values for KS test statistics uptil the critical values for CvM test statistics become grater than the critical values for KS test statistics for all α when n=50 and 100.

3.2 Modified EDF for type II censoring and unknown parameters

Suppose a type-II censored sample $\underline{X} = (X_{1:n}, X_{2:n}, \ldots, X_{r:n})$ from *EG* distribution with c.d.f. given in (1.1), then

1- The modified Kolmogorov-Smirnov statistics

$$D^{+}_{n,r} = \max_{1 \le i \le r} \{\frac{i}{n} - F_{\circ}(x_{i:n};\hat{\theta})\};$$

$$D^{-}_{n,r} = \max_{1 \le i \le r} \{F_{\circ}(x_{i:n};\hat{\theta}) - \frac{i-1}{n}\};$$

$$D_{n,r} = \max(D^{+}_{n,r}, D^{-}_{n,r}). \tag{3.4}$$

2- The modified Cramer-von Mises statistics

$$W^{2}_{n,r} = \sum_{i=1}^{r} [F_{\circ}(x_{i:n};\hat{\theta}) - \frac{2i-1}{2n}]^{2} + \frac{r}{12n^{2}} - \frac{n}{3}[\frac{r}{n} - F_{\circ}(x_{r:n})]^{3}. \tag{3.5}$$

3- The modified Anderson-Darling statistics

$$A^{2}_{n,r} = -\frac{1}{n}\sum_{i=1}^{r} (2i-1)\{\log[F_{\circ}(x_{i:n};\hat{\theta})] - \log[1 - F_{\circ}(x_{i:n};\hat{\theta})]\}$$
$$-\frac{1}{n}\{(r-n)^{2}\log[1 - F_{\circ}(x_{r:n})] - r^{2}\log F_{\circ}(x_{r:n}) + n^{2}F_{\circ}(x_{r:n})\}$$
$$-2\sum_{i=1}^{r}\log[1 - F_{\circ}(x_{i:n};\hat{\theta})]. \tag{3.6}$$

For each of the three test procedures (KS, CvM, AD), each censored sample r=8(4)24(8)40 and 80, the random sample $x_1, \ldots, x_n$ were generated from *EG* distribution. This random sample was used to estimate the shape parameter by (2.3). The resulting maximum likelihood estimator of the shape parameter ($\hat{\theta}_{MLE}$) were then used to determine $F_{\circ}(x)$, the hypothesized c.d.f.. The appropriate test statistic was calculated for the given values of n. The 80% quantiles were found from 2000 samples a total of 20 times. The quantiles reported in Table 2 are the mean of 20 individual quantiles. *i.e.* the procedure was repeated 2000 times, thus generating 2000 independent values of the appropriate test statistic. These 2000 values were ranked (in each of the 20 times), and the 80%, 85%, 90%, 95%, 99% quantiles were found, then we take the mean of 20 results, thus establishing the critical value for that particular test and sample size. Table 2 contains critical values for modified KS, CvM and AD tests.

Table 2: Critical values for modified KS, CvM and AD tests for type II censoring.

n	r	Statistic	Critical values				
			α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	8	KS	0.249591	0.262972	0.280264	0.306923	0.360002
		CvM	0.084841	0.097077	0.114370	0.149851	0.257586
		AD	0.469752	0.541402	0.656524	0.896688	1.59206
15	12	KS	0.207961	0.219618	0.234003	0.256087	0.303152
		CvM	0.092356	0.105065	0.124667	0.160525	0.259652
		AD	0.508126	0.580448	0.693829	0.911429	1.516090
20	16	KS	0.182515	0.192532	0.205248	0.225646	0.266659
		CvM	0.096876	0.110947	0.131240	0.166913	0.264041
		AD	0.523380	0.599841	0.710957	0.914153	1.530140
25	20	KS	0.164797	0.173844	0.185891	0.204940	0.242409
		CvM	0.098358	0.113482	0.133281	0.171183	0.265529
		AD	0.536128	0.612395	0.721492	0.935771	1.545500
30	24	KS	0.151380	0.159682	0.170889	0.187949	0.223790
		CvM	0.100316	0.114554	0.136357	0.175094	0.284051
		AD	0.544188	0.622535	0.741646	0.964825	1.630830
40	32	KS	0.132591	0.139830	0.149811	0.165191	0.198367
		CvM	0.102879	0.118777	0.141803	0.183012	0.293882
		AD	0.548463	0.625117	0.747316	0.964803	1.594700
50	40	KS	0.118359	0.124802	0.133416	0.147152	0.175349
		CvM	0.104840	0.120244	0.143108	0.185450	0.301429
		AD	0.551939	0.628296	0.740339	0.952618	1.617770
100	80	KS	0.085066	0.089873	0.096057	0.105810	0.126974
		CvM	0.106772	0.122937	0.146266	0.187801	0.308437
		AD	0.557521	0.634116	0.748167	0.965656	1.601720

To use this table one estimates the shape parameter by using (2.3) and inserts the estimator in (1.1) to obtain a value for $F_{\circ}(x)$. The value of $F_{\circ}(x)$ is used in calculating $D_{n,r}, W^2_{n,r}$ and $A^2_{n,r}$ respectively by equations (3.4), (3.5) and (3.6).

If the calculated value of the test statistics from a type II censored sample exceeds the tabulated value, the null hypothesis of an *EG* distribution is rejected at the chosen significance level.

Table 2 shows that all critical values for test statistics increase as the significant level (α) decreases. Also, the critical values for KS test statistics decrease as n increases. Moreover, the critical values for AD test statistics usually increase as n increases. On the other hand the critical values for CvM test statistics increase as n increases. However, we note from Table 2 that for the specific sample size the critical values for AD test statistics is the biggest for all significant levels. Also, for the specific sample size the critical values for KS test statistics is bigger than the critical values for CvM test statistics when n=10, 15 and 20 for all α and for n=25 and 30 for all α except α=0.01. Then as n increases and α decreases the critical values for CvM test statistics become grater than the critical values for KS test statistics up till the critical values for CvM test statistics become grater than the critical values for KS test statistics for all α when n=100.

4. Power Study

The power of the modified KS, AD and CvM tests was computed for sample sizes 10(5)30(10)50 and 100 for 3 selected alternative distributions. The null hypothesis of *EG* distribution with unspecified shape parameter was tested at significance levels α = 0.01, 0.05, 0.10, 0.15 and 0.20.
For each selected test, sample size, and alternative distribution, 5000 random samples were generated from the alternative distribution. The test was conducted using the critical values in this paper, and the proportion of rejections was recorded as the power for that situation. A comparison of the powers is given in Tables 3, 4 and 5 for the standard normal distribution, *i.e.*, N(0, 1), chi square distribution with one degree of freedom, *i.e.*, χ_1^2 and exponential distribution, *i.e.*, Exp(1), respectively, as alternative distributions and significance levels α = 0.01, 0.05, 0.10, 0.15 and 0.20.

Table 3: Power of tests for *EG* distribution when the alternative distribution is N(0, 1) based on complete samples.

n	Statistic	Power of the test				
		α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	KS	0.4406	0.3914	0.3006	0.2034	0.0824
	CvM	0.4782	0.4300	0.3472	0.2386	0.1138
	AD	0.4852	0.4454	0.3440	0.2372	0.0912
15	KS	0.4404	0.3666	0.2996	0.1954	0.0812
	CvM	0.4690	0.4104	0.3458	0.2380	0.1190
	AD	0.4878	0.4202	0.3454	0.2376	0.1006
20	KS	0.4244	0.3776	0.2922	0.2028	0.0780
	CvM	0.4598	0.4184	0.3262	0.2406	0.1044
	AD	0.4876	0.4304	0.3402	0.2334	0.0908
25	KS	0.4334	0.3590	0.2886	0.1978	0.0740
	CvM	0.4596	0.3980	0.3316	0.2454	0.1092
	AD	0.4774	0.4160	0.3398	0.2350	0.0910
30	KS	0.4332	0.3516	0.2850	0.1866	0.0700
	CvM	0.4814	0.4024	0.3386	0.2326	0.1038
	AD	0.4876	0.4094	0.3362	0.2254	0.0936
40	KS	0.4100	0.3660	0.2770	0.1830	0.0664
	CvM	0.4530	0.4096	0.3298	0.2314	0.1066
	AD	0.4710	0.4176	0.3336	0.2340	0.0900
50	KS	0.4268	0.3656	0.2926	0.1930	0.0678
	CvM	0.4744	0.4062	0.3502	0.2548	0.1036
	AD	0.4886	0.4244	0.3498	0.2452	0.0806
100	KS	0.4178	0.3502	0.2888	0.1826	0.0668
	CvM	0.4642	0.4058	0.3464	0.2394	0.1052
	AD	0.4800	0.4116	0.3472	0.2346	0.0920

From Table 3, we note that the modified AD test is the most powerful for α= 0.20 and 0.15. At α=0.10 some times (n=20, 25, 40 and 100) the modified AD test is the most powerful and some times (n=10, 15, 30 and 50) the modified CvM is the most powerful. But the modified CvM test is usually more powerful for α= 0.01 and 0.05. Moreover, the KS test statistics is much less powerful for all α.

Table 4: Power of tests for *EG* distribution when the alternative distribution is χ_1^2 based on complete samples.

n	Statistic	Power of the test				
		α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	KS	0.4508	0.3818	0.3066	0.1972	0.0876
	CvM	0.4954	0.4236	0.3496	0.2356	0.1186
	AD	0.5072	0.4358	0.3432	0.2266	0.0966
15	KS	0.4382	0.3738	0.2870	0.2066	0.0810
	CvM	0.4718	0.4152	0.3260	0.2434	0.1146
	AD	0.4820	0.4204	0.3324	0.2420	0.0968
20	KS	0.4254	0.3730	0.2830	0.2022	0.0826
	CvM	0.4618	0.4102	0.3228	0.2380	0.1048
	AD	0.4814	0.4186	0.3270	0.2294	0.0940
25	KS	0.4380	0.3712	0.2990	0.1998	0.0806
	CvM	0.4724	0.3984	0.3378	0.2430	0.1096
	AD	0.4830	0.4074	0.3420	0.2360	0.0938
30	KS	0.4294	0.3498	0.2816	0.1828	0.0738
	CvM	0.4766	0.3904	0.3314	0.2270	0.1064
	AD	0.4806	0.3938	0.3274	0.2202	0.0966
40	KS	0.4304	0.3624	0.2744	0.1844	0.0718
	CvM	0.4742	0.3978	0.3140	0.2312	0.1084
	AD	0.4790	0.4136	0.3186	0.2276	0.0842
50	KS	0.4250	0.3568	0.2724	0.1870	0.0618
	CvM	0.4694	0.4104	0.3336	0.2422	0.1036
	AD	0.4800	0.4190	0.3378	0.2350	0.0858
100	KS	0.4206	0.3550	0.2844	0.1838	0.0680
	CvM	0.4692	0.4030	0.3396	0.2328	0.1056
	AD	0.4818	0.4082	0.3422	0.2248	0.0950

From Table 4, we note that the modified AD test is the most powerful for α= 0.20 and 0.15. At α=0.10 some times (n=15, 20, 25, 40, 50 and 100) the modified AD test is the most powerful and some times (n=10 and 30) the modified CvM test is the most powerful. But the modified CvM test is the most powerful for α= 0.05 and 0.01. Moreover, the KS test is much less powerful for all α.

Table 5: Power of tests for *EG* distribution when the alternative distribution is Exp(1) based on complete samples.

n	Statistic	Power of the test				
		α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	KS	0.4468	0.3854	0.3030	0.2144	0.0858
	CvM	0.4712	0.4194	0.3422	0.2632	0.1152
	AD	0.4938	0.4214	0.3452	0.2578	0.0926
15	KS	0.4472	0.3748	0.2966	0.1972	0.0860
	CvM	0.4832	0.4148	0.3392	0.2398	0.1198
	AD	0.4996	0.4218	0.3510	0.2366	0.0986
20	KS	0.4340	0.3630	0.2836	0.1954	0.0786
	CvM	0.4710	0.4058	0.3194	0.2342	0.1066
	AD	0.4926	0.4150	0.3310	0.2346	0.0924
25	KS	0.4176	0.3694	0.2892	0.1906	0.0764
	CvM	0.4588	0.4052	0.3298	0.2316	0.1118
	AD	0.4730	0.4126	0.3404	0.2252	0.0892
30	KS	0.4150	0.3560	0.2778	0.1966	0.0826
	CvM	0.4580	0.4068	0.3302	0.2392	0.1130
	AD	0.4676	0.4044	0.3278	0.2306	0.0972
40	KS	0.4366	0.3570	0.2876	0.1830	0.0712
	CvM	0.4710	0.3894	0.3276	0.2294	0.1070
	AD	0.4844	0.4036	0.3342	0.2272	0.0892
50	KS	0.4290	0.3522	0.2666	0.1852	0.0754
	CvM	0.4658	0.4048	0.3210	0.2370	0.1114
	AD	0.4762	0.4132	0.3278	0.2280	0.0948
100	KS	0.4124	0.3574	0.2554	0.1764	0.0598
	CvM	0.4546	0.4024	0.3244	0.2270	0.1020
	AD	0.4718	0.4126	0.3250	0.2228	0.0846

From Table 5, we note that the modified AD test usually is the most powerful for α = 0.10, 0.15 and 0.20. But the modified CvM test usually is the most powerful for α = 0.01 and 0.05. Moreover, the KS test is much less powerful for all α.

We conclude from Tables 3, 4 and 5 in the sense of the significance levels and sample sizes the following results:
we recommend to use such distribution as alternative to *EG* distribution

1- N(0, 1) at α = 0.05, 0.10 and 0.15 for most given sample sizes.
2- χ_1^2 at α = 0.01 and 0.20 for most given sample sizes.

Hence, we prefer the standard normal distribution as an alternative for EG distribution in the case of complete samples for most significance levels studied.

In the same manner, the power of the modified KS, AD and CvM tests was computed for type II censored samples of sizes 8(4)24(8)40 and 80. A comparison of the powers is given in Tables 6, 7, 8 for the three alternative distributions and significance levels 0.01, 0.05, 0.10, 0.15 and 0.20.

Table 6: Power of tests for *EG* distribution when the alternative distribution is N(0, 1) based on type II censoring.

n	r	Statistic	Power of the test				
			α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	8	KS	0.4598	0.3740	0.3288	0.2306	0.1036
		CvM	0.5186	0.4444	0.3822	0.2732	0.1046
		AD	0.5514	0.4728	0.3902	0.2612	0.0924
15	12	KS	0.4404	0.3854	0.3110	0.2172	0.0914
		CvM	0.4996	0.4426	0.3666	0.2640	0.1182
		AD	0.5248	0.4622	0.3752	0.2634	0.1044
20	16	KS	0.4270	0.3818	0.3070	0.2132	0.0886
		CvM	0.4806	0.4372	0.3528	0.2596	0.1238
		AD	0.5162	0.4638	0.3726	0.2686	0.1094
25	20	KS	0.4426	0.3654	0.3058	0.2008	0.0836
		CvM	0.4914	0.4226	0.3650	0.2530	0.1272
		AD	0.5234	0.4460	0.3810	0.2584	0.1068
30	24	KS	0.4386	0.3650	0.2852	0.1932	0.0772
		CvM	0.4904	0.4168	0.3424	0.2540	0.1104
		AD	0.5142	0.4402	0.3590	0.2552	0.0946
40	32	KS	0.4216	0.3746	0.2872	0.1902	0.0692
		CvM	0.4792	0.4238	0.3420	0.2416	0.1044
		AD	0.5106	0.4516	0.3678	0.2534	0.1018
50	40	KS	0.4316	0.3644	0.2926	0.2004	0.0744
		CvM	0.4786	0.3984	0.3448	0.2488	0.1028
		AD	0.5098	0.4356	0.3746	0.2672	0.0970
100	80	KS	0.4314	0.3480	0.2728	0.1942	0.0700
		CvM	0.4746	0.4014	0.3248	0.2426	0.1060
		AD	0.5166	0.4430	0.3622	0.2620	0.1048

In Table 6, we note that the modified AD test is the most powerful for α = 0.10, 0.15 and 0.20. At α=0.05 the modified AD usually is more powerful and some times (n=10 and 15) the modified CvM is the most powerful. But the modified CvM test is the most powerful for α = 0.01. Moreover, the KS test is usually much less powerful for all α studied.

Table 7: Power of tests for *EG* distribution when the alternative distribution is χ_1^2 based on type II censoring.

n	r	Statistic	Power of the test				
			α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	8	KS	0.4468	0.3916	0.3186	0.2172	0.1028
		CvM	0.5026	0.4550	0.3774	0.2614	0.1092
		AD	0.5348	0.4766	0.3898	0.2492	0.0912
15	12	KS	0.4346	0.3768	0.3116	0.2086	0.0936
		CvM	0.4940	0.4390	0.3748	0.2560	0.1220
		AD	0.5178	0.4578	0.3852	0.2518	0.1036
20	16	KS	0.4322	0.3602	0.3118	0.2152	0.0890
		CvM	0.4810	0.4138	0.3598	0.2666	0.1206
		AD	0.5124	0.4442	0.3760	0.2736	0.1058
25	20	KS	0.4338	0.3716	0.2984	0.2062	0.0892
		CvM	0.4844	0.4256	0.3602	0.2608	0.1354
		AD	0.5136	0.4474	0.3748	0.2642	0.1108
30	24	KS	0.4252	0.3700	0.2966	0.2006	0.0852
		CvM	0.4890	0.4288	0.3406	0.2540	0.1156
		AD	0.5054	0.4500	0.3586	0.2504	0.0996
40	32	KS	0.4354	0.3740	0.2928	0.1904	0.0778
		CvM	0.4794	0.4280	0.3410	0.2450	0.1178
		AD	0.5094	0.4602	0.3692	0.2528	0.1054
50	40	KS	0.4284	0.3794	0.2954	0.1908	0.0770
		CvM	0.4696	0.4230	0.3434	0.2394	0.1008
		AD	0.5102	0.4578	0.3754	0.2584	0.0966
100	80	KS	0.4242	0.3702	0.2894	0.1872	0.0768
		CvM	0.4780	0.4232	0.3402	0.2438	0.1096
		AD	0.5138	0.4624	0.3720	0.2616	0.1078

Table 8: Power of tests for *EG* distribution when the alternative distribution is Exp(1)based on type II censoring.

n	r	Statistic	Power of the test				
			α=0.20	α=0.15	α=0.10	α=0.05	α=0.01
10	8	KS	0.4442	0.3922	0.3152	0.2324	0.0958
		CvM	0.4966	0.4568	0.3858	0.2648	0.1106
		AD	0.5322	0.4838	0.3976	0.2504	0.0870
15	12	KS	0.4446	0.3764	0.3180	0.2152	0.0974
		CvM	0.4992	0.4332	0.3726	0.2698	0.1254
		AD	0.5326	0.4528	0.3894	0.2628	0.1128
20	16	KS	0.4318	0.3880	0.3000	0.2110	0.0896
		CvM	0.4876	0.4408	0.3592	0.2646	0.1254
		AD	0.5176	0.4648	0.3762	0.2690	0.1066
25	20	KS	0.4228	0.3658	0.2972	0.1996	0.0896
		CvM	0.4818	0.4230	0.3586	0.2602	0.1254
		AD	0.5128	0.4484	0.3806	0.2626	0.1066
30	24	KS	0.4320	0.3866	0.2972	0.1858	0.0862
		CvM	0.4818	0.4512	0.3600	0.2466	0.1182
		AD	0.5056	0.4666	0.3754	0.2448	0.1010
40	32	KS	0.4372	0.3642	0.2824	0.1986	0.0760
		CvM	0.4998	0.4200	0.3316	0.2526	0.1152
		AD	0.5284	0.4518	0.3532	0.2644	0.1072
50	40	KS	0.4440	0.3676	0.2890	0.1930	0.0808
		CvM	0.4894	0.4104	0.3410	0.2372	0.1072
		AD	0.5278	0.4444	0.3682	0.2594	0.0976
100	80	KS	0.4296	0.3442	0.2890	0.1988	0.0722
		CvM	0.4850	0.3924	0.3402	0.2510	0.1128
		AD	0.5196	0.4338	0.3740	0.2670	0.1102

In Tables 7 and 8 we note that the modified AD test is the most powerful for $\alpha = 0.10$, 0.15 and 0.20. At α=0.05 the modified AD usually is more powerful and some times (n=10, 15 and 30) the modified CvM is the most powerful. But the modified CvM test is the most powerful for $\alpha =$ 0.01. Moreover, the modified KS test usually is much less powerful for all α studied.

We conclude from Tables 6, 7 and 8 in the sense of the significance levels and sample sizes the following results:
we recommend to use such distribution as alternative to *EG* distribution

1-N(0, 1) at $\alpha = 0.05$ for most sample sizes studied.

2- χ_1^2 at $\alpha = 0.15$ for most big sample sizes.

3-Exp(1) at $\alpha = 0.01$, 0.10 and 0.20 for most sample sizes studied and at $\alpha = 0.15$ for some sample sizes studied.

Hence, we prefer the exponential distribution as an alternative for EG distribution in the case of type II censored samples for most significance levels studied.

References

[1] Aho, M., Bain, L. J. and Engelhardt, M. (1985). Goodness-of-fit tests for the Weibull distribution with unknown parameters and heavy censoring. J. Statist. Comput. Simul., 21, 213-225.

[2] Balakrishnan, N. and Basu, A. P. (1995). The Exponential Distribution: Theory, Methods and Applications. Gordon and Breach Publishers, Amsterdam.

[3] Bush, J. G., Woodruff, B. W., Moore, A. H. and Dunne, E. J. (1983). Modified Cramer-von Mises and Anderson-Darling tests for Weibull distributions with unknown location and scale parameters. Commun. Statist. –Theory Meth., 12(21), 2465-2476.

[4] Chandra, M., Singpurwalla, N. D. and Stephens, M. A. (1981). Kolmogorov statistics for tests of fit for the extreme-value and Weibull distributions. J. A. S. A., 76(375), 729-731.

[5] D'Agostino, R. B. and Stephens, M. A. (1986). Goodness-Of-Fit Techniques. Marcel Dekker, New York.

[6] Durbin, J. (1975). Kolmogorov-Smirnov tests when parameters are estimated with applications to tests of exponentially and test on spacings. Biometrica, 62(1), 5-22.

[7] Green, J. R. and Hegazy, Y. A. S. (1976). Powerful modified EDF goodness-of-fit tests. J. A. S. A., 71, 204-209.

[8] Hassan, A. S. (2005). Goodness-of-fit for the generalized exponential distribution. InterStat, July 2005, 1-15.

[9] Lawless, J. F. (1982). Statistical Models and Methods for Lifetime Data. Wiley, New York.

[10] Lilliefors, H. W. (1967). On the Kolmogorov-Smirnov test for normality with mean and variance unknown. J. A. S. A., 62, 399-402.

[11] Lilliefors, H. W. (1969). On the Kolmogorov-Smirnov test for the exponential distribution with mean unknown. J. A. S. A., 64, 387-389.

[12] Littell, R. D., McClave, J. T. and Offen, W. W. (1979). Goodness-of-fit tests for the two-parameter Weibull distribution. Commun. Statist.-Simul. Comput., 8(3), 257-269.

[13] Mudholkar, G. S., and Srivastava, D. K. (1993). Exponentiated Weibull family for analyzing bathtub failure-rate data . IEEE Trans. Reliability, 42 (2) , 299-302.

[14] Mudholkar, G. S., Srivastava, D. K. and Freimer , M. (1995). The exponentiated Weibull family: A reanalysis of the bus-motor-failure data. Technometrics, 37(4), 436-445.

[15] Murthy, D. N. P., Xie, M. and Jiang, R. (2004). Weibull Models. Wiley, New Jersey.

[16] Shawky, A. I. and Bakoban, R. A. (2008a). Bayesian and non-Bayesian estimations on the exponentiated gamma distribution. Applied Mathematical Sciences, 2(51), 2521-2530.

[17] Shawky, A. I. and Bakoban, R. A. (2008b). Characterization from exponentiated gamma distribution based on record values. JSTA, 7(3), 263-278.

[18] Shawky, A. I. and Bakoban, R. A. (2009a). Order statistics from exponentiated gamma distribution and associated inference. The International Journal of Contemporary Mathematical Sciences, 4(2),71-91.

[19] Shawky, A. I. and Bakoban, R. A. (2009b). Conditional expectation of certain distributions of record values. Int. Journal of Math. Analysis, 3(17), 829 – 838.

[20] Shawky, A. I. and Bakoban, R. A. (2009c). Certain Characterizations of the exponentiated gamma distribution. Accepted for publication in JATA.

[21] Stephens, M. A. (1970). Use of the Kolmogorov-Smirnov, Cramer-von Mises and related statistics without extensive tables. J. Roy. Statist. Soc., B(32), 115-122.

[22] Stephens, M. A. (1974). EDF statistics for goodness of fit and some comparisons. J. A. S. A., 69, 730-737.

[23] Woodruff, B. W., Viviano, P. J., Moore, A. H. and Dunne, E. J. (1984). Modified goodness-of-fit tests for gamma distributions with unknown location and scale parameters. IEEE, Transactions on Reliability, 33(3), 241-245.

[24]Yen, V. C. and Moore, A. H. (1988). Modified goodness-of-fit test for the Laplace distribution. Commun. Statist. –Simula. Comput., 17(1), 275-281.