Penalties For Misclassification Of First Order Bilinear And Linear Moving Average Time Series Processes

By

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Abstract: Considering the similarity in the behaviour of the first order purely diagonal bilinear time series process (PDB(1)) and that of the linear moving average process of order one (MA(1)) under covariance analysis, the need arises for investigation of the penalty resulting from the misclassification of a bilinear process as a linear process and vice versa. It was discovered that the penalty function of misclassifying a PDB(1) process as an MA(1) process is non-negative while that of misclassifying an MA(1) process as a PDB(1) process is always negative. The penalty function of misclassifying a PDB(1) process as an MA(1) process is a polynomial of order 16 involving only even powers of the parameter of the PDB(1) process. On the other hand, the penalty function of misclassifying a MA(1) process as a PDB(1) process is a polynomial of order 2 or 3 with respect to the parameter of the MA(1) process.

Key Words: Diagonal bilinear time series model, Moving average model, Misclassification, Penalty, Polynomial function.

INTRODUCTION

The similarity and differences of the first, second and fourth order moments of the purely diagonal bilinear time series model of order one and that of the linear moving average process of order one has been studied by Ohakwe and Iwueze (2009).

For clarity, $X_t, \ t \in Z$ is said to be a purely diagonal bilinear time series model of order one, denoted by PDB(1), if for every $t \in Z = \{..., -1, 0, 1, ...\}$

$$X_t = \theta_1 X_{t-1} e_{t-1} + e_t \tag{1.1}$$

where $\{e_t\}$ is the purely random process with $E(e_t) = 0$ and $E(e_t^2) = \sigma_1^2 < \infty$.

For the PDB(1) process of (1.1), the first and second moments are (Ohakwe and Iwueze, 2008);

$$E(X_t) = \sigma_1^2 \theta_1 \tag{1.2}$$

$$R(k) = \begin{cases} \dfrac{\sigma_1^2\left(1 + \sigma_1^2\theta_1^2 + \sigma_1^4\theta_1^4\right)}{1 - \sigma_1^2\theta_1^2} = \dfrac{\sigma_1^2\left(1 + \lambda_1^2 + \lambda_1^4\right)}{1 - \lambda_1^2}, & k = 0 \\ \sigma_1^4\theta_1^2 = \sigma_1^2\lambda_1^2 \ , & k = 1 \\ 0 \ , & k \geq 2 \end{cases} \tag{1.3}$$

and

$$\rho_k = \begin{cases} 1, & k = 0 \\ \dfrac{\sigma_1^2\theta_1^2\left(1 - \sigma_1^2\theta_1^2\right)}{1 + \sigma_1^2\theta_1^2 + \sigma_1^4\theta_1^4} = \dfrac{\lambda_1^2\left(1 - \lambda_1^2\right)}{1 + \lambda_1^2 + \lambda_1^4}, & k = 1 \\ 0, & k \geq 2 \end{cases} \tag{1.4}$$

where $\lambda_1 = \sigma_1 \theta_1$.

The equivalent first order linear moving average process, denoted by MA(1), with same covariance structure to that of the PDB(1) process is given by

$$X_t = \beta_0 + \beta_1 u_{t-1} + u_t \tag{1.5}$$

where $E(u_t) = 0$, $E(u_t^2) = \sigma_2^2 < \infty$ and u_t may not be a purely random process.

Similarly, for the MA(1) process of (1.2) the first and second moments are (Box et al, 1994, Chatfield, 2004).

$$E(X_t) = \beta_0 \tag{1.6}$$

$$R(k) = \begin{cases} \sigma_2^2\left(1+\beta_1^2\right), & k=0 \\ \sigma_2^2 \beta_1, & k=1 \\ 0, & k \geq 2 \end{cases} \tag{1.7}$$

and

$$\rho_k = \begin{cases} 1, & k = 0 \\ \dfrac{\beta_1}{1+\beta_1^2}, & k = 1 \\ 0, & k \geq 2 \end{cases} \tag{1.8}$$

Equating corresponding moments of a PDB(1) process and that of an MA(1) process, Ohakwe and Iwueze (2009) established the following relationship

$$\sigma_2 = \sqrt{\frac{\sigma_1^2\left(1+\lambda_1^2+\lambda_1^4\right)}{\left(1+\beta_1^2\right)\left(1-\lambda_1^2\right)}} \tag{1.9}$$

where

$$\beta_1 = \frac{\left(1+\lambda_1^2+\lambda_1^4\right) - \sqrt{1+2\lambda_1^2-\lambda_1^4+10\lambda_1^6-3\lambda_1^8}}{2\lambda_1^2\left(1-\lambda_1^2\right)} \tag{1.10}$$

The PDB (1) and MA(1) processes have the same covariance structure even though some dissimilarities have been noted by Ohakwe and Iwueze (2009). The two important dissimilarities are;

(i) for the PDB(1) process, $0 < \rho_1 < 0.16$ (Ohakwe and Iwueze (2009) while for the MA(1) process, $\left|\rho_1\right| < 0.5$ (Kendall and Ord, 1990, Wei, 1990).

(ii) for the PDB(1) process, the graph of ρ_1 *against* λ_1 has 3 turning points

(-0.605, 0.1547), (0,0) and (0.605, 0.1547) and is wave like in nature while that of the MA(1) process (ρ_1 *against* β_1) looks like an arctangent curve (Boyle, 1983).

Despite these hidden dissimilarities, sameness in covariance structure will lead to misclassification of one model for the other. Both prediction problems (predicting σ_2 for known σ_1 and vice versa) and penalty for misclassification are studied.

MISCLASSIFICATION OF A PDB(1) PROCESS AS AN MA(1) PROCESS

Penalty for misclassification (wrong identification) will be measured by an increase or decrease in the error standard deviation. Hence penalty for misclassification of a PDB(1) process as an MA(1) process is given by

$$W_1 = \frac{\sigma_2 - \sigma_1}{\sigma_1} = \frac{\sigma_2}{\sigma_1} - 1 \quad (2.1)$$

Similarly, penalty for misclassifying an MA(1) process as a PDB(1) process is given by

$$W_2 = \frac{\sigma_1 - \sigma_2}{\sigma_2} = \frac{\sigma_1}{\sigma_2} - 1 \quad (2.2)$$

Naturally, it is always of interest to predict W_1 *or* W_2 having known the parameters of the PDB(1) process or MA(1) process. That is, we will be interested in two equations:

$$W_1 = f\left(\lambda_1\right) \quad (2.3)$$

$$W_2 = f\left(\beta_1\right) \quad (2.4)$$

since W_1 is a function of λ_1 (see Equation (2.5)) and W_2 is a function of β_1 (see Equation (3.3)).

This paper develops the predictive equations (2.3) and (2.4) and investigates the penalty of misclassification of a PDB(1) process as an MA(1) process and vice versa.

Furthermore, it is of great importance to note that after some algebraic manipulations, it can be verified that our penalty function (2.1) can be given as

$$W_1 = \frac{\sigma_2}{\sigma_1} - 1 = \left[\sqrt{\frac{\left(1 + \lambda_1^2 + \lambda_1^4\right)}{\left(1 + \beta_1^2\right)\left(1 - \lambda_1^2\right)}} \right] - 1 \qquad (2.5)$$

If we consider the fact that $\lambda_1 = \sigma_1 \theta_1$, it is clear that ρ_1 (see Equation (1.4)), β_1 (see Equation (1.10)) and W_1 (see Equation (2.5)) do not vary for varying values of σ_1 *and* θ_1 when λ_1 is fixed.

Having seen that ρ_1, β_1 *and* W_1 are constants independent of the variations between θ_1 *and* σ_1 that yielded the fixed λ_1, we shall now compute the values of ρ_1, β_1 *and* W_1 for $\lambda_1 = [-0.99, 0.99]$. The computations are shown in Table 1 and the plot of W_1 against $\lambda_1 = [-0.99, 0.99]$ is shown in Figure 1.

By examining Figure 1, the following comments can be made.

(1) The vertical line $\lambda_1 = -1$ *and* $\lambda_1 = 1$ are vertical asymptotes of the graph of $W_1 = f(\lambda_1)$ (Smith and Minton , 2005).

(2) The graph of W_1 *for* $\lambda_1 < 0$, is the mirror image in $W_1 - axis$ of the graph of W_1 *for* $\lambda_1 > 0$.

(3) The graph is tangent to the $\lambda_1 - axis$ at the origin $\left(W_1 = f(0) = 0\right)$ and does not cross it (Flanders and Price, 1981, Hoffmann et al, 2005).

(4) The plot is symmetric in the $W_1 - axis$ *with* $f(-\lambda_1) = f(\lambda_1)$. Here, we have an even function. It can be shown that a polynomial is an even

function if and only if, it involves only even powers of the variable (Flanders and Price, 1981).

By looking at the values of W_1 in Table 1, it can be seen that there is always an increase in the penalty and subsequently the error variance if we misclassify a PDB(1) process as an MA(1) process. That is, $W_1 \geq 0$, *for all* $-1 < \lambda_1 < 1$.

Having earlier said that we shall be interested in the predictive Equation (2.3), our next task is to establish the appropriate function that precisely explains the relationship between W_1 *and* λ_1. Figure 1 suggests a polynomial function, which can be represented as

$$W_1 = \sum_{i=0}^{p} \alpha_{2i} \lambda_1^{2i} \tag{2.6}$$

To chose p (the order of the polynomial (2.6) is $2p$), we fit polynomials of various degrees starting from $p = 1$ and then assess the best fit using the coefficient of determination, R^2 (Draper and Smith, 1981). Furthermore, the t-value would be used to check for the significance of each of the coefficients. The fitted polynomials, R^2 and t-value are shown in Table 2.

Using R^2 in Table 2, we can see that the predictive equation depends on the magnitude of the R^2, we would want to tolerate. We would use $p = 6$ if we want R^2 to be approximately 99%, while we would use $p = 8$ if we want R^2 to be approximately 100%. Ignoring the constant term which is not significant, the predictive equation of (1.11) when $R^2 \approx 100\%$ is given by

$$\hat{W}_1 = -\underset{(0.98)}{3.3747}\lambda_1^2 + \underset{(21.66)}{133.8363}\lambda_1^4 - \underset{(179.90)}{1361.6836}\lambda_1^6$$

$$+ \underset{(737.20)}{6555.2092}\lambda_1^8 - \underset{(1648.00)}{16729.3999}\lambda_1^{10} + \underset{(2047.00)}{23311.5210}\lambda_1^{12}$$

$$\underset{(1325.00)}{-16743.7149\lambda_1^{14}} + \underset{(348.60)}{4852.3196\lambda_1^{16}} \tag{2.7}$$

Table 1. An abridged table showing the computations of ρ_1, β_1, *and* W_1

S/NO.	λ_1	ρ_1	β_1	W_1	$\hat{W}_1$	$\lvert W_1 - \hat{W}_1 \rvert$
1	-0.9	0.0624	0.0627	2.5957	2.6253	0.0297
2	-0.8	0.1124	0.1139	1.3708	1.3181	0.0526
3	-0.7	0.1444	0.1476	0.8221	0.8378	0.0157
4	-0.6	0.1547	0.1586	0.5068	0.5376	0.0308
5	-0.5	0.1429	0.1459	0.3090	0.2571	0.0519
6	-0.4	0.1134	0.1149	0.1803	0.1989	0.0186
7	-0.3	0.0746	0.0750	0.0954	0.1306	0.0352
8	-0.2	0.0369	0.0369	0.0409	0.0072	0.0338
9	-0.1	0.0098	0.0098	0.0101	-0.0217	0.0317
10	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
11	0.1	0.0098	0.0098	0.0101	-0.0217	0.0317
12	0.2	0.0369	0.0369	0.0409	0.0072	0.0338
13	0.3	0.0746	0.0750	0.0954	0.1306	0.0352
14	0.4	0.1134	0.1149	0.1803	0.1989	0.0186
15	0.5	0.1429	0.1459	0.3090	0.2571	0.0519
16	0.6	0.1547	0.1586	0.5068	0.5376	0.0308
17	0.7	0.1444	0.1476	0.8221	0.8378	0.0157
18	0.8	0.1124	0.1139	1.3708	1.3181	0.0526
19	0.9	0.0624	0.0627	2.5957	2.6253	0.0297

Note: For the full Table where $\lambda_1 = [-0.99, 0.99]$, see Ohakwe (2008).

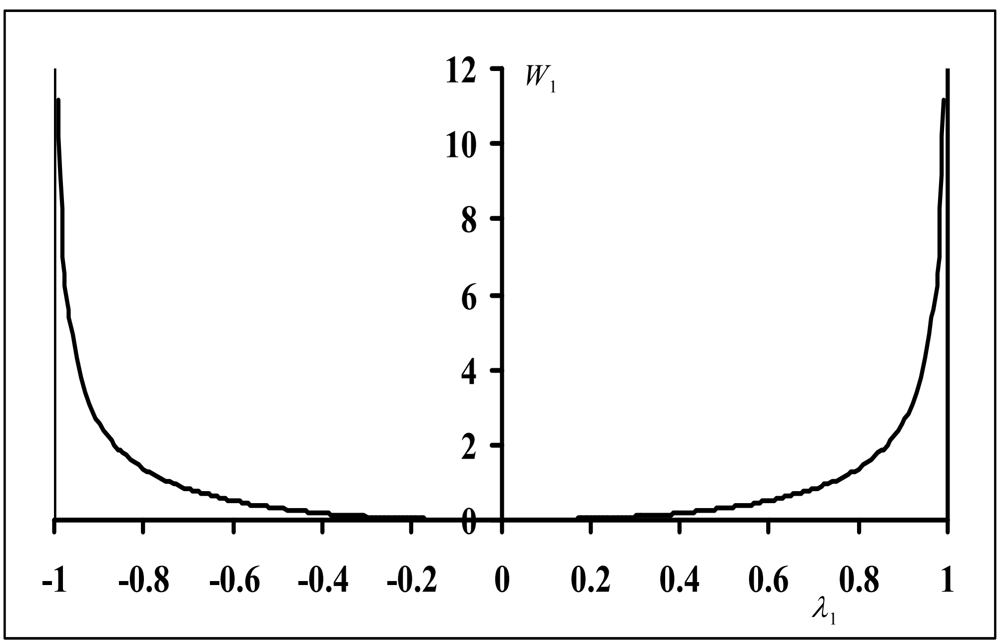

Figure 1: A plot of W_1 against $\lambda_1 = [-0.99, 0.99]$

The numbers in brackets in Equation (2.7) are the standard errors of the estimated parameters. The fitted values of W_1, denoted by $\hat{W}_1$, are also given in Table 1. It is clear from Table 1 (using the absolute values of the deviations, $\left|W_1 - \hat{W}_1\right|$), that Equation (2.7) is an excellent fit, since there were almost no differences between the original penalties(W_1) and their fits($\hat{W}_1$).

MISCLASSIFICATION OF AN MA(1) PROCESS AS A PDB(1) PROCESS

We have examined the penalty for the misclassification of a PDB(1) process as an MA(1) process and also established the predictive equation of (2.3). However, the similarity in the covariance function of a PDB(1) process and an MA(1) process may also lead to misclassifying an MA(1) process as a PDB(1) process. Prior to investigating the penalty of wrongly identifying an MA(1) process as a PDB(1) process and the predictive Equation (2.4), we shall first establish a relationship between the parameters of a given MA(1) process and its equivalent PDB(1) process.

Using the method of moments, Ohakwe and Iwueze (2009) established the following relationship:

$$\lambda_1^2 = \frac{\left(1-\beta_1+\beta_1^2\right) \pm \sqrt{\left(1-\beta_1+\beta_1^2\right)^2 - 4\beta_1\left(1+\beta_1+\beta_1^2\right)}}{2\left(1+\beta_1+\beta_1^2\right)} \qquad (3.1)$$

and

$$\sigma_1 = \sigma_2 \sqrt{\frac{\left(1+\beta_1^2\right)\left(1-\lambda_1^2\right)}{1+\lambda_1^2+\lambda_1^4}} \qquad (3.2)$$

Interestingly, the penalty function defined by (2.4) can, by use of (3.2), be represented as

$$W_2 = \frac{\sigma_1}{\sigma_2} - 1 = \sqrt{\frac{\left(1 + \beta_1^2\right)\left(1 - \lambda_1^2\right)}{1 + \lambda_1^2 + \lambda_1^4}} - 1 \qquad (3.3):$$

Having seen clearly from (3.3) that for fixed β_1 *and* $\lambda_1 = \sigma_1 \theta_1$, W_2 is constant irrespective of the variations between σ_1 *and* θ_1, we now proceed by

computing the value of W_2 for a given β_1.

If we consider the fact that $-0.5 < \rho_1 < 0.5$ for an MA(1) process (Box et al., 1994, Chatfield, 2004) and $0 < \rho_1 < 0.16$ for a PDB(1) process (Ohakwe and Iwueze, 2009), we shall here consider the points where an MA(1) process could be mistaken for the PDB(1) process. That is, we shall consider the MA(1) process with $0 < \rho_1 < 0.16$ and this requires $0 < \beta_1 < 0.16$ (Ohakwe and Iwueze, 2009). The computation of λ_1^2 [using (3.1)] and W_1 [using (3.3)] for given β_1 are shown in Table 3.

We can see from Table 3 that there are two feasible solutions for W_2, denoted by $W_2(+)$ and $W_2(-)$, which were respectively gotten from $\lambda_1^2(+)$ and $\lambda_1^2(-)$ resulting from using the "plus" and "minus" signs of (3.1) respectively. It is important to note from Table 3 that $W_2(+)$ and $W_2(-)$ are respectively defined on the segments $0.5 < \lambda_1^2 < 1.0$ and $0 < \lambda_1^2 < 0.3$. That is to say that $W_2(+)$ is defined on the segments where $-1.0 < \lambda_1 < -0.69$ and $0.69 < \lambda_1 < 1.0$, while $W_2(-)$ is defined on $-0.52 < \lambda_1 < 0$ and $0 < \lambda_1 < 0.52$. Plots of $W_2(+)$ against β_1 and $W_2(-)$ against β_1 are shown in Figure 2.

Table 2: Coefficients and their t-values for the fitted polynomials (2.4) with the corresponding R^2

α_i	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=7$	$p=8$	$p=9$	$p=10$	$p=11$
α_0	0.93 (7.90)	-0.53 (-4.82)	0.33 (3.56)	-0.22 (-2.93)	0.16 (2.59)	-0.11 (-2.33)	0.08 (2.15)	-0.06 (-2.02)	0.04 (1.91)	-0.03 (-1.82)	0.02 (1.75)	-0.01 (-1.69)
α_2		4.43 (17.90)	-4.21 (-7.21)	7.44 (8,05)	-6.38 (-5.25)	8.6590 (6.12)	-6.56 (-4.31)	8.14 (5.28)	-5.49 (-3.70)	6.71 (4.89)	-3.88 (-3.18)	5.05 (4.81)
α_4			10.18 (15.48)	-25.12 (-9.84)	51.67 (8.69)	-80.06 (-7.51)	112.34 (6.90)	-140.46 (-6.36)	165.31 (6.02)	-180.68 (-5.69)	189.56 (5.48)	-187.93 (-5.25)
α_6				26.16 (14.07)	-108.36 (-10.81)	291.13 (9.54)	-590.93 (-8.52)	1028.98 (7.90)	-1571.13 (-7.38)	2190.47 (7.01)	-2813.15 (-6.69)	3389.81 (6.45)
α_8					72.82 (13.54)	-417.56 (-11.41)	1398.32 (10.13)	-3518.45 (-9.29)	7291.59 (8.67)	-13119.15 (-8.19)	21112.48 (7,81)	-31017.23 (-7.51)
α_{10}						209.30 (13.47)	-1504.58 (-11.68)	6122.70 (10.66)	-18193.46 (-9.89)	43824.51 (9.32)	-90324.94 (-8.87)	164636.19 (8.51)
α_{12}							603.95 (13,40)	-5241.71 (-12.02)	24964.88 (11.10)	-85431.37 (-10.42)	233859.30 (9.89)	-541890.29 (-9.46)
α_{14}								1754.61 (13.45)	-17732.12 (-12.31)	96530.33 (11.50)	-372837.40 (-10.89)	1140129.21 (10.40)
α_{16}									5095.13 (13.56)	-58549.08 (-12.60)	357649.50 (11.88)	-1534718.89 (-11.33)
α_{18}										14743.92 (13.72)	-189244.25 (-12.89)	1278264.36 (12.25)
α_{20}											42429.20 (13.91)	-599823.67 (-13.18)
α_{22}												121230.59 (44.13)
R^2	0.0	62.1	83.0	91.6	95.8	97.8	98.9	99.4	99.7	99.9	99.9	100.0

Note: The numbers in brackets are the t-values of the coefficients.

It is seen from Figure 2 that the relationship between W_2 and β_1 in both segments are precisely explained by polynomial functions of order q given by

$$W_2 = \sum_{i=0}^{q} \phi_i \beta_1^i \qquad (3.4)$$

Thus, the new task here is to establish the appropriate value of q and the corresponding Equation (3.4). Similar to the method of Section 2, we would also use R^2 and t-values to assess the best value of q and the significance of the coefficients respectively. The computations are given in Table 4 for $W_2(+)$ and $W_2(-)$. By looking at the values of $W_2(+)$ and $W_2(-)$ in Table 3, it can be seen that there is always a decrease in the penalty and subsequently the error variance if we misclassify an MA(1) process as a PDB(1) process. That is, $W_2(+) \leq 0$, $W_2(-) \leq 0$, for all $0 < \beta_1 < 0.16$.

It is seen from Table 4 that for $W_2(+)$, $q = 2$ when we want $R^2 \approx 99\%$. However, if we require $R^2 \approx 100\%$ for $W_2(+)$, then $q = 3$. Furthermore, from Table 4, $q = 2$ when $R^2 \approx 100\%$ is required for $W_2(-)$. Therefore the predictive Equation (1.12) for $W_2(+)$ and $W_2(-)$ based on $q = 3$ are

$$W_2(+) = -\underset{(0.01)}{0.9801} + \underset{(0.47)}{6.6646}\beta_1 - \underset{(7.45)}{52.2899}\beta_1^2 + \underset{(32.62)}{214.0281}\beta_1^3 \qquad (3.5)$$

and

$$W_2(-) = -\underset{(0.07)}{1.2196}\beta_1 + \underset{(1.34)}{5.1962}\beta_1^2 - \underset{(6.39)}{54.3510}\beta_1^3 \qquad (3.6)$$

where (3.6) is a fit gotten by ignoring the constant term which is not significant. Again the numbers in brackets in Equations (3.5) and (3.6) are the standard errors of the estimated parameters It is clear from Table 3 that Equations (3,5) and (3.6) are excellent fits, since there were almost no differences (using the absolute values of the deviations, $\left|W_2(+) - \hat{W}_2(+)\right|$ and $\left|W_2(-) - \hat{W}_2(-)\right|$) between the original penalties ($W_2(+)$ and $W_2(-)$) and their fits [$W_2(+)$ and $W_2(-)$].

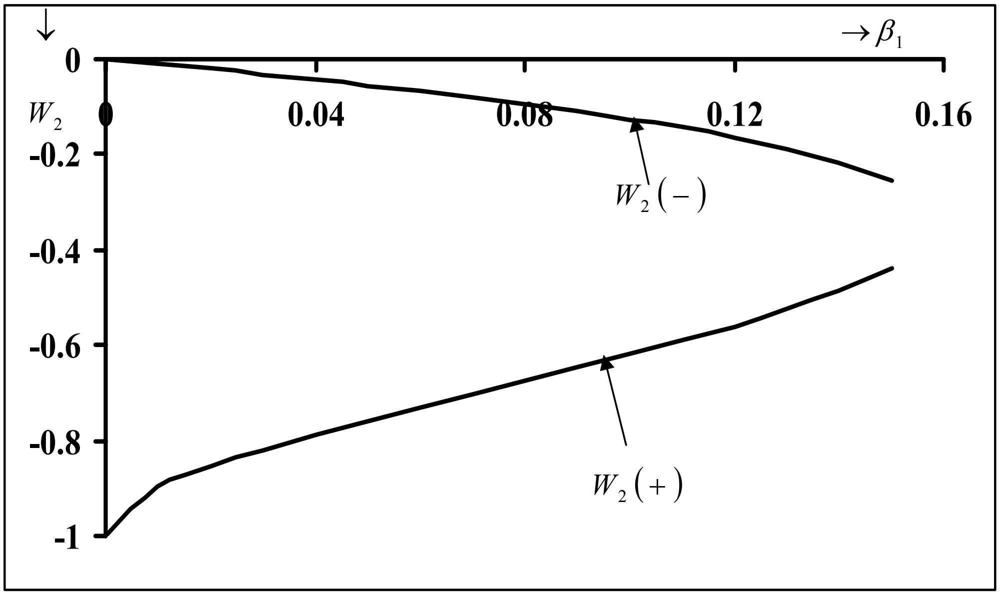

Figure 2: A plot of W_2 against β_1

CONCLUSION

In this study, we have established the penalty for the misclassification of a PDB(1) process as an MA(1) process and vice versa. Furthermore, we established predictive equations that can be used for predicting penalty function associated with the misclassification of one process as the other. In other words, we can predict σ_2 *or* σ_1, if one of them is known for a given β_1 *or* λ_1.

From the results, it was found that misclassifying a PDB(1) process as an MA(1) process leads to an increase in the error variance ($W_1 \geq 0$, *for all* $-1 < \lambda_1 < 1$) while that of an MA(1) process as a PDB(1) process will result in a decrease in the error variance ($W_2(+) \leq 0, W_2(-) \leq 0$, for all

Table 3. Computations showing (i) ρ_1 and the parameter λ_1 for a PDB(1) process (ii) the penalties (W_2) of misclassifying an MA(1) process as a PDB(1) process.

S/NO.	β_1	ρ_1	$\lambda_1^2(+)$	$\lambda_1^2(-)$	$W_2(+)$	$W_2(-)$	$\hat{W}_2(+)$	$\hat{W}_2(-)$	$\lvert W_2(+)-\hat{W}_2(+)\rvert$	$\lvert W_2(-)-\hat{W}_2(-)\rvert$
1	0.00	0.0000	1.0000	0.0000	-1.0000	0.0000	-0.9800	0.0000	0.0200	0.0000
2	0.01	0.0100	0.9700	0.0102	-0.8985	-0.0102	-0.9184	-0.0117	0.0200	0.0016
3	0.02	0.0200	0.9399	0.0209	-0.8541	-0.0207	-0.8660	-0.0227	0.0118	0.0021
4	0.03	0.0300	0.9098	0.0320	-0.8184	-0.0315	-0.8214	-0.0334	0.0030	0.0018
5	0.04	0.0399	0.8795	0.0437	-0.7867	-0.0429	-0.7835	-0.0439	0.0033	0.0011
6	0.05	0.0499	0.8490	0.0560	-0.7573	-0.0547	-0.7508	-0.0548	0.0065	0.0001
7	0.06	0.0598	0.8182	0.0689	-0.7292	-0.0671	-0.7722	-0.0662	0.0070	0.0009
8	0.07	0.0697	0.7870	0.0827	-0.7018	-0.0802	-0.6964	-0.0786	0.0054	0.0017
9	0.08	0.0795	0.7552	0.0975	-0.6745	-0.0942	-0.6720	-0.0921	0.0026	0.0021
10	0.09	0.0893	0.7227	0.1134	-0.6471	-0.1092	-0.6478	-0.1073	0.0007	0.0019
11	0.10	0.0990	0.6891	0.1307	-0.6191	-0.1254	-0.6225	-0.1243	0.0034	0.0011
12	0.11	0.1087	0.6541	0.1499	-0.5899	-0.1433	-0.5948	-0.1436	0.0049	0.0003
13	0.12	0.1183	0.6170	0.1715	-0.5590	-0.1634	-0.5634	-0.1654	0.0044	0.0020
14	0.13	0.1278	0.5768	0.1965	-0.5252	-0.1867	-0.5271	-0.1901	0.0019	0.0035
15	0.14	0.1373	0.5313	0.2272	-0.4867	-0.2151	-0.4846	-0.2180	0.0021	0.0030
16	0.15	0.1467	0.4746	0.2696	-0.4378	-0.2541	-0.4345	-0.2495	0.0032	0.0046
17	0.16	0.1560	*	*	*	*	*	*	*	*
18	0.17	0.1652	*	*	*	*	*	*	*	*

Note: * means that computations outside the feasible limits, $0 < \beta_1 < 0.16$, is not possible

Table 4. The fitted polynomials and the corresponding R^2 and the t-values of the coefficients for $W_2(+)$ and $W_2(-)$

ϕ_i	$W_2(+)$						$W_2(-)$			
	$q=1$	$q=2$	$q=3$	$q=4$	$q=5$	$q=6$	$q=1$	$q=2$	$q=3$	$q=4$
ϕ_0	-0.94 (-92.57)	-0.95 (-70.74)	-0.98 (-123.95)	-0.99 (-140.40)	-1.00 (-236.97)	-1.00 (-335.15)	0.02 (2.55)	-0.01 (-1.66)	0.00 (1.14)	0.00 (-0.79)
ϕ_1	3.25 (28.26)	3.87 (9.30)	6.66 (14.12)	8.3029 (11.92)	10.78 (17.02)	12.4629 (19.14)	-1.57 (-19.63)	-0.55 (-4.90)	-1.32 (-11.95)	-0.78 (-8.40)
ϕ_2		-4.13 (-1.55)	-52.27 (-7.01)	-105.19 (-5.30)	-235.41 (-8.28)	-367.54 (-8.60)		-6.83 (-9.53)	6.50 (3.72)	-10.80 (-4.06)
ϕ_3			213.94 (6.56)	773.85 (3.83)	3186.74 (6.40)	6932.68 (6.13)			-59.25 (-7.75)	123.74 (4.57)
ϕ_4				-1866.36 (-2.80)	-20207.59 (-5.46)	-68256.87 (-4.86)				-609.96 (-6.82)
ϕ_5					48909.94 (4.98)	33412.25 (4.06)				
ϕ_6						-632227.35 (-3.48)				
R^2	98.3	98.5	99.7	99.8	99.9	100.00	96.5	99.6	99.9	100.0

Note: The numbers in brackets are the t-values of the coefficients.

$0 < \beta_1 < 0.16$). The result with respect to $W_2(+)$ *and* $W_2(-)$ is in support of the well known result in statistical modeling that decrease in error variance does not imply a better fit.

REFERENCES

Box, G. E. P., G. M. Jenkins,. and G. C. Reinsel, 1994. Time Series Analysis, Forecasting and Control. 3rd ed; Prentice-Hall, Englewood Cliffs N.J.

Boyle, P. J., 1983. Trigonometry with Applications, Harper & Row, Publishers, New York.

Chatfield, C., 2004. The Analysis of Time Series: An Introduction, Chapman and Hall/CRC Press, Boca Raton.

Draper, N. R. and H. Smith, 1981. Applied Regression Analysis, 2nd ed; John Wiley and Sons Inc, New York.

Flanders, H. and J.J. Price, 1981. Algebra. 2nd ed. Saunders College Publishing, Philadelphia.

Hoffmann, L. D., G. L. Bradley, and K.H. Rosen, 2005. Applied Calculus for Business, Economics, and the Social and Life Sciences, McGraw Hill, Boston.

Ohakwe, J. (2008). Tables and moments of the purely diagonal bilinear time series model. Ph.d Thesis, Abia State University, Uturu, Nigeria.

Ohakwe, J. and I.S. Iwueze, 2009. Characterizations of the moments of the purely diagonal bilinear time series model of order one. In Press. InterStat Journal of Statistics

Smith, R. T. and R.B. Minton, 2005. Calculus, 2nd ed; McGraw Hill, New York.

Kendall, M. G. and J.K. Ord, 1990. Time Series, 3rd Ed., Charles Griffin, London

Wei, W. W. S. (1990). Time Series Analysis: Univariate and Multivariate Method, Addisin – Wesley, California