Prediction for the Future Record Values from Logistic Distribution

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Abstract

In this paper, we deal with some point and interval prediction methods for future record values. In particular, having observed a sequence of record values from logistic distribution, we consider maximum likelihood predictor (MLP), best linear unbiased predictor (BLUP), and median unbiased predictor (MUP). The predictive likelihood equations cannot be solved to obtain closed form for the MLP. Either BLUP or MUP has explicit form and is quite easy to compute. Monte Carlo simulations were performed to estimate MSEs of these predictors. It is shown that efficiency of MUP relative to BLUP is high for most values of *m* and *n* considered in this paper. Some approaches are proposed for constructing prediction interval for the future record values. These approaches depend on location and scale invariant statistics, pivotal quantity and conditional distribution of $(X_{u(n)} - X_{u(m)})$ given $X_{u(m)}, n > m$. We have determined some percentage points of the statistics considered in this paper through Monte Carlo simulations (based on 10,000 runs). With the help of these points, one could easily construct 100(1- γ)% prediction intervals for the future record value. A comparison among prediction intervals for future record values is made via extensive Monte Carlo simulations. In most simulation cases the prediction interval based on conditional distribution of $(X_{u(n)} - X_{u(m)})$ given $X_{u(m)}$ is superior in terms of larger estimated coverage probabilities and shorter estimated average lengths.

Keywords: Logistic distribution; Upper record values; Point prediction; Interval prediction; Maximum likelihood predictor; Best linear unbiased predictor; Median unbiased predictor; Location-scale invariant statistic; Monte Carlo simulation.

1- Introduction

A random variable X is said to have a logistic distribution $L(\mu,\sigma^2)$ if its probability density function is

$$f(x,\mu,\sigma) = \frac{\pi}{\sigma\sqrt{3}} \frac{e^{-\pi(x-\mu)/\sigma\sqrt{3}}}{\{1+e^{-\pi(x-\mu)/\sigma\sqrt{3}}\}^2} \qquad -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0 \tag{1}$$

and the corresponding cumulative distribution function is

$$F(x,\mu,\sigma) = \left[1+e^{-\pi(x-\mu)/\sigma\sqrt{3}}\right]^{-1} \tag{2}$$

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Also, its hazard function and the corresponding cumulative hazard function are respectively;

$$h(x,\mu,\sigma)=\frac{\pi}{\sigma\sqrt{3}}\left[1+e^{-\pi(x-\mu)/\sigma\sqrt{3}}\right]^{-1}, \tag{3}$$

$$H(x,\mu,\sigma)=-\ln\left[1+e^{\pi(x-\mu)/\sigma\sqrt{3}}\right]^{-1} \tag{4}$$

The logistic distribution has several important biological, actuarial, industrial, engineering and life testing applications. Several theoretical methodological relating to the logistic distribution are addressed in great detail and explained with many numerical examples in Balakrishnan (1992). While a lot of work has been done in order statistics, not much has been done in record values. Record values arise naturally in many real life applications involving data relating to weather, sports, economics and life tests. For applications of the record values see Nevzorov(2001).

To define the upper record values, let $\{X_n\}$, $n \geq 1$ be a sequence of independent and identically distributed logistic random variables. Set $Y_n = \max\{X_n\}$ for $n \geq 1$, we say X_j is an upper record value of $\{X_n\}$ if $Y_j > Y_{j-1}$. By definition X_1 is an upper record value. In this paper we denote the j th upper record value by $X_{u(j)}$.

The problem of prediction of future record values from exponential distribution has been discussed by Ahsanullah (1980), Dunsmore (1983), Kaminsky and Rohdin (1985) and Awad and Raqab (2000). Best linear invariant predictor for future records from generalized extreme value distribution has been derived by Ahsanullah and Holland (1994). Berred (1998) discussed the problem of prediction under suitable assumptions on the tail of distribution function. A Bayesian prediction of record values has been introduced by Nagaraja (1984), Balakrishnan and Chan (1994), Doganaksoy and Balakrishnan (1997), Wesolowski and Ahsanullah (2001), Al.Hussaini and Ahmed (2003), Madi and Raqab (2004), Ahmadi *et al*. (2005) and Klimczak (2006).

In this paper we consider prediction for future upper record values from logistic distribution. In section 2 we discuss the MLP, BLUP and MUP. In Section 3, we give some approaches for constructing prediction interval for future upper record value. These approaches depend on location and scale invariant statistics, pivotal quantity and conditional distribution of $(X_{u(n)} - X_{u(m)})$ given $X_{u(m)}$, $n > m$. In section 4, we carry out a numerical study to compare the performance of MUP with BULP in terms of their mean square errors. Also a comparison among prediction intervals, considered in this paper, is made via extensive Monte Carlo simulations.

2- Point Prediction

In this section we derive some point predictors, maximum likelihood predictor (MLP), best linear unbiased predictor (BLUP) and median unbiased predictor (MUP).

2-1 Maximum likelihood predictor

Let $X^* = \{ X_{u(1)}, X_{u(2)}, \ldots\ldots, X_{u(m)} \}$ be the first observed m upper record values from logistic distribution . Our aim is to predict the n th record value $X_{u(n)}$, $n > m$. Let $x^* = \{x_{u(1)}, x_{u(2)}, \ldots\ldots, x_{u(m)}\}$ with $x_{u(1)} < x_{u(2)} \ldots\ldots\ldots < x_{u(m)}$. The predictive likelihood function of $X_{u(n)}$, μ and σ is given by

$$L = L\left(x^*, x_{u(n)}, \quad \mu, \sigma \right) = \prod_{i=1}^{m} h(x_{u(i)}) \frac{\{H(x_{u(n)}) - H(x_{u(m)})\}^{n-m-1}}{\Gamma(n-m)} f(x_{u(n)})$$

$$= \prod_{i=1}^{m} \left[\frac{\pi}{\sigma\sqrt{3}} \left(1 + e^{-\pi(x_{u(i)} - \mu)/\sigma\sqrt{3}} \right)^{-1} \right] \frac{1}{\Gamma(n-m)}$$

$$\text{x} \left[\ln\left(1 + e^{\pi(x_{u(m)} - \mu)/\sigma\sqrt{3}} \right)^{-1} - \ln\left(1 + e^{\pi(x_{u(n)} - \mu)/\sigma\sqrt{3}} \right)^{-1} \right]^{n-m-1} \quad (5)$$

$$\text{x} \; \frac{\pi}{\sigma\sqrt{3}} \frac{e^{-\pi(x_{u(n)} - \mu)/\sigma\sqrt{3}}}{\left(1 + e^{-\pi(x_{u(n)} - \mu)/\sigma\sqrt{3}} \right)^2}$$

The likelihood equations are given by

$$\frac{\partial \ln L}{\partial X_{u(n)}} = \frac{\pi}{\hat{\sigma}\sqrt{3}} \left(1 + e^{-\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right)^{-1}$$

$$\text{x} \left[\frac{\text{n - m - 1}}{\ln\left(1 + e^{\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right) - \ln\left(1 + e^{\pi(x_{u(m)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right)} - \left(1 - e^{-\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right) \right] = 0 \quad (6)$$

$$\frac{\partial \ln L}{\partial \mu} = \frac{\pi}{\hat{\sigma}\sqrt{3}} \left[\begin{array}{l} -\sum_{i=1}^{m} \left(1 + e^{\pi(x_{u(i)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right)^{-1} \\ + \dfrac{(n-m-1)\left(\left\{ 1 + e^{-\pi(x_{u(m)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right\}^{-1} - \left\{ 1 + e^{-\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right\}^{-1} \right)}{\ln\left(1 + e^{\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right) - \ln\left(1 + e^{\pi(x_{u(m)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right)} \\ - \left\{ 1 - e^{\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right\} \left\{ 1 + e^{\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right\}^{-1} \end{array} \right] = 0 \quad (7)$$

$$\frac{\partial \ln L}{\partial \sigma} = -\sum_{i=1}^{n} \left[\frac{1}{\hat{\sigma}} + \frac{\pi}{\hat{\sigma}^2\sqrt{3}} \left(x_{u(i)} - \hat{\mu} \right) \left(1 + e^{\pi\left(x_{u(i)} - \hat{\mu} \right)/\hat{\sigma}\sqrt{3}} \right)^{-1} \right]$$

$$+ \frac{(n-m-1)\left(\dfrac{\pi}{\hat{\sigma}^2\sqrt{3}}\right)(x_{u(m)} - \hat{\mu}) \left(1 + e^{-\pi\left(x_{u(m)} - \hat{\mu} \right)} \right)^{-1} + (\hat{x}_{u(n)} - \hat{\mu}) \left(1 + e^{-\pi\left(\hat{x}_{u(n)} - \hat{\mu} \right)} \right)^{-1}}{\ln\left(1 + e^{\pi(\hat{x}_{u(n)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right) - \ln\left(1 + e^{\pi(x_{u(m)} - \hat{\mu})/\hat{\sigma}\sqrt{3}} \right)}$$

$$- \left[\frac{1}{\hat{\sigma}} + \frac{\pi}{\hat{\sigma}^2\sqrt{3}} \left(\hat{x}_{u(n)} - \hat{\mu} \right)^{-1} \left(1 - e^{\pi\left(\hat{x}_{u(n)} - \hat{\mu} \right)/\hat{\sigma}\sqrt{3}} \right) \left(1 + e^{\pi\left(\hat{x}_{u(n)} - \hat{\mu} \right)/\hat{\sigma}\sqrt{3}} \right)^{-1} \right] = 0 \quad (8)$$

The likelihood equations in (6), (7) and (8) do not admit explicit solutions. The maximum likelihood predictor of $X_{u(n)}$ and predictive maximum likelihood estimator of μ and σ may be obtained by solving (6), (7) and (8) iteratively.

2-2 Best linear unbiased predictor

For a location-scale family with location parameter μ and scale parameter σ, the best linear unbiased predictor of $X_{u(n)}$ is considered by Arnold *et al*. (1998) and is given by

$$\tilde{X}_{u(n)} = \tilde{\mu} + \alpha_n \tilde{\sigma} + W'B^{-1}(X^* - \tilde{\mu}.\mathrm{I} - \tilde{\sigma}\alpha) \tag{9}$$

where $\tilde{\mu}$ and $\tilde{\sigma}$ are the best linear unbiased estimators of μ and σ, respectively, I is vector of 1's, α is the vector of means of record values from the standard distribution, B is the dispersion matrix of standard record values and $W' = (\beta_{1n}, \beta_{2n}, \ldots\ldots, \beta_{nn})$, $\beta_{in} = \mathrm{C}ov(X_{u(i)}, X_{u(n)})$. In our prediction problem (see Balakrishnan *et al*. 1995);

$$\tilde{\mu} = \sum_{i=1}^{m} a_i X_{u(i)}, \qquad \tilde{\sigma} = \sum_{i=1}^{m} b_i X_{u(i)} \tag{10}$$

with $$a = \frac{\alpha' B^{-1} \alpha \mathrm{I}' B^{-1} - \alpha' B^{-1} \mathrm{I} \alpha' B^{-1}}{(\alpha' B^{-1} \alpha)(\mathrm{I}' B^{-1} \mathrm{I}) - (\alpha' B^{-1} \mathrm{I})^2},$$

$$b = \frac{\mathrm{I}' B^{-1} \mathrm{I} \alpha' B^{-1} - \mathrm{I}' B^{-1} \alpha \mathrm{I}' B^{-1}}{(\alpha' B^{-1} \alpha)(\mathrm{I}' B^{-1} \mathrm{I}) - (\alpha' B^{-1} \mathrm{I})^2}$$

with $\alpha' = (\alpha_1, \alpha_2, \ldots\ldots, \alpha_m)$,

$$\alpha_n = n - \sum_{k=1}^{\infty} \frac{1}{k(k+1)^n},$$

$B = ((\beta_{ij}))$, $\quad 1 \le i \le j \le m$, with

$$\beta_{ij} = i\left\{ \sum_{k=1}^{\infty} \frac{1}{k^{i+1}} + \sum_{k=1}^{\infty} \frac{1}{k^{j+1}} - 1 \right\} - \sum_{k=1}^{\infty} \frac{1}{k(k+1)^i} \sum_{k=1}^{\infty} \frac{1}{k(k+1)^j}$$

$$+ \sum_{k=1}^{\infty} \frac{1}{k(k+1)^{j-i}} \sum_{\iota=1}^{\infty} \frac{1}{\iota(\iota+1+k)^i}$$

2-3 Median unbiased predictor

Takada (1991) introduced a median unbiased predictor in an invariant prediction problem for the location-scale family. In our prediction problem, this predictor takes the form

$$\breve{\tilde{X}}_{u(n)} = \tilde{X}_{u(n)} + \tilde{\sigma}.\mathrm{Med}\left(\frac{X_{u(n)} - \tilde{X}_{u(n)}}{\tilde{\sigma}}\right) \quad (11)$$

where, $\tilde{X}_{u(n)}$ is the best linear unbiased predictor of $X_{u(n)}$ and $\tilde{\sigma}$ is the BLUE of the scale parameter σ. For our prediction problem, we note that, derivation of distribution for quantity, $\frac{X_{u(n)} - \tilde{X}_{u(n)}}{\tilde{\sigma}}$, is more difficult. Therefore, we determine the median for this quantity through Monte Carlo simulations (based on 10,000 runs). The simulated medians are presented in Table 1 for m = 2(1)9, n = 3(1)10.

3- Prediction interval

In this section we propose different prediction intervals for the future record values from logistic distribution .

3-1 Prediction interval based on the location and scale invariant statistic

This prediction interval for $X_{u(n)}$ based on the location and scale invariant statistic

$$T = \frac{X_{u(n)} - X_{u(m)}}{\tilde{\sigma}}$$

Where, $\tilde{\sigma}$ is the BLUE of σ based on the first m upper record values. Through Monte Carlo simulations (based on 10,000 runs) some percentage points of T are determined for m = 2(1)9 and n = 3(1)10 and presented in Table 2. It follows that the prediction limits for $X_{u(n)}$ can be written as

$$L_1 = X_{u(m)} + T_{\gamma/2}\tilde{\sigma} \quad \text{and} \quad U_1 = X_{u(m)} + T_{1-\gamma/2}\tilde{\sigma} \quad (12)$$

Another prediction interval for $X_{u(n)}$ is introduced. It is also based on location and scale invariant statistics

$$Q = \frac{X_{u(n)} - \tilde{\mu}}{\tilde{\sigma}}$$

where $\tilde{\mu}$ and $\tilde{\sigma}$ are again BLUEs of μ and σ, respectively, based on the first m upper record values. Also, Monte Carlo simulations (based on 10,000 runs) are used to evaluate some percentage points of the statistic Q for m = 2(1)9, n = 3(1)10. These simulated percentage points are presented in Table 3. From Table 3 one can construct a (1- γ)100% prediction interval for the future record values as follows

$$L_2 = \tilde{\mu} + Q_{\gamma/2}\tilde{\sigma} \quad \text{and} \quad U_2 = \tilde{\mu} + Q_{1-\gamma/2}\tilde{\sigma} \quad (13)$$

3-2 Pivot method

We consider a pivotal quantity, P which is a function of upper record $X_{u(n)}$ and BLUP of $X_{u(n)}$. Its distribution does not depend on $X_{u(n)}$

$$P = \frac{X_{u(n)} - \tilde{X}_{u(n)}}{\tilde{\sigma}}$$

Some percentage points of the statistic P are determined using Monte Carlo simulations (based on 10,000 runs). These points are tabulated in Table 4 for m = 2(1)9, n = 3(1)10. Thus a 100(1- γ) % prediction interval for $X_{u(n)}$ takes the form (L_3, U_3) where,

$$L_3 = \tilde{X}_{u(n)} + P_{\gamma/2}\tilde{\sigma} \qquad \text{and} \qquad U_3 = \tilde{X}_{u(n)} + P_{1-\gamma/2}\tilde{\sigma} \qquad (14)$$

3-3 Prediction interval based on conditional distribution

Using the fact that ($X_{u(n)}$ – $X_{u(m)}$) and $X_{u(m)}$ are independent(see Awad and Raqab(2000)), the conditional probability distribution of ($X_{u(n)}$ – $X_{u(m)}$) given $X_{u(m)}$ can be derived from the joint pdf of $X_{u(m)}$ and $X_{u(n)}$ and then make a transformation, $V = X_{u(n)} - X_{u(m)}$, $Y = X_{u(m)}$, and integrate out Y. We obtain the joint pdf of V and Y as follows:

$$f(v,y) = [\Gamma(n)\Gamma(n-m)]^{-1}\left[-\ln\left(1+e^{\pi(y-\mu)/\sigma\sqrt{3}}\right)^{-1}\right]^{m-1}$$

$$\text{x} \left[\ln\left(1+e^{\pi(y-\mu)/\sigma\sqrt{3}}\right)^{-1} - \ln\left(1+e^{\pi(v+y-\mu)/\sigma\sqrt{3}}\right)^{-1}\right]^{n-m-1}$$

$$\text{x} \ \frac{\pi^2}{3\sigma^2}\left(1+e^{-\pi(y-\mu)/\sigma\sqrt{3}}\right)^{-1}\frac{e^{-\pi(v+y-\mu)/\sigma\sqrt{3}}}{\left(1+e^{-\pi(v+y-\mu)/\sigma\sqrt{3}}\right)^2} \qquad (15)$$

We do not get a closed form expression for pdf of V on integrating (15). Numerical methods must be needed to obtain integration . So, of course it is more difficult to obtain the percentage points of V. Therefore we use a Monte Carlo simulation based on 10,000 runs to obtain some of these percentage points for m = 2(1)9, n = 3(1)10 as will be described in section (4).These values presented in Table 5. From Table 5, a 100(1- γ) % prediction interval for $X_{u(n)}$ is (L_4, U_4) where are

$$L_4 = X_{u(m)} + V_{\gamma/2} \quad \text{and} \quad U_4 = X_{u(m)} + V_{1-\gamma/2} \qquad (16)$$

4- Numerical comparisons

The aim of this section is to compare the performance of the MUP, and BLUP in terms of their MSEs. Monte Carlo simulations were performed to estimate MSEs of these predictors. Also, a comparison of prediction intervals based on *T*, *Q*, *P* and conditional distribution of ($X_{u(n)} - X_{u(m)}$)given $X_{u(m)}$ is made. Monte Carlo Simulations were performed to estimate the coverage probabilities and average widths of the intervals. The simulations procedure is described below:

Step 1: A random sample (X_1, X_2, …………) was generated from $L(0, 1)$ and 10 upper record values ($X_{u(1)}$, $X_{u(2)}$, … .., $X_{u(10)}$) were obtained.

Step 2: Given m = 2, … , 9, n = 3, 4, …, 10, $X_{u(m)}$ and $X_{u(n)}$ were obtained and the values of BLUP and MUP were computed.

Step 3: A 90%, 95%, and 99% prediction intervals based on *T*, *Q*, *P* and conditional distribution of ($X_{u(n)} - X_{u(m)}$)given $X_{u(m)}$ were computed.

Step 4: Steps 1, 2 and 3 were repeated 1000 times. The values of mean square prediction error of BLUP and MUP were obtained. The average interval lengths from 1000 samples were computed and the proportions of intervals containing $X_{u(n)}$ among 1000 samples were obtained as the mean interval widths and coverage probabilities respectively.

All simulations studies presented here were obtained via GAUSS 6.0 programs. Table 6 gives simulations results for the values of BULP and MUP and relative efficiency of MUP. From Table 6 we observe that the efficiency of MUP relative to BLUP is high for most values of *m* and *n* considered in this paper.

Tables 7, 8 and 9 give simulation results for the comparison between prediction interval based on *T*, *Q* and *P* and conditional distribution ($X_{u(n)} - X_{u(m)}$) given $X_{u(m)}$ at nominal coverage 1- γ = 0.90, 0.95 and 0.99, respectively. The average widths of interval depend on conditional distribution of ($X_{u(n)} - X_{u(m)}$)given $X_{u(m)}$ are shorter and the coverage probabilities are close to the nominal levels for most of the cases studied. The differences are not large among the other approaches. All these approaches deliver average widths of interval increase as the future record value far from the present one.

Table 1: Simulated Medians of

$$\frac{X_{u(n)} - \tilde{X}_{u(n)}}{\tilde{\sigma}}$$

m \ *n*	3	4	5	6	7	8	9	10
2	**2.7381**	**4.0044**	**5.0696**	**6.2194**	**7.3793**	**8.5165**	**9.9879**	**12.2029**
3		**3.6525**	**4.6737**	**5.7044**	**6.7476**	**7.8526**	**9.1234**	**11.0026**
4			**4.6689**	**5.6900**	**6.6579**	**7.7388**	**9.0170**	**10.9017**
5				**5.6985**	**6.6834**	**7.7812**	**9.0443**	**11.0138**
6					**6.7408**	**7.8005**	**9.0701**	**11.0102**
7						**7.8134**	**9.0018**	**10.8650**
8							**8.9718**	**10.7454**
9								**10.4183**

Table 2: Simulated Percentage Points of

$$T = \frac{X_{u(n)} - X_{u(m)}}{\tilde{\sigma}}$$

m	*n*	0.005	0.01	0.025	0.05	0.1	0.90	0.95	0.975	0.990	0.995
2	3	0.0053	0.0113	0.0305	0.0638	0.1314	9.1770	18.3903	37.2666	99.428	188.3279
	4	0.0771	0.1122	0.1894	0.3038	0.4800	16.6385	32.6060	67.7852	197.5602	344.9652
	5	0.2149	0.2787	0.4215	0.5760	0.8435	24.6265	48.7242	102.3916	285.0806	476.5191
	6	0.4147	0.5283	0.6962	0.8968	1.2145	31.5886	64.0613	136.9936	294.9645	524.9448
	7	0.5102	0.6482	0.8996	1.1873	1.6267	38.3045	83.0415	155.2066	406.9030	735.1114
	8	0.7427	0.9006	1.2171	1.5459	2.0335	44.7923	98.5938	204.0459	493.0511	1001.2248
	9	1.0092	1.1796	1.5178	1.9092	2.5314	54.5407	113.2789	245.2669	599.3627	1243.1385
	10	1.2985	1.5344	1.9301	2.4593	3.2024	68.5064	139.3293	277.2616	723.8805	1442.9027
3	4	0.0051	0.0092	0.0277	0.0561	0.1119	3.9979	6.3530	9.1045	15.6852	22.6053
	5	0.0894	0.1252	0.2126	0.3247	0.4880	7.0034	10.8453	15.5604	25.2264	39.1296
	6	0.2346	0.3164	0.4740	0.6360	0.9165	9.9808	14.8746	21.2912	32.5675	45.0003
	7	0.4632	0.5939	0.8169	1.0689	1.4002	13.0056	19.3486	27.6854	43.6820	61.8408
	8	0.6935	0.8522	1.1130	1.4358	1.9018	16.2616	24.3104	36.3002	57.5854	90.5054
	9	1.1055	1.2742	1.6120	1.9522	2.5086	20.0321	28.5429	42.0414	65.4158	90.2711
	10	1.4784	1.6652	201234	2.6609	3.3460	25.7582	38.2794	55.9333	94.4389	124.1539
4	5	0.0055	0.0100	0.0244	0.0541	0.1103	2.9979	4.5543	6.4269	9.5478	13.4355
	6	0.0984	0.1351	0.2124	0.3001	0.4729	5.4669	7.6902	10.4138	13.8806	18.0142
	7	0.2658	0.3639	0.5069	0.6902	0.9618	7.8007	10.9946	14.3543	20.7218	24.9681
	8	0.4887	0.6117	0.8632	1.1114	1.4947	10.2279	13.8518	19.0455	26.2877	32.2833
	9	0.8804	1.0391	1.3819	1.7245	2.1881	13.0568	17.2496	22.7849	32.2810	40.4654
	10	1.3682	1.5924	2.0025	2.4898	3.1156	17.9499	23.6801	30.8341	44.7181	58.0359
5	6	0.0044	0.0089	0.0235	0.0475	0.0994	2.7798	3.9337	5.2328	7.1367	9.1920
	7	0.0857	0.1396	0.2122	0.3185	0.4928	5.0931	6.9080	8.9259	12.1359	15.0215
	8	0.3065	0.3871	0.5593	0.7659	1.0362	7.2715	9.5144	12.0562	16.3693	20.9338
	9	0.5849	0.7212	1.0253	1.3209	1.7076	9.8857	12.7490	15.9816	22.3313	27.4790
	10	1.1165	1.3674	1.7419	2.1598	2.7456	13.7552	17.8060	22.3025	28.5115	34.2171
6	7	0.0041	0.0086	0.0224	0.0502	0.1056	2.7562	3.7637	4.8073	6.6077	8.1366
	8	6.1144	0.1608	0.2443	0.3566	0.5328	4.9727	6.5135	8.0874	10.4577	12.6995
	9	0.3502	0.4479	0.6460	0.8832	1.1999	7.3684	9.3988	11.8450	15.1784	18.2734
	10	0.7783	0.9779	1.3090	1.6802	2.1908	11.2034	14.3321	17.2914	22.1505	26.6210
7	8	0.0063	0.0125	0.0282	0.0614	0.1288	2.8398	3.9282	4.9653	6.5319	7.6530
	9	0.1216	0.1872	0.2864	0.4231	0.6208	5.2848	6.7749	8.2596	10.8267	12.8201
	10	0.4695	0.6167	0.8708	1.1096	1.5075	8.8396	11.1208	13.1659	16.5102	19.7672
8	9	0.0085	0.0176	0.0394	0.0752	0.1532	3.2134	4.1596	5.2600	6.8921	8.1012
	10	0.1668	0.2549	0.4015	0.5901	0.9002	6.8330	8.5951	10.6699	13.7756	16.1423
9	10	0.0095	0.0200	0.0452	0.0972	0.2055	4.4445	5.7598	7.2899	9.3212	10.9333

Table3: Simulated Percentage Points of
$$Q=(X_{u(n)}-\tilde{\mu})/\tilde{\sigma}$$

m	*n*	0.005	0.01	0.025	0.05	0.1	0.90	0.95	0.975	0.990	0.995
2	3	1.6502	1.6562	1.6754	1.7087	1.7764	10.8219	20.0352	38.9115	101.0477	189.9729
	4	1.7220	1.7571	1.8343	1.9487	2.1249	18.2835	34.2509	69.43016	199.2051	346.6101
	5	1.8598	1.9236	2.0664	2.2210	2.4884	26.2715	50.3691	104.0365	286.7255	478.1641
	6	2.0596	2.1732	2.3411	2.5418	2.8594	33.2335	65.7063	138.6386	296.6095	526.5898
	7	2.1552	2.2931	2.5445	2.8322	3.2716	39.9495	84.6864	156.8515	408.5480	736.7563
	8	2.3877	2.5455	2.8620	3.1909	3.6785	46.4372	100.2387	205.6908	494.6960	1002.8697
	9	2.6541	2.8246	3.1627	3.5541	4.1763	56.1856	114.9239	246.9118	601.0077	1244.7835
	10	2.9434	3.1793	3.5751	4.1042	4.8474	70.1513	140.9742	278.9065	725.5255	1444.5476
3	4	2.6364	2.6723	2.7420	2.8128	2.9191	6.8570	9.1483	11.8661	18.3376	25.7286
	5	2.7905	2.8633	3.0016	3.1276	3.3099	9.8555	13.7668	18.4156	27.8972	41.8454
	6	2.9948	3.0932	3.2559	3.4746	3.7331	12.7777	17.7076	24.1579	35.5728	47.8969
	7	3.2306	3.3673	3.6151	3.8809	4.2336	15.8995	22.1304	30.4989	46.4975	64.7527
	8	3.4641	3.6759	3.9370	4.2630	4.7125	19.1326	27.1570	39.2317	60.4325	93.1228
	9	3.8742	4.0776	4.3939	4.7669	5.3376	22.8094	31.5363	44.9797	68.1423	93.2713
	10	4.2377	4.4893	4.9611	5.4905	6.1838	28.6917	41.2676	58.8679	97.5445	126.9933
4	5	3.4857	3.5470	3.6610	3.7747	3.9317	6.9583	8.5240	10.3603	13.4517	17.5179
	6	3.7303	3.8150	3.9642	4.1105	4.3517	9.4298	11.6124	14.3277	18.0538	21.7802
	7	4.0188	4.0990	4.3221	4.5585	4.8306	11.7616	14.9970	18.2853	24.6981	29.1718
	8	4.2269	4.4057	4.6886	4.9728	5.3721	14.1450	17.7827	22.8652	30.2059	36.3026
	9	4.6247	4.8751	5.2475	5.5804	6.0503	16.9900	21.1125	26.6641	36.0796	44.5543
	10	5.1845	5.4103	5.8415	6.3386	7.0026	21.8870	27.4853	34.6477	48.3500	61.6966
5	6	4.3007	4.4007	4.5363	4.6927	4.8794	7.7463	8.9265	10.2066	12.1545	14.1277
	7	4.5747	4.6891	4.8840	5.0809	5.3393	10.0820	11.8717	13.8549	17.1181	19.8775
	8	4.9254	5.0614	5.3329	5.5656	5.8934	12.2853	14.5472	17.0168	21.3424	25.8092
	9	5.3251	5.4864	5.8111	6.1499	6.5992	14.8423	17.8152	20.8962	27.3531	32.8371
	10	5.8941	6.1415	6.6071	7.0095	7.6027	18.6978	22.7305	27.1826	33.46.79	39.3765
6	7	5.1505	5.2885	5.4624	5.6310	5.8571	8.7514	9.7495	10.7574	12.6406	14.2276
	8	5.5087	5.6448	5.8849	6.1159	6.1159	10.9810	12.4835	14.0419	16.4530	18.6915
	9	5.9484	6.1130	6.3911	6.6735	7.0527	13.3715	15.4255	17.9281	21.2139	24.4756
	10	6.5026	6.6697	7.0943	7.4924	8.0451	17.1982	20.3100	23.4149	28.0979	32.8574
7	8	6.0589	6.1972	6.4336	6.6260	6.8497	9.8697	10.9299	12.0507	13.6722	14.8408
	9	6.4449	6.6149	6.8933	7.1363	7.4499	12.2789	13.8108	15.3189	17.7242	19.9424
	10	7.0530	7.2430	7.6273	7.9496	8.3941	15.8342	18.1368	20.2120	23.4153	26.9264
8	9	6.9887	7.1211	7.3710	7.5971	7.8827	11.2688	12.2132	13.2639	14.8125	16.0571
	10	7.5163	7.7025	8.0297	8.3710	8.7564	14.8697	16.6855	18.6471	21.9647	24.2007
9	10	8.0246	8.1998	8.4793	8.7354	9.0300	13.5116	14.8742	16.3513	18.3890	19.9352

Table 4: Simulated Percentage Points of

$$P = (X_{u(n)} - \tilde{X}_{u(n)} / \tilde{\sigma}$$

m	*n*	0.005	0.01	0.025	0.05	0.1	0.90	0.95	0.975	0.990	0.995
2	3	-1.1967	-1.1908	-1.1716	-1.1382	-1.0706	7.9749	17.1882	36.0645	98.2007	187.1259
	4	-2.2073	-2.1722	-2.0950	-1.9806	-1.8044	14.3542	30.3216	65.5008	195.2758	342.6808
	5	-3.1064	-3.0426	-2.8998	-2.7453	-2.4778	21.3052	45.4028	99.0703	281.7593	473.1978
	6	-3.9240	-3.8104	-3.6424	-3.4418	-3.1241	27.2500	59.7227	132.6550	290.6259	520.6062
	7	-4.8368	-4.6988	-4.4474	-4.1597	-3.7203	32.9717	77.6945	149.8596	401.5560	729.7644
	8	-5.6083	-5.4505	-5.134	-4.8051	-4.3175	38.4412	92.2427	197.6948	486.7000	994.8737
	9	-6.3439	-6.1735	-5.8353	-5.4439	-4.8217	47.1876	105.9259	237.9138	592.0096	1235.7855
	10	-7.0556	-6.8197	-6.4239	-5.8948	-5.1516	60.1523	130.9752	268.9075	715.5266	1434.5486
3	4	-1.0790	-1.0734	-1.0532	-1.0272	-0.9704	2.9213	5.2642	8.0186	14.5953	21.5336
	5	-2.0333	-1.9943	-1.9079	-1.7947	-1.6303	4.8795	8.7212	13.4389	23.0974	37.0031
	6	-2.9059	-2.8144	-2.6650	-2.4989	-2.2227	6.8339	11.7252	18.1517	29.4409	41.8669
	7	-3.6837	-3.5567	-3.3369	-3.0809	-2.7440	8.8632	15.1881	23.5258	39.5350	57.7152
	8	-4.4477	-4.2958	-4.0342	-3.7179	-3.2498	11.1143	19.1562	31.1519	52.4364	85.3406
	9	-5.0548	-4.8856	-4.5366	-4.1992	-3.6397	13.8733	22.4006	35.9039	59.2564	84.1400
	10	-5.6891	-5.4820	-5.0301	-4.4981	-3.8067	18.5986	31.1114	48.7872	87.3050	117.0027
4	5	-1.0326	-1.0273	-1.0124	-0.9829	-0.9279	1.9606	3.5180	5.3924	8.5045	12.3966
	6	-1.9577	-1.9195	-1.8452	-1.7517	-1.5828	3.4112	5.6439	8.36211	11.8390	11.8390
	7	-2.7977	-2.6977	-2.5605	-2.3695	-2.1037	4.7418	7.9365	11.2995	17.6482	21.9135
	8	-3.5789	-3.4567	-3.2030	-2.9574	-2.5729	6.1698	9.7829	14.9683	22.2168	28.2307
	9	-4.1909	-4.0337	-3.6912	-3.3467	-2.8858	7.9873	12.1767	17.7138	27.2007	35.4107
	10	-4.6991	-4.4730	-4.0768	-3.5770	-2.9557	11.8832	17.6065	24.7705	38.6386	51.9944
5	6	-1.0132	-1.0086	-0.9948	-0.9697	-0.9187	1.7646	2.9132	4.2104	6.1188	8.1691
	7	-1.9399	-1.8909	-1.8131	-1.7065	-1.5335	3.0662	4.8800	6.8943	10.1122	12.9920
	8	-2.7205	-2.6425	-2.4691	-2.2668	-1.9945	4.2387	6.4843	9.0295	13.3447	17.9082
	9	-3.4473	-3.3118	-3.0109	-2.7118	-2.3207	5.8537	8.7194	11.9581	18.3003	23.4487
	10	-3.9168	-3.6703	-3.2900	-2.8752	-2.2894	8.7209	12.7648	17.2677	23.4821	29.1836
6	7	-1.0043	-1.0001	-0.9855	-0.9593	-0.9028	1.7463	2.7550	3.7990	5.5997	7.1288
	8	-1.9025	-1.8546	-1.7684	-1.6564	-1.4791	2.9608	4.5016	6.0772	8.4453	10.6908
	9	-2.6666	-2.5655	-2.3693	-2.1324	-1.8160	4.3529	6.3877	8.8307	12.168	15.2585
	10	-3.2393	-3.0362	-2.7092	-2.3348	-1.8255	7.1870	10.3190	13.2718	18.1360	22.6075
7	8	-0.9975	-0.9914	-0.9761	-0.9433	-0.8756	1.8361	2.9226	3.9601	5.5275	6.6489
	9	-1.8833	-1.8182	-1.7217	-1.5834	-1.3854	3.2778	4.7692	6.2526	8.8202	10.8125
	10	-2.5358	-2.3919	-2.1372	-1.8991	-1.4993	5.8344	8.1141	10.1592	13.5048	16.7593
8	9	-0.9938	-0.9843	-0.9630	-0.9269	-0.8483	2.2113	3.1582	4.2565	5.8898	7.0982
	10	-1.8372	-1.7482	-1.6008	-1.4135	-1.1019	4.8296	6.5918	8.6650	11.7726	14.1395
9	10	-0.9915	-0.9813	-0.9558	-0.9038	-0.7955	3.4438	4.7590	6.2883	8.3203	9.9322

Table 5: Simulated Percentage Points based on Conditional Dist. of $(X_{u(n)} - X_{u(m)})$ Given $X_{u(m)}$

m	*n*	0.005	0.01	0.025	0.05	0.1	0.90	0.95	0.975	0.990	0.995
2	3	0.0030	0.0062	0.0154	0.0299	0.0595	1.0987	1.4096	1.6662	2.0272	2.3220
	4	0.0523	0.0787	0.1269	0.1898	0.2776	1.6705	1.9500	2.2120	2.5405	2.8121
	5	0.1882	0.2430	0.3255	0.4199	0.5593	2.1233	2.3975	2.6459	2.9658	3.2262
	6	0.3548	0.4551	0.5846	0.7042	0.8688	2.5573	2.8509	3.1343	3.4395	3.6311
	7	0.5693	0.6829	0.8521	0.9960	1.2005	2.9717	3.2417	3.5010	3.8031	4.0702
	8	0.8309	0.9556	1.1780	1.3608	1.5701	3.4230	3.7391	4.0196	4.3488	4.5191
	9	1.1798	1.2830	1.5406	1.7528	2.0040	3.9797	4.2956	4.6004	4.9373	5.1480
	10	1.6025	1.7940	2.0761	2.2865	2.5781	4.9637	5.4120	5.8328	6.4328	6.8157
3	4	0.0027	0.0046	0.0121	0.0267	0.0523	0.9322	1.1514	1.3668	1.6631	1.8904
	5	0.0476	0.0718	0.1176	0.1652	0.2503	1.4653	1.7283	1.9543	2.2184	2.4200
	6	0.1572	0.2069	0.3874	0.5105	0.5105	1.9332	2.1865	2.4012	2.6735	2.8358
	7	0.3256	0.4037	0.5220	0.6580	0.8191	2.3735	2.6524	2.8898	3.1684	3.3606
	8	0.5760	0.6610	0.8083	0.9706	1.1572	2.8770	3.1496	3.3970	3.6657	3.8621
	9	0.9057	1.0054	1.1835	1.3582	1.5661	3.4309	3.7297	3.9806	4.2621	4.4572
	10	1.2809	1.4211	1.6574	1.8694	2.1331	4.4197	4.8542	5.2682	5.7645	6.2746
4	5	0.0022	0.0048	0.0116	0.0239	0.0485	0.8740	1.1027	1.2908	1.5414	1.6755
	6	0.0536	0.0693	0.1122	0.1567	0.2304	1.4131	1.6580	1.8726	2.1075	2.3134
	7	0.1620	0.2100	0.2891	0.3702	0.4986	1.9042	2.1495	2.3658	2.6542	2.8579
	8	0.3107	0.3980	0.5147	0.6389	0.7961	2.3860	2.6414	2.8770	3.1327	3.3079
	9	0.6123	0.7021	0.8503	1.0111	1.2021	2.9802	3.2868	3.5277	3.8748	4.0714
	10	0.9282	1.0919	1.3034	1.4837	1.7404	3.9828	4.4258	4.8438	5.4418	5.7616
5	6	0.0021	0.0043	0.0108	0.0208	0.0433	0.8605	1.0891	1.3021	1.5136	1.7182
	7	0.0428	0.0645	0.1052	0.1523	0.2262	1.4231	1.6678	1.8874	2.1357	2.3117
	8	0.1609	0.2074	0.2973	0.3812	0.5078	1.9442	2.2127	2.4432	2.7643	2.9297
	9	0.3395	0.4123	0.5488	0.6815	0.8556	2.5740	2.8671	3.1280	3.3992	3.6349
	10	0.6851	0.7794	0.9790	1.1580	1.3807	3.5776	3.9965	4.3639	4.8430	5.2737
6	7	0.0012	0.0037	0.0099	0.0209	0.0440	0.8712	1.1150	1.3137	1.5680	1.7384
	8	0.0542	0.0746	0.1173	0.1637	0.2395	1.4951	1.7557	1.9763	2.2768	2.4479
	9	0.1991	0.2353	0.3286	0.4196	0.5612	2.1217	2.4167	2.70812	3.0320	3.2391
	10	0.4256	0.5113	0.6626	0.8272	1.0459	3.1802	3.6182	3.9634	4.3962	4.8869
7	8	0.0027	0.0051	0.0127	0.0254	0.0524	0.9359	1.1810	1.4253	1.6766	1.8534
	9	0.0568	0.0826	0.1313	0.1907	0.2761	1.6720	1.9581	2.2100	2.5231	2.7641
	10	0.2455	0.2992	0.4045	0.5152	0.6930	2.7541	3.1622	3.5779	4.0983	4.4540
8	9	0.0038	0.0073	0.0166	0.0328	0.0637	1.0963	1.3578	1.6279	1.9837	2.2119
	10	0.0715	0.1125	0.1798	0.2598	0.3920	2.2968	2.7235	3.1522	3.6506	4.1099
9	10	0.0040	0.0080	0.0187	0.0430	0.0873	1.6203	2.0042	2.4342	3.0142	3.3916

Table 6: MSEs of BULP and MUP of $X_{u(n)}$ and Relative efficiency of MUP

Record *m*	*n*	MSEs of BULP	MSEs of MUP	Relative efficiency of MUP
3	**4**	**0.2523**	**0.2126**	**1.1867**
	5	**0.6555**	**0.5369**	**1.2209**
	6	**1.2217**	**1.0351**	**1.1803**
	7	**1.9442**	**1.7212**	**1.1296**
	8	**2.7850**	**2.6127**	**1.0660**
	9	**3.7205**	**3.8973**	**0.9546**
	10	**5.0708**	**6.4272**	**0.7890**
4	**5**	**0.1909**	**0.1708**	**1.1177**
	6	**0.4790**	**0.4061**	**1.1795**
	7	**0.8483**	**0.7223**	**1.1744**
	8	**1.3171**	**1.1888**	**1.1079**
	9	**1.8932**	**1.9047**	**0.9940**
	10	**2.8983**	**3.3515**	**0.8648**
5	**6**	**0.1667**	**0.1601**	**1.0412**
	7	**0.3990**	**0.3586**	**1.1127**
	8	**0.7101**	**0.6671**	**1.0645**
	9	**1.1099**	**1.1304**	**0.9819**
	10	**2.0653**	**2.1860**	**0.9448**
6	**7**	**0.1584**	**0.1601**	**0.9894**
	8	**0.3800**	**0.3695**	**1.0284**
	9	**0.6960**	**0.7019**	**0.9916**
	10	**1.6462**	**1.5230**	**1.0809**
7	**8**	**0.1719**	**0.1821**	**0.9440**
	9	**0.4450**	**0.4438**	**1.0027**
	10	**1.3546**	**1.0908**	**1.2418**
8	**9**	**0.2380**	**0.2434**	**0.9778**
	10	**0.7134**	**0.5645**	**1.2638**
9	**10**	**1.0861**	**0.8024**	**1.3536**

Relative efficiency of MUP=MSE of BLUP/MSE of MUP

Table 7: Simulation results for prediction intervals at nominal coverage

$1 - \gamma = 0.90$

m	*n*	Average width				Coverage probability			
		T	*Q*	*P*	*V*	*T*	*Q*	*P*	*V*
3	4	2.7035	2.7011	2.7201	1.1247	0.903	0.905	0.895	0.898
	5	4.7272	4.7250	4.7804	1.5631	0.897	0.896	0.898	0.920
	6	6.1434	6.1372	6.1410	1.7990	0.901	0.899	0.896	0.889
	7	7.8377	7.8331	7.8247	1.9944	0.912	0.9110	0.9090	0.917
	8	10.1584	10.1582	10.1671	2.1791	0.896	0.894	0.894	0.8850
	9	11.3385	11.3423	11.4146	2.3715	0.896	0.896	0.901	0.898
	10	15.5911	15.5872	15.6605	2.9848	0.913	0.915	0.908	0.903
4	5	1.9010	1.9013	2.0062	1.0789	0.890	0.889	0.892	0.895
	6	3.2003	3.2027	3.2487	1.5013	0.9060	0.9050	0.9010	0.888
	7	4.3794	4.3801	4.4364	1.7793	0.909	0.909	0.902	0.897
	8	5.2506	5.2506	5.2793	2.0025	0.902	0.902	0.897	0.896
	9	6.4954	6.4947	6.4983	2.2757	0.896	0.896	0.889	0.910
	10	8.8677	8.8648	8.8494	2.9421	0.900	0.900	0.904	0.917
5	6	1.5761	1.5747	1.7170	1.0683	0.918	0.916	0.900	0.918
	7	2.7471	2.7458	2.8310	1.5155	0.904	0.904	0.912	0.902
	8	3.5898	3.5908	3.6854	1.8315	0.889	0.890	0.900	0.895
	9	4.5958	4.5971	4.6912	2.1856	0.895	0.896	0.899	0.900
	10	6.3490	6.3465	6.3794	2.8385	0.899	0.898	0.895	0.897
6	7	1.4992	1.4995	1.6627	1.0941	0.921	0.922	0.926	0.915
	8	2.4653	2.4657	2.5496	1.5920	0.900	0.899	0.899	0.906
	9	3.4554	3.4573	3.5513	1.9971	0.896	0.896	0.912	0.902
	10	5.0856	5.0866	5.1524	2.7910	0.893	0.893	0.896	0.897
7	8	1.5597	1.5593	1.7360	1.1556	0.899	0.899	0.899	0.907
	9	2.5206	2.5209	2.6486	1.7674	0.895	0.895	0.906	0.903
	10	3.9947	3.9955	4.0649	2.6470	0.896	0.896	0.897	0.903
8	9	1.6279	1.6282	1.8398	1.3250	0.899	0.900	0.891	0.897
	10	3.2274	3.2275	3.3522	2.4637	0.918	0.918	0.923	0.910
9	10	2.3333	2.3333	2.5295	1.9612	0.882	0.882	0.886	0.893

Coverage probability is proportion of intervals containing $X_{u(n)}$ among 1000 samples

Table 8: Simulation results for prediction intervals at nominal coverage

$1 - \gamma = 0.95$

m	*n*	Average width				Coverage probability			
		T	*Q*	*P*	*V*	*T*	*Q*	*P*	*V*
3	4	3.9864	3.9843	4.0072	1.3547	0.947	0.944	0.942	0.956
	5	6.7023	6.7018	6.7312	1.8367	0.945	0.948	0.948	0.950
	6	9.2251	9.2249	9.2627	2.1149	0.952	0.953	0.955	0.950
	7	11.9535	11.9508	11.9602	2.3678	0.942	0.943	0.946	0.943
	8	15.0604	15.0599	15.1064	2.5887	0.963	0.961	0.957	0.965
	9	17.8752	17.8801	17.9444	2.7971	0.940	0.940	0.941	0.947
	10	23.5796	23.5829	23.6221	3.6109	0.953	0.952	0.953	0.951
4	5	2.6779	2.6789	2.8021	1.2792	0.956	0.954	0.948	0.952
	6	4.3428	4.3454	4.4119	1.7603	0.953	0.954	0.948	0.946
	7	5.8490	5.8543	5.8979	2.0767	0.950	0.952	0.954	0.951
	8	7.6385	7.6340	7.6362	2.3624	0.951	0.951	0.955	0.945
	9	8.8785	8.8793	8.8841	2.6773	0.958	0.957	0.954	0.941
	10	12.3397	12.3464	12.3288	3.5405	0.950	0.950	0.952	0.946
5	6	2.1542	2.1525	2.3448	1.2913	0.951	0.953	0.953	0.952
	7	3.6034	3.6008	3.7097	1.7822	0.957	0.957	0.968	0.958
	8	4.6741	4.6748	4.7501	2.1459	0.945	0.945	0.948	0.954
	9	6.2196	6.2250	6.2732	2.5792	0.956	0.956	0.957	0.960
	10	8.4282	8.4270	8.4343	3.3849	0.943	0.943	0.936	0.939
6	7	1.9077	1.8901	2.1111	1.3073	0.945	0.944	0.943	0.949
	8	3.1828	3.1827	3.3101	1.8590	0.942	0.942	0.946	0.938
	9	4.5811	4.5816	4.7194	2.3795	0.950	0.951	0.947	0.948
	10	6.5007	6.5002	6.6383	3.3008	0.950	0.950	0.949	0.951
7	8	2.0049	2.0045	2.2810	1.4126	0.943	0.944	0.944	0.947
	9	3.1830	3.1834	3.3636	2.079	0.955	0.956	0.950	0.953
	10	4.8949	4.8954	5.0102	3.1735	0.957	0.957	0.959	0.962
8	9	2.1298	2.1293	2.4041	1.6113	0.950	0.950	0.949	0.951
	10	4.1525	4.1515	4.2937	2.9724	0.955	0.955	0.951	0.950
9	10	2.9932	2.9930	3.2524	2.4155	0.948	0.948	0.950	0.943

Coverage probability is proportion of intervals containing $X_{u(n)}$ among 1000 samples

Table 9: Simulation results for prediction intervals at nominal coverage

1 - γ = 0.99

m	*n*	Average width				Coverage probability			
		T	***Q***	***P***	***V***	***T***	***Q***	***P***	***V***
3	**4**	**9.5869**	**9.5219**	**9.7956**	**1.8877**	**0.986**	**0.988**	**0.987**	**0.986**
	5	**16.8978**	**16.8962**	**16.9042**	**2.3723**	**0.988**	**0.988**	**0.990**	**0.994**
	6	**18.8390**	**18.8420**	**18.8964**	**2.6786**	**0.9860**	**0.9860**	**0.9850**	**0.980**
	7	**27.0661**	**27.0755**	**27.1299**	**3.0350**	**0.989**	**0.989**	**0.991**	**0.991**
	8	**40.2658**	**40.2552**	**40.1971**	**3.2861**	**0.989**	**0.988**	**0.990**	**0.986**
	9	**38.8004**	**38.8131**	**38.9011**	**3.5515**	**0.988**	**0.988**	**0.989**	**0.990**
	10	**54.9999**	**55.0072**	**55.0358**	**4.9937**	**0.993**	**0.994**	**0.994**	**0.989**
4	**5**	**5.6781**	**5.6777**	**5.9326**	**1.6733**	**0.992**	**0.994**	**0.997**	**0.985**
	6	**7.4715**	**7.4675**	**7.5274**	**2.2598**	**0.989**	**0.989**	**0.981**	**0.991**
	7	**10.6024**	**10.6062**	**10.7958**	**2.6959**	**0.987**	**0.987**	**0.984**	**0.988**
	8	**13.5332**	**13.5596**	**13.6730**	**2.9972**	**0.997**	**0.997**	**0.996**	**0.995**
	9	**16.4605**	**16.4675**	**16.6038**	**3.4591**	**0.990**	**0.990**	**0.991**	**0.989**
	10	**23.6607**	**23.6590**	**23.5832**	**4.8334**	**0.990**	**0.990**	**0.989**	**0.990**
5	**6**	**3.7373**	**3.7352**	**3.9974**	**1.7161**	**0.992**	**0.990**	**0.991**	**0.991**
	7	**6.1044**	**6.1028**	**6.2544**	**2.2688**	**0.992**	**0.991**	**0.990**	**0.993**
	8	**8.4825**	**8.4831**	**8.5879**	**2.7688**	**0.993**	**0.993**	**0.993**	**0.991**
	9	**11.0572**	**11.0580**	**11.3113**	**3.2955**	**0.996**	**0.996**	**0.994**	**0.993**
	10	**13.8069**	**13.8068**	**13.9661**	**4.5885**	**0.995**	**0.995**	**0.993**	**0.995**
6	**7**	**3.2910**	**3.2913**	**3.6733**	**1.7367**	**0.992**	**0.991**	**0.990**	**0.997**
	8	**5.0844**	**5.0877**	**5.3258**	**2.3937**	**0.988**	**0.988**	**0.990**	**0.991**
	9	**7.2620**	**7.2628**	**7.5067**	**3.0400**	**0.989**	**0.989**	**0.992**	**0.984**
	10	**10.4538**	**10.4555**	**10.6609**	**4.4613**	**0.993**	**0.993**	**0.990**	**0.990**
7	**8**	**3.0566**	**3.0565**	**3.5104**	**1.8507**	**0.981**	**0.981**	**0.983**	**0.985**
	9	**5.1726**	**5.1715**	**5.4981**	**2.7072**	**0.991**	**0.991**	**0.991**	**0.994**
	10	**7.7303**	**7.7293**	**7.9610**	**4.2084**	**0.993**	**0.993**	**0.995**	**0.986**
8	**9**	**3.2508**	**3.2506**	**3.6428**	**2.2081**	**0.989**	**0.989**	**0.991**	**0.9851**
	10	**6.4974**	**6.4979**	**6.7857**	**4.0384**	**0.990**	**0.990**	**0.992**	**0.992**
9	**10**	**4.5181**	**4.5181**	**4.9262**	**3.3876**	**0.993**	**0.993**	**0.990**	**0.993**

Coverage probability is proportion of intervals containing $X_{u(n)}$ among 1000 samples

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