

SIMULATED SAMPLING DISTRIBUTION OF PRICE INDEX NUMBERS

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Abstract

Index numbers represent change over two different situations. They give an indication of the scenario of the economic pressure related to market situations. The sampling distribution is helpful in explaining the variability of estimates due to sampling. This paper presents an examination of sampling distribution of price index numbers. Graphical plots of various price indices showing the concentration of index estimates around the expected value are included. Several different weighted aggregate price indices are discussed in detail in the form of sampling distributions using numerical support. A comparison among them is made using simulation method and some useful conclusions related to moments are also drawn.

Keywords : Index Number, Sampling Distribution, Prices, Quantity, SRSWOR, Raw Moments and Central Moments.

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1.0 Introduction:

Index numbers are indicators of the average change in a group of related variables over varying time, space or situations. They are referred to as a measure of change, a device to measure change or a series representing the process of change. Most of the time they are helpful in providing framework for decision making and to forecast future of economy. According to Croxton and Cowden (1967) Index numbers can be classified into following three categories: **(a)** Price Index Number **(b)** Quantity Index Number and **(c)** Value Index Number. More elaborative properties and applications about indices are described in Dubois (1964).

Further, Gupta et.al. (2004) , Dasgupta (2004), and Kotwal and Shinde (2005) have taken into consideration the probability distributions with their relative properties. A limitation of price indices is that their calculation, for all practical purposes, are based on single sample data. If this data varies, the index computation changes accordingly. Therefore, it is important to consider the sampling distributions of these price indices. This paper gives emphasis only on price index number with different formulae, and database sampling distribution is derived to examine the moment properties.

2.0 Sample based simulation scheme:

Following Ahmed et. al. (2006) a simulation scheme for sampling distribution of price index number is describe as under;

Step - I: Draw a random sample of size n by Simple Random Sampling without Replacement SRSWOR.

Step -II: Calculate price index numbers using different methods like (Laspayre's, Paasche's, Marshall-Edgeworth, Dorbish-Bowley's, Fisher's etc.) on each sample.

Step- III: Compare these values over a large number of samples.

Step-IV : Tabulate the value using class interval and frequencies. Convert

frequencies into probabilities by $p_i = \left(\frac{f_i}{\sum f_i} \right)$.

Step- V: Draw the curve between the class intervals and probability frequencies.

Step VI: Observe the tendency of the curve and compute moments.

3.0 Price Index Formulae Used:

As per Croxton and Cowden (1967), and Dubois (1964) different price indices are:

3.1 - Laspayre's Price Index Number:

Let P_0 be the price of the base year, P_1 be the price of the current year, Q_0 be the quantity of the base year, Q_1 be the quantity of the current year, then

$$P_{o1}^L = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

3.2 - Paasche's Price Index Number:

$$P_{o1}^P = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

3.3 - Dorbish - Bowley's Price Index Number:

$$P_{o1}^D = \frac{1}{2} \left[\frac{\sum P_1 Q_0}{\sum P_0 Q_0} + \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \right] \times 100$$

3.4 - Fisher Price Index Number:

$$P_{o1}^F = \left[\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \right] \times 100$$

4.0 Empirical Study:

The data of 15 commodities are taken and the prices and quantities of base and current year are given and respective values for different formulas of Laspayre's, Paasche's Fisher's and Dorbish and Bowley's are calculated. A sample of 97 units with 7 commodities together is drawn for which the details are provided in tables 4.1 and 4.2. The frequency distribution is prepared and the details related to it are appended in table 4.3.

Table 4.1 : List of 15 commodities are taken and the details are as under :

S. No	Prices of Base And Current Year		Quantities of Base And Current Year		Product of Prices And Quantity of Base Year	Product of Prices of Base And Quantity of Current Year	Product of Prices of Current And Quantity of Base Year	Product of Prices And Quantity of Current Year
	P_0	P_1	Q_0	Q_1	P_0Q_0	P_0Q_1	P_1Q_0	P_1Q_1
1	4	8	10	15	40	60	80	120
2	1.5	20	2	3	3	4.5	40	60
3	11	16	15	20	165	220	240	320
4	2	2.5	400	450	800	900	1000	1125
5	5	4	300	400	1500	2000	1200	1600
6	10	12	126	240	1260	2400	1512	2880
7	8	6.5	200	250	1600	2000	1300	1625
8	0.9	1.6	10	50	9	45	16	80
9	1.35	2.5	110	60	148.5	81	275	150
10	3.2	4.4	40	100	128	320	176	440
11	2.12	3.15	16	12	33.92	25.44	50.4	37.8
12	5	6	10	15	50	75	60	90
13	1.75	1.9	35	41	61.25	71.75	66.5	77.9
14	6	4	15	30	90	180	60	120
15	1.55	1.7	14	20	21.7	31	23.8	34

TABLE 4.2 – Total 97 Samples drawn each of 7 commodities from the list of 15 commodities.

Sr. No.	Samples	P_0Q_0	P_0Q_1	P_1Q_0	P_1Q_1	Laspayre's	Paasche's	Fisher's	Dorbish & Bowley's
1	1 to 7	5368	7584.5	5372	7730	100.07	101.92	100.99	101.00
2	2 to 8	5337	7569.5	5308	7690	99.46	101.59	100.52	100.52
3	3 to 9	5482.5	7646	5543	7780	101.10	101.75	101.43	101.43
4	1,2,4,7,9,11,13	2686.67	3142.69	2811.9	3195.7	104.66	101.69	103.16	103.17
5	1 to 6 and 8	3777	5629.5	4088	6185	108.23	109.87	109.05	109.05
6	1 to 6 and 9	3916.5	5665.5	4347	6255	110.99	110.41	110.70	110.70
7	1 to 6 and 10	3896	5904.5	4248	6545	109.03	110.85	109.94	109.94
8	1 to 6 and 11	3801.92	5609.94	4122.4	6142.8	108.43	109.50	108.96	108.96
9	1 to 6 and 12	3818	5659.5	4132	6195	108.22	109.46	108.84	108.84
10	1 to 6 and 13	3829.25	5656.25	4138.5	6182.9	108.08	109.31	108.69	108.69
11	1 to 6 and 14	3858	5764.5	4132	6225	107.10	107.99	107.54	107.55
12	1 to 6 and 15	3789.7	5615.5	4095.8	6139	108.08	109.32	108.70	108.70
13	2 to 7 & 9	5476.5	7605.5	5567	7760	101.65	102.03	101.84	101.84
14	2 to 7 & 10	5456	7844.5	5468	8050	100.22	102.62	101.41	101.42
15	2 to 7 & 11	5361.92	7549.94	5342.4	7647.8	99.64	101.30	100.46	100.47
16	2 to 7 & 12	5378	7599.5	5352	7700	99.52	101.32	100.42	100.42
17	2 to 7 & 13	5389.25	7596.25	5358.5	7687.9	99.43	101.21	100.31	100.32
18	2 to 7 & 14	5418	7704.5	5352	7730	98.78	100.33	99.55	99.56

19	2 to 7 & 15	5349.7	7555.5	5315.8	7644	99.37	101.17	100.26	100.27
20	3-8 &10	5462	7885	5444	8070	99.67	102.35	101.00	101.01
21	3-8,11	5367.92	7590.44	5318.4	7667.8	99.08	101.02	100.04	100.05
22	3-8,12	5384	7640	5328	7720	98.96	101.05	100.00	100.00
23	3-8,13	5395.25	7636.75	5334.5	7707.9	98.87	100.93	99.90	99.90
24	3-8,14	5424	7745	5328	7750	98.23	100.06	99.14	99.15
25	3-8,15	5355.7	7596	5291.8	7664	98.81	100.90	99.85	99.85
26	4 to 10	5445.5	7746	5479	7900	100.62	101.99	101.30	101.30
27	4 to 9,11	5351.42	7451.44	5353.4	7497.8	100.04	100.62	100.33	100.33
28	4-9,12	5367.5	7501	5363	7550	99.92	100.65	100.28	100.28
29	4-9,13	5378.75	7497.75	5369.5	7537.9	99.83	100.54	100.18	100.18
30	4-9,14	5407.5	7606	5363	7580	99.18	99.66	99.42	99.42
31	4-9,15	5339.2	7457	5326.8	7494	99.77	100.50	100.13	100.13
32	5to 11	4679.42	6871.44	4529.4	6812.8	96.79	99.15	97.96	97.97
33	5-10,12	4695.5	6921	4539	6865	96.67	99.19	97.92	97.93
34	5-10,13	4706.75	6917.75	4545.5	6852.9	96.57	99.06	97.81	97.82
35	5-10,14	4735.5	7026	4539	6895	95.85	98.14	96.99	96.99
36	5-10,15	4667.2	6877	4502.8	6809	96.48	99.01	97.74	97.74
37	6 to12	3229.42	4946.44	3389.4	5302.8	104.95	107.20	106.07	106.08
38	6 to11and 13	3240.67	4943.19	3395.9	5290.7	104.79	107.03	105.90	105.91

39	6 to 11 ,14	3269.42	5051.44	3389.4	5332.8	103.67	105.57	104.62	104.62
40	6 to 11 ,15	3201.12	4902.44	3353.2	5246.8	104.75	107.02	105.88	105.89
41	7 to 13	2030.67	2618.19	1943.9	2500.7	95.73	95.51	95.62	95.62
42	7 To 12,14	2059.42	2726.44	1937.4	2542.8	94.08	93.26	93.67	93.67
43	7 To 12, 15	1991.12	2577.44	1901.2	2456.8	95.48	95.32	95.40	95.40
44	8 to 14	520.67	798.19	703.9	995.7	105.19	104.74	109.86	109.97
45	8 to 13,15	452.37	649.19	667.7	909.7	104.60	100.13	103.82	103.86
46	9 to 15	533.37	784.19	711.7	949.7	103.43	101.11	107.12	107.27
47	1 to 4 & 6 to 8	3877	5629.5	4188	6210	108.02	110.31	109.16	109.17
48	1 to 4 & 6,7,9	4016.5	5665.5	4447	6280	110.72	110.85	110.78	110.78
49	1 to 4 & 6,7,10	3996	5904.5	4348	6570	108.81	111.27	110.03	110.04
50	1 to 4 & 6,7 11	3901.92	5609.94	4222.4	6167.8	108.21	109.94	109.08	109.08
51	1 to 4 & 6,7,12	5458.2	7732	5486.8	7854	100.52	101.58	101.05	101.05
52	1 to 4 & 6,7,13	5436.5	7701	5463	7820	100.49	101.55	101.01	101.02
53	1 to 4 & 6,7,14	3958	5764.5	4232	6250	106.92	108.42	107.67	107.67
54	1 to 4 & 6,7,15	3889.7	5615.5	4195.8	6164	107.87	109.77	108.81	108.82
55	2-5, 7,8, 9	4580.92	6865.44	4314.4	6752.8	94.18	98.36	96.25	96.27
56	2-5, 7,8, 10	4592.17	6862.19	4320.9	6740.7	94.09	98.23	96.14	96.16
57	2-5, 7,8, 11	4620.92	6970.44	4314.4	6782.8	93.37	97.31	95.32	95.34
58	2-5, 7,8, 12	4552.62	6821.44	4278.2	6696.8	93.97	98.17	96.05	96.07

59	2-5, 7,8, 13	4530.92	6790.44	4254.4	6662.8	93.90	98.12	95.99	96.01
60	2-5, 7,8, 14	4167	5349.5	3856	4930	92.54	92.16	92.35	92.35
61	2-5, 7,8, 15	4098.7	5200.5	3819.8	4844	93.20	93.14	93.17	93.17
62	3-6,8,9,10	3162.67	4698.19	3279.9	4940.7	103.71	105.16	104.43	104.43
63	3-6,8,9,11	3191.42	4806.44	3273.4	4982.8	102.57	103.67	103.12	103.12
64	3-6,8,9,12	3123.12	4657.44	3237.2	4896.8	103.65	105.14	104.39	104.40
65	3-6,8,9,13	3101.42	4626.44	3213.4	4862.8	103.61	105.11	104.36	104.36
66	3-6,8,9,14	3101.42	4626.44	3213.4	4862.8	103.61	105.11	104.36	104.36
67	3-6,8,9,15	3101.42	4626.44	3213.4	4862.8	103.61	105.11	104.36	104.36
68	3-6,9,10,11	3218.75	4852.75	3289.5	5022.9	102.20	103.51	102.85	102.85
69	3-6,9,10,12	3150.45	4703.75	3253.3	4936.9	103.26	104.96	104.11	104.11
70	3-6,9,10,13	3128.75	4672.75	3229.5	4902.9	103.22	104.93	104.07	104.07
71	3-6,9,10,14	3128.75	4672.75	3229.5	4902.9	103.22	104.93	104.07	104.07
72	3-6,9,10,15	3128.75	4672.75	3229.5	4902.9	103.22	104.93	104.07	104.07
73	3-6,10,11,12	3190.45	4808.75	3253.3	4966.9	101.97	103.29	102.63	102.63
74	3-6,10,11,13	3168.75	4777.75	3229.5	4932.9	101.92	103.25	102.58	102.58
75	3-6,10,11,14	3168.75	4777.75	3229.5	4932.9	101.92	103.25	102.58	102.58
76	3-6,10,11,15	3168.75	4777.75	3229.5	4932.9	101.92	103.25	102.58	102.58
77	3-6,8,10,11	3202.67	4803.19	3279.9	4970.7	102.41	103.49	102.95	102.95
78	3-6,8,10,12	3134.37	4654.19	3243.7	4884.7	103.49	104.95	104.22	104.22

79	3-6,8,10,13	3112.67	4623.19	3219.9	4850.7	103.44	104.92	104.18	104.18
80	3-6,8,10,14	3952	6065	4204	6565	106.38	108.24	107.31	107.31
81	3-6,8,10,15	3883.7	5916	4167.8	6479	107.32	109.52	108.41	108.42
82	4-7, 9,10,11	2086.75	2772.75	1953.5	2582.9	93.61	93.15	93.38	93.38
83	4-7, 9,10,12	2018.45	2623.75	1917.3	2496.9	94.99	95.17	95.08	95.08
84	4-7, 9,10,13	1996.75	2592.75	1893.5	2462.9	94.83	94.99	94.91	94.91
85	4-7, 9,10,14	5526.5	7881	5523	7940	99.94	100.75	100.34	100.34
86	4-7, 9,10,15	5458.2	7732	5486.8	7854	100.52	101.58	101.05	101.05
87	5-8,10,11,12	4580.92	6865.44	4314.4	6752.8	94.18	98.36	96.25	96.27
88	5-8,10,11,13	4592.17	6862.19	4320.9	6740.7	94.09	98.23	96.14	96.16
89	5-8,10,11,14	4620.92	6970.44	4314.4	6782.8	93.37	97.31	95.32	95.34
90	5-8,10,11,15	4552.62	6821.44	4278.2	6696.8	93.97	98.17	96.05	96.07
91	6-9,11,12,13	3162.67	4698.19	3279.9	4940.7	103.71	105.16	104.43	104.43
92	6-9,11,12,14	3191.42	4806.44	3273.4	4982.8	102.57	103.67	103.12	103.12
93	6-9,11,12,15	3123.12	4657.44	3237.2	4896.8	103.65	105.14	104.39	104.40
94	7-10,12,13,14	2086.75	2772.75	1953.5	2582.9	93.61	93.15	93.38	93.38
95	7-10,12,13,15	2018.45	2623.75	1917.3	2496.9	94.99	95.17	95.08	95.08
96	2,4,5,6,8,10,14	3790	5849.5	4004	6305	105.65	107.79	106.71	106.72
97	2,4,5,6,8,10,15	3721.7	5700.5	3967.8	6219	106.61	109.10	107.85	107.85

Table 4.3: Frequency and probability Distribution of price Index numbers

S. N.	Class Interval	Mid Value	Frequencies											
			Laspeyre's	Prob.= (freq) / total freq.	Cum. Freq.	Paasche's	Prob.= (freq) / total freq.	Cum. Freq.	Fisher's	Prob. = (freq) / total freq.	Cum. Freq.	Dorbish & Bowley's	Prob.= (freq) / total freq.	Cum. Freq.
1	90-92	91	0	0	0	0	0	0	0	0	0	0	0	0
2	92-94	93	6	0.0619	0.0619	8	0.0825	0.0825	7	0.072	0.072	7	0.072	0.072
3	94-96	95	8	0.0825	0.1444	2	0.0206	0.1035	5	0.051	0.123	5	0.052	0.124
4	96-98	97	7	0.0722	0.2166	3	0.0309	0.1345	5	0.051	0.175	5	0.052	0.176
5	98-100	99	10	0.1031	0.3197	8	0.0825	0.217	6	0.061	0.237	6	0.062	0.238
6	100-102	101	16	0.1649	0.4847	16	0.1649	0.382	24	0.247	0.484	24	0.247	0.484
7	102-104	103	2	0.0206	0.5057	12	0.1237	0.506	3	0.030	0.515	3	0.031	0.515
8	104-106	105	7	0.0722	0.5779	3	0.0309	0.537	3	0.030	0.546	3	0.031	0.546
9	106-108	107	12	0.1237	0.7019	4	0.0412	0.5782	6	0.061	0.608	6	0.062	0.608
10	108-110	109	21	0.2165	0.9184	23	0.2371	0.8152	29	0.299	0.907	29	0.299	0.907
11	110-112	111	8	0.0825	1.0009	18	0.1856	1.0008	9	0.092	1	9	0.093	1
12	112-114	113	0	0	1.0009	0	0	1.0008	0	0	1	0	0.093	0

5.0 Sampling Distribution of Price Index Numbers:

Table 4.3 helps us in finding out values of moments, Skewness and Kurtosis as under;

5.1 Moments Skewness and Kurtosis of Index Number:

After calculating the values we calculate the values of moments about any point, central moments, Skewness and Kurtosis for different Index number as under:

5.1.1: For Laspayre's Index Number;

The moments about any points calculated with the help of table 4.3 for Laspayre's Index Number are as under;

$$\mu'_1 = 2.144; \mu'_2 = 36.29; \mu'_3 = 169.89; \mu'_4 = 1510.27 .$$

The central moments, Skewness and Kurtosis for Laspayre's Index Number are

$$\mu_2 = 31.71; \quad \mu_3 = -43.50; \quad \mu_4 = 990.25 .$$

$$\beta_1 = 0.0593; \quad \beta_2 = 0.985; \quad \gamma_1 = 0.243; \quad \gamma_2 = -2.015$$

5.1.2: For Paasche's Index Number;

The moments about any points calculated with the help of table 4.3 for Paasche's Index Number are as under;

$$\mu'_1 = 3.28; \mu'_2 = 43.09; \mu'_3 = 269.44; \mu'_4 = 3264.16$$

The central moments, Skewness and Kurtosis for Paasche's Index Number are

$$\mu_2 = 32.33; \mu_3 = -83.98; \mu_4 = 2163.35$$

$$\beta_1 = 0.2087; \quad \beta_2 = 2.0697; \quad \gamma_1 = 0.457; \quad \gamma_2 = -0.9303$$

5.1.3: For Fisher's Ideal Index Number;

The moments about any points calculated with the help of table 4.3 for Fisher's Ideal Index Number are as under;

$$\mu'_1 = 2.66; \mu'_2 = 38.80; \mu'_3 = 209.57; \mu'_4 = 2617.57$$

The central moments skewness and Kurtosis are

$$\mu_2 = 31.724; \mu_3 = -62.414; \mu_4 = 1884.48$$

$$\beta_1 = 0.0667; \quad \beta_2 = 1.872; \quad \gamma_1 = 0.258; \quad \gamma_2 = -1.127$$

5.1.4: For Dorbish & Bowley's Index Number;

The values of the parameters are same as we obtained the values of Fisher's Ideal Index Number. $\mu'_1 = 2.66; \mu'_2 = 38.80; \mu'_3 = 209.57; \mu'_4 = 2617.57$ are the moments about any point.

The central moment, skewness and kurtosis are

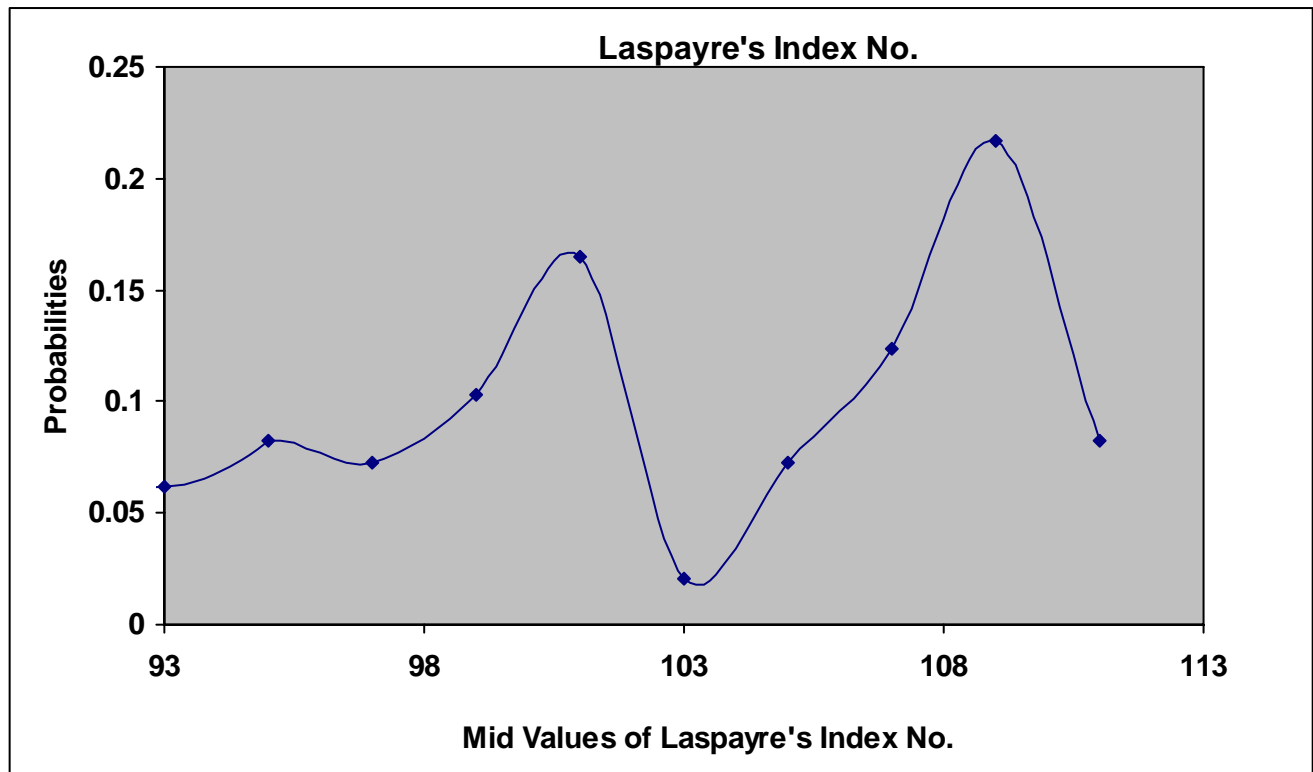
$$\mu_2 = 31.724; \mu_3 = -62.414; \mu_4 = 1884.48$$

$$\beta_1 = 0.0667; \quad \beta_2 = 1.872; \quad \gamma_1 = 0.258; \quad \gamma_2 = -1.127$$

6.0 Chart drawn between the mid value and probabilities of the class intervals

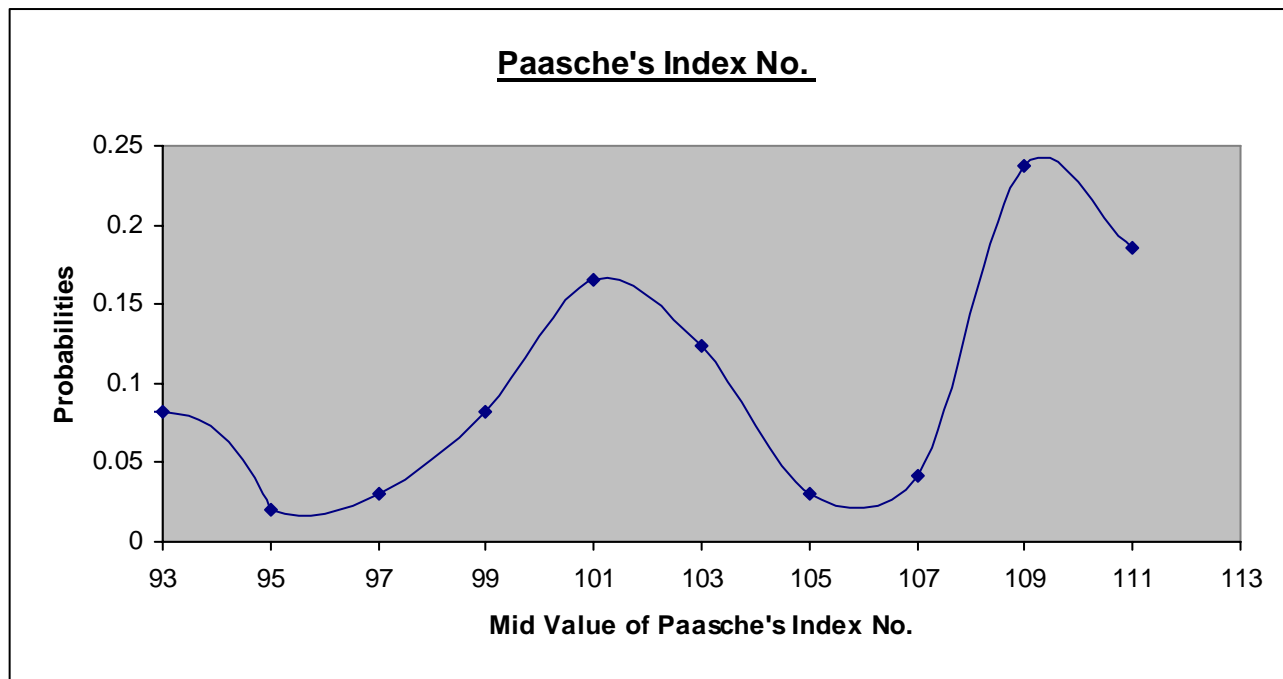
6.1.1 Laspayre's Index Number

S.No.	Mid Value	Laspeyre's	Probability
1	93	6	0.0619
2	95	8	0.0825
3	97	7	0.0722
4	99	10	0.1031
5	101	16	0.1649
6	103	2	0.0206
7	105	7	0.0722
8	107	12	0.1237
9	109	21	0.2165
10	111	8	0.0825



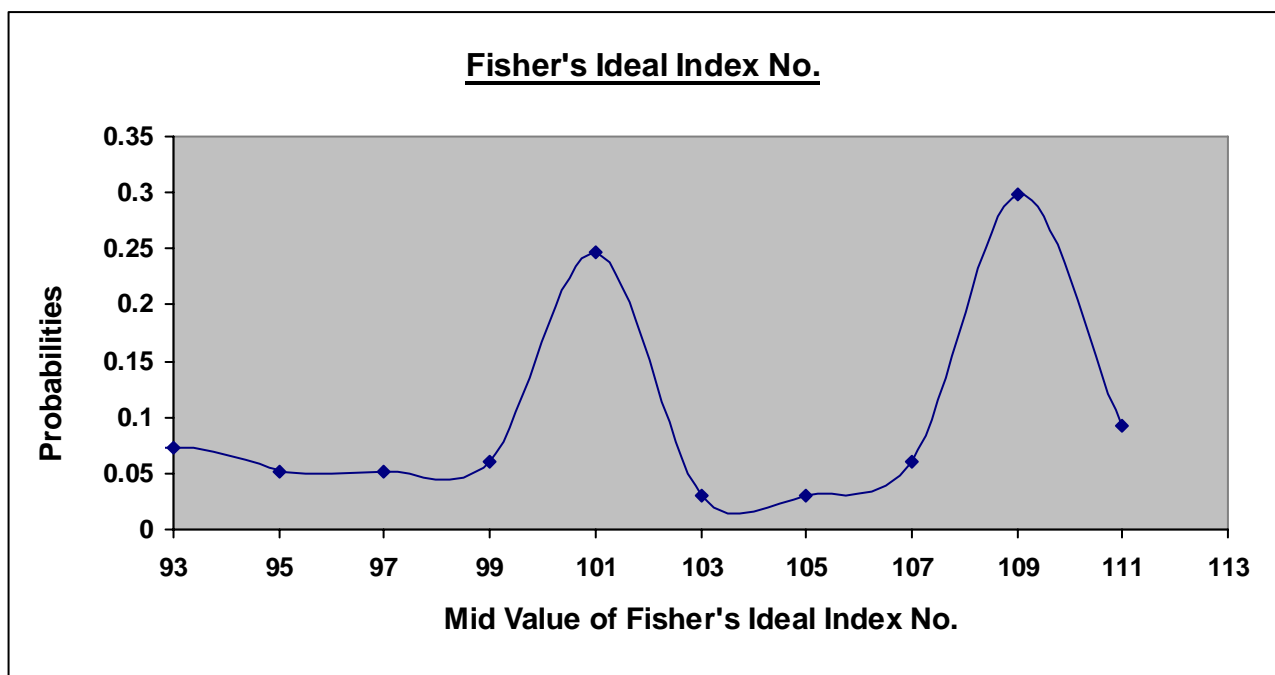
6.1.2 Paasche's Index Number

S.No.	Mid Value	Paasche's	Probability
1	93	8	0.0825
2	95	2	0.0206
3	97	3	0.0309
4	99	8	0.0825
5	101	16	0.1649
6	103	12	0.1237
7	105	3	0.0309
8	107	4	0.0412
9	109	23	0.2371
10	111	18	0.1856



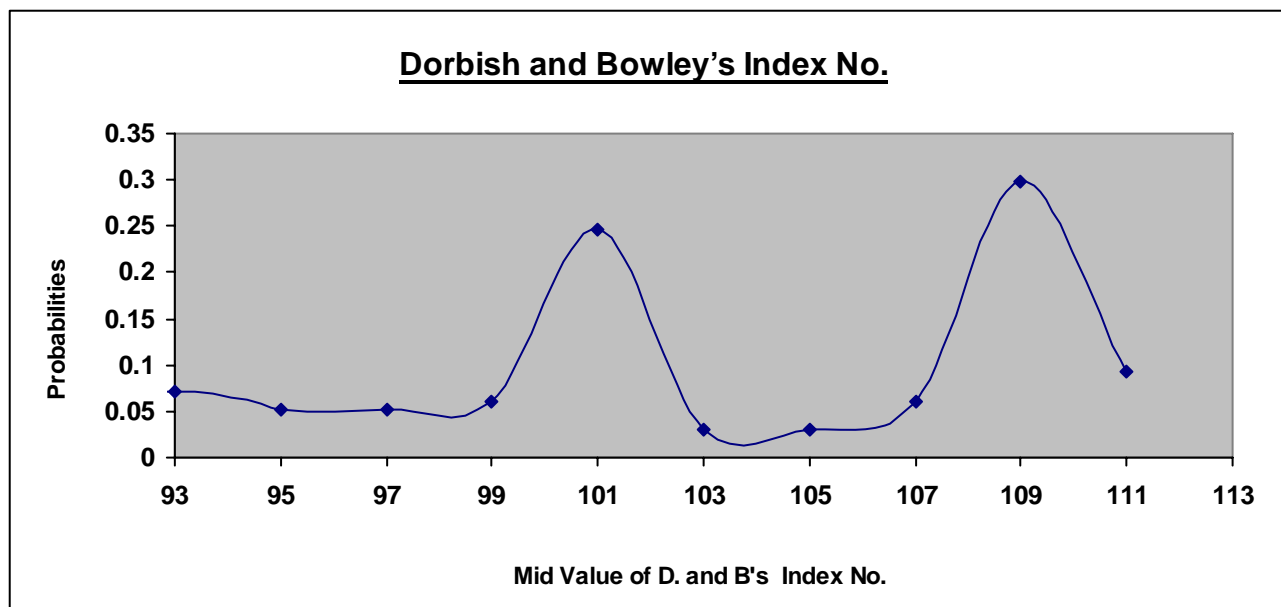
6.1.3 Fisher's Index Number

S.No	Mid Value	Fisher's	Probability
1	93	7	0.072
2	95	5	0.051
3	97	5	0.051
4	99	6	0.061
5	101	24	0.247
6	103	3	0.030
7	105	3	0.030
8	107	6	0.061
9	109	29	0.299
10	111	9	0.092



6.1.4 Dorbish and Bowley's Index Number

S.No.	Mid Value	Dorbish Bowley's	Probability
1	93	7	0.072
2	95	5	0.051
3	97	5	0.051
4	99	6	0.061
5	101	24	0.247
6	103	3	0.030
7	105	3	0.030
8	107	6	0.061
9	109	29	0.299
10	111	9	0.092



7.0 Concluding Remark:

With the help of table given in section 4.3 the moment about any point, central moments, skewness and kurtosis are calculated. The mean of Laspayre's Index number is 2.144 and variance is 31.71 whereas in Paasche's Index number the average value comes to 3.28 and variance is 32.33. The Fisher's Ideal Index number and Dorbish and Bowley's Index number gives the value of mean equal to 2.66 with the variability of 31.724. From the figure obtained we can conclude that the variability in Laspayre's Index number is minimum as compared to other Index number and is supposed to be more consistent. And the mean value of Paasche's Index number is more than the other Index number under consideration.

The value of skewness of all the Index number is calculated which provides the lack of symmetry in the curve and the nature of the curve is negatively skewed. The same thing was observed in the kurtosis and result into platy-kurtic giving value less than 3.

The numerical illustration is also supported by finding out the probabilities and with the curve drawn between the mid value of the class interval and probabilities. The explanation is discussed in section 5.0.

By observing the solution of the problem one can opt any Index number among the Fisher's Ideal Index number and Dorbish and Bowley's Index number because both provides the same values for the parameter.

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