Generalized Quantitative Randomized Response Model

Zawar Hussain and Javid Shabbir

Department of Statistics, Quaid-i-Azam University, 45320, Islamabad, 44000 Pakistan

Summary

In this study we proposed a generalized extension of Ryu et al. (2005) randomized response model (RRM) to estimate the mean of a sensitive quantitative variable. It has been shown that the proposed estimator has advantages over the estimators of Ryu et al. (2005) in terms of efficiency as well as the privacy protection. We also studied the performance of proposed structure in stratified random sampling protocol and it has been observed that results are better than in the case of the Ryu et al. (2005) stratified mean estimator. Completely truthful responses are assumed.

Key Words: Randomized response technique; Estimation of mean of sensitive character; Stratified random sampling.

1. Introduction

Greenberg et al. (1971) extended the Warner's (1965) idea of randomized response models to the estimation of mean of sensitive quantitative variables. Since then, a large number of randomized response methods have been developed to estimate the mean of quantitative variables. Randomized response models are used to decrease the evasive answer bias and to provide privacy protection to the respondents in order to increase the response rate. Some of the methods allowing for scrambling of actual responses are Eichhorn and Hayre (1983), Gupta et al. (2002) and Ryu et al. (2005) method. All these methods have some limitations in one sense or the other. For example, the disadvantage of the Eichhorn and Hayre (1983) method is the provision of a single possibility for reporting the response. Similarly, Gupta et al. (2002) method provides two possibilities for reporting the response; a particular respondent can either report actual response 'A' to sensitive question or the "scrambled" response 'AB,' where "B" is a random number with known mean and variance. Ryu et al. (2005) proposed a simple extension of Gupta et al. (2002) model in two stages. Borrowing the idea from Ryu et al. (2005) RRM we present a general model that is more efficient than those of Greenberg et al.'s (1971), Gupta et al.'s (2002) and Ryu et al. (2005) RRMs under the assumption of completely truthful reporting. We also studied stratified random sampling with the proposed model. As we intend to compare the proposed RRM to Ryu et al. (2005) RRM, we give a short review of Ryu et al. (2005) RRM in Section 2, followed by our proposed randomized response model in Section 3. In Section 4 we made the efficiency comparison. Section 5 is devoted to the application of both the Ryu et al. (2005) RRM and the proposed RRM in stratified random sampling.

2. Ryu et al. RRM

Extending the idea of Gupta et al. (2002) and using the Mangat and Singh (1990) RRM, Ryu et al. (2005) proposed a two stage RRM. A simple random sample with replacement of size n is taken. The i^{th} respondent selected in the sample is requested to use the randomization device R_1 which consists of two statements: (i) "Report the true response A of sensitive question" and (ii) "Go to randomization device R_2 in the second stage" represented with probabilities Q_1 and $1-Q_1$. The randomization device R_2 consists of two statements: (i) "Report the true response A of sensitive question" and (ii) "Report the scrambled response AB of sensitive question" represented by probabilities Q_2 and $1-Q_2$ respectively. Using the assumption of known distribution of scrambling variable B such that $\mu_B = 1$ and $\sigma_B^2 = \psi^2$, the response of i^{th} respondent can be written as

$$U_i = \alpha A_i + (1-\alpha)\left[\beta A_i + (1-\beta) A_i B_i\right], \tag{2.1}$$

where $\alpha = 1$, if a respondent is randomly assigned to the statement (i) in R_1, and $\alpha = 0$, if a respondent is randomly assigned to statement (ii) in R_1. Further, $\beta = 1$, if a respondent is randomly assigned to statement (i) in R_2, and $\beta = 0$, if a respondent is randomly assigned statement (ii) in R_2. The expected value of the observed response is

$$E(U_i) = E\left\{\alpha A_i + (1-\alpha)\left[\beta A_i + (1-\beta) A_i B_i\right]\right\}$$

$$E(U_i) = E(\alpha) E(A_i) + E(1-\alpha)\left[E(\beta) E(A_i) + E(1-\beta) E(A_i B_i)\right]$$

$$= Q_1 \mu_A + (1-Q_1)\left\{Q_2 \mu_A + (1-Q_2)\mu_A \mu_B\right\} = \mu_A, \tag{2.2}$$

where α and β are Bernoulli random variables with means Q_1 and Q_2, and variances $Q_1(1-Q_1)$, $Q_2(1-Q_2)$ respectively. Ryu et al (2005) suggested an unbiased estimator of the mean μ_A as

$$\hat{\mu}_R = \frac{1}{n}\sum_{i=1}^{n} U_i . \tag{2.3}$$

Its variance is given by

$$Var(\hat{\mu}_R) = \frac{1}{n}\left\{\sigma_A^2 + \left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)\psi^2\right\}. \tag{2.4}$$

3. Proposed Model

Proposed Model is based on the idea of distributing the probability of reporting on 'A' into $k(>2)$ stages. In the proposed model a simple random sample of size n is drawn with replacement from the finite population of interest. An individual respondent in the sample is instructed to use the randomization device R_1 which consists of two statements: (i) "Report the true response A of a sensitive question" and (ii)" Go to randomization device R_2 in the second stage" represented with the probabilities Q_1 and $1-Q_1$. The randomization device R_2 consists of two statements: (i) "Report the true response A of sensitive question" and (ii) "Go to randomization device R_3 in the third stage" represented with probabilities Q_2 and $1-Q_2$, and so on up to k^{th} stage to use R_k containing two statements : (i) "Report the true response A of sensitive question" and (ii) "Report the scrambled response AB of a sensitive question" represented with probabilities Q_k and $1-Q_k$. In order to protect his/her privacy the respondents are requested to keep the number of particular randomization device have used as a secret to an interviewer.

Let S_i be the response of the i^{th} respondent then it can be shown that

$$E\left(S_i\right)=\mu_A, \tag{3.1}$$

and variance of S_i is given by

$$Var\left(S_i\right)=\sigma_A^2+\left(\mu_A^2+\sigma_A^2\right)\left(1-Q_1\right)\left(1-Q_2\right)\ldots\left(1-Q_k\right)\psi^2. \tag{3.2}$$

The estimator of μ_A based on proposed model is simply the sample mean ($\bar{S}$) of the reported values with variance

$$Var\left(\bar{S}\right)=\frac{\sigma_A^2}{n}+\frac{1}{n}\left(\mu_A^2+\sigma_A^2\right)\left(1-Q_1\right)\left(1-Q_2\right)\ldots\left(1-Q_k\right)\psi^2. \tag{3.3}$$

Actually, in proposed model we distributed the total probability of reporting on 'A' into 'k' stages. By distributing the probability of reporting on 'A' into 'k' stages we perceived the following advantages: (i) inability of a clever respondent to correctly guess the total probability on reporting 'A' and may feel more secure to disclose his/her actual response 'A'. (ii) provision of more protection against the privacy of the respondents, and therefore making the interviewer unable to know at which stage respondents actually reported his response. (iii) there are more $(k-1)$ degrees of freedom to set the values for design parameters $Q_1, Q_2, ..., Q_k$ in order to keep the total probability of reporting on A at some desired level, where total probability of reporting on A is given by

$$P_r(A) = Q_1 + (1-Q_1)Q_2 + (1-Q_1)(1-Q_2)Q_3 + ... + \{(1-Q_1)(1-Q_2)...(1-Q_k)Q_k\}. \qquad (3.4)$$

For $k=1$, proposed model is essentially the Gupta et al. (2002) RRM and for $k=2$, it reduces to Ryu et. (2005) RRM. For $k=2$ results are given in Section 2. In the lines to follow we illustrate the working of proposed RRM for $k=3$. Following these lines we can easily derive the generalize results given by (3.1), (3.2) and (3.4).

For the purpose of illustration of the idea suppose we have three different transparent boxes(B_1, B_2, B_3) containing green and red beads with Q_1, Q_2, Q_3 proportions of red beads respectively. A respondent is requested to draw a bead at random from the box 'B_1'. If a red bead is drawn, he/she is requested to report on 'A', otherwise go to second box 'B_2'. At this stage , again, he/she is requested to randomly draw a bead from the box 'B_2', and report on 'A' if red bead is drawn, otherwise go to third box 'B_3', and randomly draw a bead from the third box. Then report on 'A' if a red bead is drawn otherwise report 'AB'. Now total probability of reporting on 'A' is

$$P_r(A) = P_r(B_1)P_r(A/B_1) + P_r(B_2)P_r(A/B_2) + P_r(B_3)P_r(A/B_3)$$

$$= 1(Q_1) + (1-Q_1)Q_2 + (1-Q_1)(1-Q_2)Q_3. \qquad (3.5)$$

If there is a single box then it would be

$$P_r(A) = P_r(B_1)P_r(A/B_1) = 1(Q). \tag{3.6}$$

It may be harder for an ordinary respondent to guess $P_r(A)$ in (3.5) which is a sum of three terms, than guessing a single term in (3.6). Now suppose we want to set a total probability of reporting on 'A', $P_r(A) = 0.784$. If we are using a single stage RRM we have to set Q_1 equal to 0.784 (i.e. 784 red beads out of a 1000 balls). It may be easier for an ordinary respondent to guess that probability of reporting on 'A' is apparently large and he may refuse to respond at all. On the other hand if we use three stages, we may distribute $P_r(A)$ into three boxes by setting $Q_1 = 0.4$, $Q_2 = 0.4$, and $Q_3 = 0.4$ to have a total probability of reporting on 'A' equal to 0.784.

By using the assumption that both A and B are positive valued random variables, and that $\mu_B = 1$ and $\sigma_B^2 = \psi^2$, we now give the results for $k = 3$. The i^{th} respondent selected in the sample of size n, drawn by using simple random sampling with replacement (SRSWR), is requested to report the value

$$S_i = \alpha A_i + (1-\alpha)\left[\beta A_i + (1-\beta)\left\{\gamma A_i + (1-\gamma)A_i B_i\right\}\right], \tag{3.7}$$

where $\alpha = 1$, if a respondent is randomly assigned to statement (i) in R_1, and $\alpha = 0$, if assigned to statement (ii) in R_1. Also, $\beta = 1$, if a respondent is assigned to statement (i) in R_2, and $\beta = 0$, if assigned to statement (ii) in R_2. Finally $\gamma = 1$, if statement (i) in R_3, and $\gamma = 0$, if statement (ii) in R_3. The expected value of the observed response is

$$E(S_i) = Q_1\mu_A + (1-Q_1)\left[Q_2\mu_A + (1-Q_2)\left\{Q_3\mu_A + (1-Q_3)\mu_A\mu_B\right\}\right] = \mu_A, \tag{3.8}$$

where α is Bernoulli random variable with mean Q_1 and variance $Q_1(1-Q_1)$, β is a Bernoulli random variable with $E(\beta)=Q_2$ and $Var(\beta)=Q_2(1-Q_2)$, and γ is a Bernoulli random variable with $E(\gamma)=Q_3$ and $Var(\gamma)=Q_3(1-Q_3)$.

Theorom 3.1. An unbiased estimator of the population mean μ_A is given by

$$\hat{\mu}_Z = \frac{1}{n}\sum_{i=1}^{n} S_i \ . \tag{3.9}$$

Theorem 3.2.. The variance of the proposed estimator μ_A is given by

$$Var(\hat{\mu}_Z) = \frac{1}{n}\left\{\left(\mu_A^2+\sigma_A^2\right)\left[1+(1-Q_1)(1-Q_2)(1-Q_3)\psi^2\right]-\mu_A^2\right\}. \tag{3.10}$$

Proof. Since

$$E\left(S_i^2\right)=E\left(\alpha^2\right)E\left(A_i^2\right)+E(1-\alpha)^2\,E\left[\beta A_i+(1-\beta)\left\{\gamma A_i+(1-\gamma)A_iB_i\right\}\right]^2$$

$$=E\left(\alpha^2\right)E\left(A^2\right)$$
$$+E(1-\alpha)^2\left[E\left(\beta^2\right)E\left(A^2\right)+E(1-\beta)^2\left\{E\left(\gamma^2\right)E\left(A^2\right)+E(1-\gamma)^2\,E\left(AB\right)^2\right\}\right]$$

$$=Q_1\left(\mu_A^2+\sigma_A^2\right)+(1-Q_1)\left[Q_2\left(\mu_A^2+\sigma_A^2\right)+(1-Q_2)\left\{Q_3\left(\mu_A^2+\sigma_A^2\right)+(1-Q_3)\left(\mu_A^2+\sigma_A^2\right)\left(\mu_B^2+\sigma_B^2\right)\right\}\right]$$
$$=\left(\mu_A^2+\sigma_A^2\right)\left\{(1-Q_1)(1-Q_2)(1-Q_3)\psi^2\right\}.$$

Thus variance of the estimator $\hat{\mu}_Z$ is given by

$$Var(\hat{\mu}_Z)=\frac{\sigma_Z^2}{n}=\frac{1}{n}\left[E\left(Z_i^2\right)-\left(E\left(Z_i\right)\right)^2\right]$$

$$=\frac{1}{n}\left[\sigma_A^2+\left(\mu_A^2+\sigma_A^2\right)(1-Q_1)(1-Q_2)(1-Q_3)\psi^2\right].$$

4. Efficiency Comparison

If $Q_3 = 0$, the proposed RRM reduces to Ryu et al. (2005) RRM and in this case variance of $\hat{\mu}_Z$ is given by (2.4).

So the relative efficiency (*RE*) of $\hat{\mu}_Z$ with respect to $\hat{\mu}_R$ is

$$RE_1 = \frac{Var(\hat{\mu}_R)}{Var(\hat{\mu}_Z)} = \frac{\left[\sigma_A^2 + \left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)\psi^2\right]}{\left[\sigma_A^2 + \left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)\psi^2\right]} = 1.$$

And if $Q_3 = 1$, then the variance of $\hat{\mu}_Z$ is

$$Var(\hat{\mu}_Z) = \frac{\sigma_A^2}{n}.$$

The relative efficiency of $\hat{\mu}_Z$ with respect to $\hat{\mu}_R$ is given by

$$RE_1 = \frac{Var(\hat{\mu}_R)}{Var(\hat{\mu}_Z)} = \frac{\left[\sigma_A^2 + \left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)\psi^2\right]}{\sigma_A^2} \geq 1.$$

If $0 < Q_3 < 1$ then in order to compute the relative efficiency of $\hat{\mu}_Z$ with respect to $\hat{\mu}_R$, we compare $Var(\hat{\mu}_Z)$ and $Var(\hat{\mu}_R)$ as follows:

$$\begin{aligned} Var(\hat{\mu}_R) - Var(\hat{\mu}_Z) &= \frac{1}{n}\left[\left(\mu_A^2 + \sigma_A^2\right)\left\{1 + (1-Q_1)(1-Q_2)\left(1+\psi^2\right)\right\} - \mu_A^2\right] \\ &\quad - \frac{1}{n}\left\{\sigma_A^2 + \left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)(1-Q_3)\left(1+\psi^2\right)\right\} \\ &= \frac{1}{n}\left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)Q_3\psi^2 \geq 0. \qquad (4.1) \end{aligned}$$

For general case of k stages we can see that

$$\begin{aligned} Var(\hat{\mu}_R) - Var(\hat{\mu}_Z) &= \frac{1}{n}\left[\left(\mu_A^2 + \sigma_A^2\right)\left\{1 + (1-Q_2)(1-Q_3)\left(1+\psi^2\right)\right\} - \mu_A^2\right] \\ &\quad - \frac{1}{n}\left\{\sigma_A^2 + \left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)\ldots(1-Q_k)\psi^2\right\} \\ &= \frac{1}{n}\left(\mu_A^2 + \sigma_A^2\right)(1-Q_1)(1-Q_2)\left\{1 - (1-Q_3)(1-Q_4)\ldots(1-Q_k)\right\}\psi^2 \geq 0. \qquad (4.2) \end{aligned}$$

Based on these results, we conclude that the proposed mean estimator is more efficient than Ryu et al (2005) mean estimator $\hat{\mu}_R$.

5. Proposed RRM When Stratification

Suppose the population is partitioned into H strata, and a sample is selected by simple random sampling with replacement from each stratum. Using the results in section 3 we can show that for h^{th} stratum the estimator of μ_{Ah} is given by

$$\hat{\mu}_{Z_h} = \frac{1}{n_h}\sum_{i=1}^{n_h} S_{hi} . \tag{5.1}$$

Its variance is given by

$$Var\left(\hat{\mu}_{Z_h}\right) = \frac{1}{n_h}\left[\sigma_{A_h}^2 + \left(\mu_{A_h}^2 + \sigma_{A_h}^2\right)\left(1-Q_{1h}\right)\left(1-Q_{2h}\right)...\left(1-Q_{kh}\right)\psi_h^2\right]. \tag{5.2}$$

Since the selections in different strata are made independently, the mean estimators for individual strata can be added together to obtain a mean estimator for the whole population. The mean estimator of μ_A is:

$$\hat{\mu}_{ZA} = \sum_{h=1}^{H} W_h \hat{\mu}_{A_h} = \sum_{h=1}^{H} W_h \left[\frac{1}{n_h}\sum_{i=1}^{n_h} S_{hi}\right] = \sum_{h=1}^{H} \frac{W_h}{n_h}\left[\sum_{i=1}^{n_h} S_{hi}\right], \tag{5.3}$$

where N is the number of units in the whole population, N_h is the total number of units in stratum h, and $W_h = \frac{N_h}{N}$ for $h = 1, 2, ..., k$, so that $\sum_{h=1}^{k} W_h = 1$.

Theorem 4.1 The proposed mean estimator $\hat{\mu}_{ZA}$ is an unbiased estimate for the population mean μ_A .

Proof. As each mean estimator $\hat{\mu}_{Z_h}$ is unbiased for μ_{A_h} , the expected value of $\hat{\mu}_{ZA}$ is:

$$E\left(\hat{\mu}_{ZA}\right)=E\left(\sum_{h=1}^{H}W_h\hat{\mu}_{Z_h}\right)=\sum_{h=1}^{H}W_hE\left(\hat{\mu}_{Z_h}\right)=\sum_{h=1}^{k}W_h\mu_{A_h}=\mu_A\,.$$

This proves the theorem.

Theorem 4.2. The variance of the mean estimator $\hat{\mu}_{ZA}$ is:

$$Var\left(\hat{\mu}_{ZA}\right)=\sum_{h=1}^{H}\frac{W_h^2}{n_h}\left[\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})...(1-Q_{kh})\psi_h^2\right]. \tag{5.4}$$

Proof. Since each unbiased mean estimator $\hat{\mu}_{Z_h}$ has its own variance, the variance of $\hat{\mu}_{ZA}$ is given by

$$Var\left(\hat{\mu}_{ZA}\right)=Var\left(\sum_{h=1}^{H}W_h\hat{\mu}_{Z_h}\right)=\sum_{h=1}^{H}W_h^2Var\left(\hat{\mu}_{Z_h}\right)$$

$$=\sum_{h=1}^{H}\frac{w_h^2}{n_h}\left[\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})...(1-Q_{kh})\psi_h^2\right].$$

This proves the theorem.

Theorem 4.3 The optimal allocation of n to $n_1,n_2,...,n_{k-1}$ and n_k to derive the minimum variance of $\hat{\mu}_{ZA}$ subject to $n=\sum_{h=1}^{H}n_h$ is approximately given by:

$$\frac{n_h}{n}=\frac{W_h\left\{\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})...(1-Q_{kh})\psi_h^2\right\}^{1/2}}{\sum_{h=1}^{H}W_h\left\{\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})...(1-Q_{kh})\psi_h^2\right\}^{1/2}}\,. \tag{5.5}$$

Proof. Using the result in Section 5.5 of Cochran (1977) for the minimum variance for fixed total sample size we have

$$n_h\propto N_h\left\{\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})...(1-Q_{kh})\psi_h^2\right\}^{1/2}.$$

Thus

$$\frac{n_h}{n}=\frac{W_h\left\{\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})\ldots(1-Q_{kh})\psi_h^2\right\}^{1/2}}{\sum_{h=1}^{H}W_h\left\{\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})\ldots(1-Q_{kh})\psi_h^2\right\}^{1/2}}.$$

So the proportion of the total sample size which is allocated to each sample is given by (5.5).

Theorem4.4. The minimal variance of the estimator $\hat{\mu}_{ZA}$ is given by:

$$\frac{1}{n}\left[\sum_{h=1}^{H}W_h\left\{\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})\ldots(1-Q_{kh})\psi_h^2\right\}^{1/2}\right]^2. \qquad (5.6)$$

Proof. By using (5.5) in (5.4) we get (5.6).

Application of Ryu at al. (2005) model in stratified sampling with fixed total sample size and optimum allocation of sample sizes in different strata yields the following mean estimator, as also suggested by Ryu at al (2005)

$$\hat{\mu}_{RA}=\sum_{h=1}^{H}W_h\hat{\mu}_{R_h}, \qquad (5.7)$$

with minimal variance

$$Var\left(\hat{\mu}_{RA}\right)=\frac{1}{n}\left[\sum_{h=1}^{H}W_h\left\{\sigma_{A_h}^2+\left(\mu_{A_h}^2+\sigma_{A_h}^2\right)(1-Q_{1h})(1-Q_{2h})\psi_h^2\right\}^{1/2}\right]^2. \qquad (5.8)$$

Our proposed stratified mean estimator is more efficient than the Ryu at al (2005) stratified mean estimator if

$$Var\left(\hat{\mu}_{RA}\right)-Var\left(\hat{\mu}_{ZA}\right)\geq 0.$$

That is

$$\frac{1}{n}\left[\sum_{h=1}^{H} W_h \left\{\sigma_{A_h}^2 + \left(\mu_{A_h}^2 + \sigma_{A_h}^2\right)\left(1-Q_{1h}\right)\left(1-Q_{2h}\right)\psi_h^2\right\}^{1/2}\right]^2$$

$$-\frac{1}{n}\left[\sum_{h=1}^{H} W_h \left\{\sigma_{A_h}^2 + \left(\mu_{A_h}^2 + \sigma_{A_h}^2\right)\left(1-Q_{1h}\right)\left(1-Q_{2h}\right)\ldots\left(1-Q_{kh}\right)\psi_h^2\right\}^{1/2}\right]^2 \geq 0$$

or

$$\left[\sum_{h=1}^{H} W_h \left\{\sigma_{A_h}^2 + \left(\mu_{A_h}^2 + \sigma_{A_h}^2\right)\left(1-Q_{1h}\right)\left(1-Q_{2h}\right)\psi_h^2\right\}^{1/2}\right]^2$$

$$-\left[\sum_{h=1}^{H} W_h \left\{\sigma_{A_h}^2 + \left(\mu_{A_h}^2 + \sigma_{A_h}^2\right)\left(1-Q_{1h}\right)\left(1-Q_{2h}\right)\ldots\left(1-Q_{kh}\right)\psi_h^2\right\}^{1/2}\right]^2 \geq 0\,. \tag{5.9}$$

If for each stratum $Var\left(\hat{\mu}_{Z_h}\right) \leq Var\left(\hat{\mu}_{R_h}\right)$, then above inequality is always true. Using (4.2) for each stratum, we can see that $Var\left(\hat{\mu}_{Z_h}\right) \leq Var\left(\hat{\mu}_{R_h}\right)$. Thus (5.9) is always true. Hence the proposed stratified mean estimator is more efficient than the Ryu et al. (2005) RRM. Ryu et al. (2205) has shown that their estimator is efficient than Greenberg et al. (1971) RRM, Gupta et al. (2002) RRM. Therefore, we can safely conclude that our proposed estimator is more efficient than the estimators proposed by Greenberg et al. (1971) and Gupta et al. (2002).

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