Estimation and Optimal Design in Step-Stress Partially Accelerated Life Test Plans for Pareto Distribution of the Second Kind with Type-II Censoring

by

Abdalla A. Abdel-Ghaly[1] Eman H. El-Khodary[2] Ali A. Ismail [3]

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ABSTRACT

This paper considers simple time-step-stress Partially Accelerated Life Testing (PALT). The lifetimes of test items are assumed to follow a two-parameter Pareto lifetime distribution of the second kind. The experiment is subject to type-II censoring. Maximum likelihood estimates (MLE) of PALT model parameters are obtained. Also, confidence interval estimation of the parameters is presented. Moreover, optimum plans for simple time-step-stress PALT are developed. Such plans minimize the generalized asymptotic variance (GAV) of the MLE of the model parameters. For illustration, numerical examples are presented.

1. Professor of Statistics, Department of Statistics, Faculty of Economics & Political Science, Cairo University, Orman, Giza, Egypt.
2. Associate Professor of Statistics, Department of Statistics, Faculty of Economics & Political Science, Cairo University, Orman, Giza, Egypt.
3. Assistant Professor of Statistics, Department of Statistics, Faculty of Economics & Political Science, Cairo University, Orman, Giza, Egypt.

1. INTRODUCTION

Testing the lifetime of some products or materials under normal usage often requires long periods of time. In order to shorten the testing period, all or some of test units may be subjected to conditions more severe than normal. Such accelerated life testing (ALT) or partially accelerated life testing (PALT) results in shorter lives than would be observed under normal conditions. In ALT, test items are run only at accelerated conditions, while in PALT they are run at both normal and accelerated conditions. According to Nelson (1990), the stress can be applied in various ways. One way to accelerate failure is step-stress, which increases the stress applied to test product in a specified discrete sequence. This paper considers simple time-step-stress PALT which uses two test-conditions. Under step-stress approach a test item is first run at use condition and, if it does not fail for a specified time τ, then it is run at accelerated condition until failure occurs or the observation is censored. The aim of such experiment is to collect more failure data in a limited time without necessarily using a high stress to all test units. As Bhattacharyya and Soejoeti (1989) indicated, step-stress PALT is practical for many problems of life testing where the test process requires a long time if the test is simply carried out under the use condition.

For an overview of such time-step-stress PALT with type-I censoring, see Abdel-Ghaly et al (2002). Under type-II censoring, Abdel-Ghani (1998) considered only the estimation problem of the Weibull distribution parameters and the acceleration factor. This paper will investigate designing step-stress PALT using type-II censoring under the Pareto distribution of the second kind.

The outline of the paper is as follows. Besides this introductory section, the paper includes six sections. In Section 2, we introduce a two-parameter Pareto distribution as a lifetime model. Section 3 deals with the derivation of the maximum likelihood estimators of the acceleration factor and the parameters of Pareto distribution. The confidence limits of the parameters are presented in Section 4. Section 5 studies the optimal design of time-step-stress PALT. For illustration, simulation studies are given in Section 6. Finally, Section 7 is devoted to the concluding remarks.

2. THE MODEL

Notations:

n	total number of test items in a PALT
$n_u,\ n_a$	number of items failed at use and accelerated conditions, respectively
$Y_{(r)}$	the time of the rth failure at which the experiment is terminated (r = n_u+ n_a)
T	lifetime of an item at use condition
Y	total lifetime of an item in a step PALT
$f(t)$	probability density function
$R(t)$	reliability function at time t at use condition
$h(t)$	hazard (failure) rate at time t at use condition

β acceleration factor (β>1)
τ stress change-time in a step PALT ($\tau < Y_{(r)}$)
P_u probability that an item fails at use condition in a step PALT
P_a probability that an item fails at accelerated condition in a step PALT
$\wedge$ implies a maximum likelihood estimator
θ Pareto scale parameter ($\theta > 0$)
α Pareto shape parameter ($\alpha > 0$)
Y_i observed value of the total lifetime Y of item i , $i = 1, \ldots, n$

The Pareto Distribution of the Second Kind:

In this paper, it is assumed that the lifetime distribution is the Pareto distribution of the second kind with two-parameters. The Pareto distribution has been widely used in economic studies. Many socio-economic phenomena such as city population sizes, occurrence of natural resources, stock price fluctuations, size of firms, and personal incomes are distributed according to certain statistical distribution with very long right tails. The Pareto distribution has played a major role in the investigation of previous phenomena [Johnson and Kotz (1970)]. Smith (1989) applied Pareto distribution to the study of ozone levels in the upper atmosphere. Grimshaw (1993) used Pareto distribution to model tensile-strength data from a random sample of nylon carpet fibers. Recently, it has been used in connection with reliability theory and survival analysis [Davis & Feldstein (1979) and Abdel-Ghaly et al (1998)].

The Pareto density with shape parameter α and scale parameter θ is given by:

$$f_T(t;\theta,\alpha) = \frac{\alpha\theta^{\alpha}}{(\theta + t)^{\alpha+1}} \quad ; t>0,\ \theta>0,\ \alpha>0. \tag{1}$$

The reliability function of the Pareto distribution takes the form:

$$R(t) = \frac{\theta^{\alpha}}{(\theta + t)^{\alpha}} \quad , \tag{2}$$

and the corresponding hazard rate is as follows:

$$h(t) = \frac{\alpha}{\theta + t} \quad ; \tag{3}$$

which is a decreasing function as $t > 0$, representing the early failure region or the initial failure region in the bathtub shape. This type of failures may be due to initial weakness or defects, weak parts, bad assembly or poor fits, etc. [Martz (1992)]. Therefore, Pareto distribution may be used as a reliability growth model [Abdel-Ghaly et al (1998)].

3. MAXIMUM LIKELIHOOD ESTIMATION

As indicated by Grimshaw (1993), the ML method is commonly used for most theoretical models and kinds of censored data. Although the exact sampling distribution of maximum likelihood estimators (MLE) is sometimes unknown, MLE have the desirable properties of being consistent and asymptotically normal for large samples under appropriate regularity conditions. Also, Bugaighis (1988) pointed out that the maximum likelihood procedure generally yields efficient estimators. However, these estimators do not always exist in closed form, so, numerical techniques are used to compute them.

In the time-step PALT, the test item is first run at use condition and if it does not fail for a specified time τ, it is run at accelerated condition until the item fails or the observation is censored. That is, $Y = T$, if $T \leq \tau$, and $Y = \tau + \beta^{-1} (T-\tau)$, if $T > \tau$. In this paper, the lifetime of the test item is assumed to follow the two-parameter Pareto distribution with scale parameter θ and shape parameter α. Therefore, the probability density function of total lifetime Y of an item in a step PALT is given by:

$$f(y) = \begin{cases} 0 & y \leq 0 \\ \dfrac{\alpha\theta^{\alpha}}{(\theta + y)^{\alpha+1}} & 0<y\leq\tau \\ \dfrac{\beta\alpha\theta^{\alpha}}{(\theta + \tau + \beta(y - \tau))^{\alpha+1}} & y > \tau \end{cases} \qquad (4)$$

The observed values of the total lifetime Y are given by:

$$y_{(1)} \leq \ldots\ldots \leq y_{(nu)} \leq \tau \leq y_{(nu+1)} \leq \ldots\ldots \leq y_{(r)}$$

Let δ_{1i} and δ_{2i} be indicator functions such that $\delta_{1i} \equiv I(Y_i \leq \tau)$, and $\delta_{2i} \equiv I(\tau < Y_i \leq y_{(r)})$; where $i = 1,\ldots,n$. Since the total lifetimes $y_1, \ldots, y_n$ of n items are independent and identically distributed random variables, then the total likelihood function for them is given by:

$$L(\beta,\theta,\alpha,Y_i,\delta_{1i},\delta_{2i}) = \prod_{i=1}^{n}[\frac{\alpha\theta^{\alpha}}{(\theta+y_i)^{\alpha+1}}]^{\delta_{1i}}.[\frac{\beta\alpha\theta^{\alpha}}{(\theta+\tau+\beta(y_i-\tau))^{\alpha+1}}]^{\delta_{2i}}$$

$$.[\frac{\theta^{\alpha}}{(\theta+\tau+\beta(y_{(r)}-\tau))^{\alpha}}]^{\bar{\delta}_{1i}\bar{\delta}_{2i}}, \qquad (5)$$

where

$\bar{\delta}_{1i} = 1-\delta_{1i}$ and $\bar{\delta}_{2i} = 1-\delta_{2i}$.

It is usually easier to maximize the natural logarithm of the likelihood function rather than the likelihood function itself. The natural logarithm of the likelihood function is given by:

$$\ln L = r\ln\alpha + n\alpha\ln\theta + n_a \ln\beta - (n-r)\alpha\ln(\theta+\tau+\beta(y_{(r)}-\tau))$$

$$-(\alpha+1)[\sum_{i=1}^{n}\delta_{1i}\ln(\theta+y_i) + \sum_{i=1}^{n}\delta_{2i}\ln(\theta+\tau+\beta(y_i-\tau))] \qquad (6)$$

The first derivatives of the natural logarithm of the total likelihood function in (6) with respect to β, θ and α are given by:

$$\frac{\partial\ln L}{\partial\beta} = \frac{n_a}{\beta} - \frac{(n-r)\alpha(y_{(r)}-\tau)}{\psi_r} - (\alpha+1)\sum_{i=1}^{n}\delta_{2i}\frac{(y_i-\tau)}{\psi_i} \qquad (7)$$

where

$\psi_r = \theta+\tau+\beta(y_{(r)}-\tau)$ and $\psi_i = \theta+\tau+\beta(y_{(i)}-\tau)$.

$$\frac{\partial\ln L}{\partial\theta} = \frac{n\alpha}{\theta} - \frac{(n-r)\alpha}{\psi_r} - (\alpha+1)[\sum_{i=1}^{n}\frac{\delta_{1i}}{(\theta+y_i)} + \sum_{i=1}^{n}\frac{\delta_{2i}}{\psi_i}] \qquad (8)$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{r}{\alpha} + n \ln \theta - (n - r) \ln \psi_r - \sum_{i=1}^{n} \delta_{1i} \ln(\theta + y_i) - \sum_{i=1}^{n} \delta_{2i} \ln \psi_i \tag{9}$$

From equation (9) the maximum likelihood estimate of α is given by the following equation:

$$\hat{\alpha} = \frac{r}{(n - r) \ln \psi_r - n \ln \hat{\theta} + \sum_{i=1}^{n} \delta_{1i} \ln(\hat{\theta} + y_i) + \sum_{i=1}^{n} \delta_{2i} \ln \psi_i} \tag{10}$$

By substituting for α into the two equations (7) and (8) and equating each of them to zero, the system equations are reduced into the following two non-linear equations:

$$\frac{n_a}{\hat{\beta}} - \frac{r(n - r)(y_{(r)} - \tau)}{Q_1 \psi_r} - (\frac{r}{Q_1} + 1).\sum_{i=1}^{n} \delta_{2i} \frac{(y_i - \tau)}{\psi_i} = 0 \tag{11}$$

and

$$\frac{nr}{Q_2} - \frac{r(n - r)}{Q_1 \psi_r} - (\frac{r}{Q_1} + 1)[\sum_{i=1}^{n} \frac{\delta_{1i}}{(\hat{\theta} + y_i)} + \sum_{i=1}^{n} \frac{\delta_{2i}}{\psi_i}] = 0 \tag{12}$$

where

$$Q_1 = (n - r) \ln \psi_r - n \ln \hat{\theta} + \sum_{i=1}^{n} \delta_{1i} \ln(\hat{\theta} + y_i) + \sum_{i=1}^{n} \delta_{2i} \ln \psi_i ,$$

and

$$Q_2 = \hat{\theta}(n - r) \ln \psi_r - n \ln \hat{\theta} + \sum_{i=1}^{n} \delta_{1i} \ln(\hat{\theta} + y_i) + \sum_{i=1}^{n} \delta_{2i} \ln \psi_i .$$

It is not easy to obtain a closed-form solutions for the two equations (11) and (12). So, iterative procedures must be used to solve these equations, numerically. The Newton-Raphson method is applied for the simultaneous numerically solution of these non-linear equations. Thus, once the values of β and θ are determined, an estimate of α is easily obtained from (10). In relation to the asymptotic variance-covariance matrix of the ML estimators of the parameters, it can be approximated by numerically inverting the Fisher-information matrix composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the ML estimates. The asymptotic Fisher-information matrix can be written as following:

$$F = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \beta^2} & -\frac{\partial^2 \ln L}{\partial \beta \partial \theta} & -\frac{\partial^2 \ln L}{\partial \beta \partial \alpha} \\ -\frac{\partial^2 \ln L}{\partial \theta \partial \beta} & -\frac{\partial^2 \ln L}{\partial \theta^2} & -\frac{\partial^2 \ln L}{\partial \theta \partial \alpha} \\ -\frac{\partial^2 \ln L}{\partial \alpha \partial \beta} & -\frac{\partial^2 \ln L}{\partial \alpha \partial \theta} & -\frac{\partial^2 \ln L}{\partial \alpha^2} \end{bmatrix} \downarrow (\hat{\beta}, \hat{\theta}, \hat{\alpha}) \tag{13}$$

The elements of the above matrix F can be expressed by the following equations:

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -\frac{n_a}{\beta^2} + \frac{(n-r)\alpha(y_{(r)} - \tau)^2}{\psi_r^2} + (\alpha+1)\sum_{i=1}^{n} \delta_{2i} \frac{(y_i - \tau)^2}{\psi_i^2} \tag{14}$$

$$\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{n\alpha}{\theta^2} + \frac{(n-r)\alpha}{\psi_r^2} + (\alpha+1)\left[\sum_{i=1}^{n} \frac{\delta_{1i}}{(\theta + y_i)^2} + \sum_{i=1}^{n} \frac{\delta_{2i}}{\psi_i^2}\right] \tag{15}$$

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -\frac{r}{\alpha^2} \tag{16}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \theta} = \frac{(n-r)\alpha(y_{(r)} - \tau)}{\psi_r^2} + (\alpha+1)\sum_{i=1}^{n} \delta_{2i} \frac{(y_i - \tau)}{\psi_i^2} \tag{17}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \alpha} = -\frac{(n-r)(y_{(r)} - \tau)}{\psi_r} - \sum_{i=1}^{n} \delta_{2i} \frac{(y_i - \tau)}{\psi_i} \tag{18}$$

$$\frac{\partial^2 \ln L}{\partial\theta\partial\alpha} = \frac{n}{\theta} - \frac{(n-r)}{\psi_r} - [\sum_{i=1}^{n} \frac{\delta_{1i}}{(\theta + y_i)} + \sum_{i=1}^{n} \frac{\delta_{2i}}{\psi_i}] \tag{19}$$

Consequently, the maximum likelihood estimators of β, θ and α have an asymptotic variance-covariance matrix defined by inverting the Fisher information matrix defined before.

4. CONFIDENCE INTERVAL ESTIMATION

Confidence intervals based on the asymptotic normal distribution of ML estimators are widely used. As indicated by Vander Wiel and Meeker (1990), the most common method to set confidence bounds for the parameters is to use the large-sample normal distribution of the ML estimators.

To construct a confidence interval for a population parameter λ; assume that $L_\lambda = L_\lambda (y_1, \ldots, y_n)$ and $U_\lambda = U_\lambda(y_1, \ldots, y_n)$ are functions of the sample data $y_1, \ldots, y_n$ such that

$$P_\lambda(L_\lambda \le \lambda \le U_\lambda)=\gamma \ , \tag{20}$$

where the interval $[L_\lambda, U_\lambda]$ is called a two-sided γ100 % confidence interval for λ. L_λ and U_λ are the lower and upper confidence limits for λ, respectively. The random limits L_λ and U_λ enclose λ with probability γ.

Asymptotically, the maximum likelihood estimators, under appropriate regularity conditions, are consistent and normally distributed. Therefore, the two-sided approximate γ 100 % confidence limits for a population parameter λ can be constructed such that:

$$\mathrm{P}[\,-z \le \frac{\hat{\lambda}-\lambda}{\sigma(\hat{\lambda})} \le z] \cong \gamma \quad , \tag{21}$$

where z is the $[100(1-\gamma/2)]^{\text{th}}$ standard normal percentile. Therefore, the two-sided approximate γ 100 % confidence limits for β, θ and α are given, respectively, as follows:

$$\left.\begin{array}{ll} L_\beta = \hat{\beta} - z\sigma(\hat{\beta}) & U_\beta = \hat{\beta} + z\sigma(\hat{\beta}) \\ L_\theta = \hat{\theta} - z\sigma(\hat{\theta}) & U_\theta = \hat{\theta} + z\sigma(\hat{\theta}) \\ L_\alpha = \hat{\alpha} - z\sigma(\hat{\alpha}) & U_\alpha = \hat{\alpha} + z\sigma(\hat{\alpha}) \end{array}\right] . \tag{22}$$

5. OPTIMAL DESIGN

Optimal Stress Change-Time τ^*:

This section considers the problem of optimally designing a simple time-step-stress PALT, which terminates after a pre-specified number of failures r occurs. Optimum test plan for products having a compound two-parameter Pareto lifetime distribution is developed. That is, the optimal stress change-time τ^* should be determined such that the GAV of the MLE of the model parameters at use stress is minimized.

Generalized Asymptotic Variance: *an optimality criterion*

The GAV of the MLE of the model Parameters is the reciprocal of the determinant of *F* [Bai, Kim & Chun (1993)]. That is,

$$\text{GAV}\,(\hat{\beta}, \hat{\theta}, \hat{\alpha}) = \frac{1}{|F|}$$

The Newton-Raphson method is applied to obtain the optimal stress-change time τ^* which minimize the GAV as defined before. Therefore, the corresponding optimal number of items failed at use and accelerated conditions are, respectively, given by:

$$nP_u \equiv n[1 - \frac{(\hat{\theta})^{\hat{\alpha}}}{(\hat{\theta} + \tau^*)^{\hat{\alpha}}}] \quad \text{and} \quad nP_a \equiv n[\frac{(\hat{\theta})^{\hat{\alpha}}}{(\hat{\theta} + \tau^*)^{\hat{\alpha}}}.(1 - \frac{(\hat{\theta})^{\hat{\alpha}}}{\{\hat{\theta} + \hat{\beta}(y_{(r)} - \tau^*)\}^{\hat{\alpha}}})]$$

6. SIMULATION STUDIES

In our simulation study we generated several data sets from Pareto distribution of the second kind for different combinations of the true parameter values of β, θ and α and for sample sizes 100, 200, 300, 400 and 500 using 500 replications for each sample size. The true parameter values used in this study were (2, 5, 0.5) and (4, 7, 0.7). Computer programs were prepared and the Newton-Raphson method was used for the practical application of the ML estimators of β, θ and α. Therefore, the derived nonlinear logarithmic likelihood equations (11) and (12) were solved iteratively. Once the values of β and θ are determined, an estimate of the shape parameter α is easily obtained from equation (10). For different sample sizes and true values of the parameters, the average numbers of items, n_u and n_a, failed at use and accelerated conditions, respectively, the MLE, their estimated variances and confidence limits were reported in Tables 1and 3. While in Tables 2 and 4 the optimal stress change-time, the optimal number of items failed at use and accelerated conditions and the optimal GAV of the MLE of the model parameters were included.

Results of simulation studies provide insight into the sampling behavior of the estimators. The numerical results indicated that the ML estimates approximate the true values of the parameters when the sample size *n* increases. Also, as shown from the numerical results, the asymptotic variances of the estimators decrease as the sample size *n* is getting to be large. The equations in (22) were used to construct the approximated confidence limits for the three parameters β, θ and α, with results shown in Tables 1 and 3. These Tables present 2-sided approximated confidence bounds based on 95% confidence degree for the parameters. As seen from the results, the intervals of the parameters appear to be narrow as the sample size *n* increases.

Also, we developed optimum test plans, numerically. It can be noted from the numerical results presented in Tables 2 and 4 that about 75 % of the items failed at use condition and 25 % tend to fail at accelerated condition. Moreover, Tables 2 and 4 show the optimal GAV of the MLE of the model parameters which was obtained numerically with τ^* in place of τ for different sized samples. As indicated from the results, the optimal GAV of the MLE of the model parameters decreases as the sample size *n* increases.

Table (1)

The ML estimates, estimated asymptotic variances of the ML estimators and confidence bounds of the parameters (β, θ, α) set at (2, 5, 0.5), respectively, given $\tau = 3$ and $r = 0.75$ n for different sized samples under type-II censoring

n	n_u	n_a	Parameter	Estimate	Variance	Lower bound	Upper bound
100	21	54	β	2.1965	0.9205	0.3160	4.0770
			θ	6.5970	18.1621	-1.7559	14.9499
			α	0.5923	0.0660	0.0887	1.0959
200	42	108	β	2.0130	0.3235	0.8982	3.1278
			θ	5.6117	3.7061	1.8384	9.3849
			α	0.5440	0.0161	0.2950	0.7930
300	64	161	β	2.0235	0.2119	1.1212	2.9257
			θ	5.2263	1.9289	2.5041	7.9484
			α	0.5205	0.0084	0.3406	0.7005
400	84	216	β	2.0604	0.1672	1.2590	2.8617
			θ	5.2734	1.4584	2.9064	7.6404
			α	0.5194	0.0062	0.3654	0.6735
500	105	270	β	2.0267	0.1253	1.3328	2.7205
			θ	5.1967	1.0862	3.1540	7.2394
			α	0.5119	0.0045	0.3810	0.6429

Table (2)

The results of optimal design of the life test with parameters (β, θ, α) set at (2, 5, 0.5), respectively, given $\tau = 3$ and $r = 0.75$ n, for different sized samples under type-II censoring

n	τ^*	nP_u	nP_a	Optimal GAV
100	44.9754	57	18	0.0102
200	44.3638	115	35	0.0002
300	44.1801	172	53	0.0001
400	43.0958	230	70	0.0000
500	44.2391	286	89	0.0000

Table (3)

The ML estimates, estimated asymptotic variances of the ML estimators and confidence bounds of the parameters (β, θ, α) set at (4, 7, 0.7), respectively, given τ= 3 and r = 0.75 n for different sized samples under type-II censoring

n	n_u	n_a	Parameter	Estimate	Variance	Lower bound	Upper bound
100	22	53	β θ α	4.2949 9.2210 0.8640	3.0797 58.7585 0.3849	0.8553 -5.8032 -0.3519	7.7345 24.2452 2.0799
200	44	106	β θ α	4.1675 8.1088 0.7724	1.3200 14.5020 0.0807	1.9156 0.6449 0.2157	6.4194 15.5728 1.3291
300	66	159	β θ α	4.0498 7.4989 0.7380	0.8177 5.5860 0.0319	2.2775 2.8665 0.3879	5.8222 12.1313 1.0882
400	88	212	β θ α	4.0348 7.4850 0.7305	0.5951 3.8913 0.0211	2.5228 3.6186 0.4457	5.5469 11.3513 1.0153
500	110	265	β θ α	4.0400 7.4197 0.7286	0.4775 3.0586 0.0168	2.6857 3.9918 0.4749	5.3944 10.8475 0.9823

Table (4)

The results of optimal design of the life test with parameters (β, θ, α) set at (4, 7, 0.7), respectively, given τ= 3 and r = 0.75 n, for different sized samples under type-II censoring

n	τ^*	nP_u	nP_a	Optimal GAV
100	14.2840	57	18	0.3530
200	14.3466	114	36	0.0165
300	14.4359	171	54	0.0016
400	14.4687	228	72	0.0006
500	14.2659	287	88	0.0003

7. SUMMARY AND CONCLUDING REMARKS

This paper deals with the problems of estimation and optimally designing simple time-step-stress PALT for Pareto distribution of the second kind under type-II censored data. The maximum likelihood estimates of the model parameters were obtained, numerically. Also, optimal test plans were developed under the assumptions of Pareto lifetimes of test units and type-II censoring. The minimization of the GAV of the MLE of the model parameters was adopted as an optimality criterion. As observed from the results, the PALT model is appropriate to somewhat where about 75 % of the tested items failed at use condition and the remaining items tend to fail at accelerated condition. That is, testing at both use and accelerated conditions. Finally, the usefulness of optimal designs lies in the fact that they can serve as benchmarks with which to compare other designs.

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