A FUZZY - STATISTICAL COMPARATIVE APPROACH TO VENDOR SELECTION PROBLEM

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Abstract

In the process of vendor selection, the most important issue is to determine a suitable decision-making method and select the right vendor. Essentially the vendor selection problem is a multi-criteria decision making problem under an uncertain environment. Fuzzy set theory best handles these uncertainties. In this paper an integration of standard scores (fuzzy and crisp) and linear programming is proposed to consider both tangible and intangible factors in choosing the best vendors and placing the optimum order quantities among them such that the total value of purchasing (TVP) is maximized. We use Andrews' plot for plotting multivariate data in two dimensions. Finally a numerical example illustrates our methodology.

Keywords

Vendor Selection Problem; Z Score; Linear programming; Fuzzy numbers; Andrews plot.

The organization of the paper is as follows: In section one which is the introduction we review the literature on Vendor Selection Problem. In section two we give the methodology with the concept of Z score. The LPP model in section three follows this. Euclidean distance is briefed in section four. In section five a numerical example explains our methodology while in six we explain Andrew's plot and its application in our problem. Finally section seven gives the conclusions and suggestions for future work.

1.0 Introduction

Evaluation of the company's vendors is considered an effective tool for rectification of defects, improving their ability to serve more satisfactorily and as a basis for making future purchasing decisions. A Vendor selection problem typically consists of four phases namely: problem definition (recognition of the need for a new dealer), formulation of criteria, qualification of suitable suppliers and final selection of the ultimate suppliers **(De Boer et al. 2001**). The evaluation of vendors is done on a periodic basis and includes written evaluation aspects relating to quality, quantity, price, service etc. as obtained from the buyer, user and quality control and other concerned staff.

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Dickson 1966, in one of the early works on supplier selection, identified over 20 supplier attributes which managers trade off when choosing a supplier. Since then a number of conceptual and empirical articles on supplier selection have appeared. The conceptual articles are examples of publications emphasizing the strategic importance of the supplier selection process. The articles highlight the trade-off among quality, cost and delivery performance measures in the supplier selection process. **Weber et al**.**1991** reviewed the literature surrounding vendor selection criteria, identified various vendor selection criteria and identified several basic techniques or models that appeared in studies over the previous 25 years. They found that the vast majority were linear weighting models, mathematical models such as economic order quantity (EOQ) and a few probabilistic models. Since 1991, other techniques have been applied to the problem: Analytic Hierarchy Process (**Nydick and Hill 1992**; **Barbarosoglu & Yazgac 1997),** Multiobjective Programming (**Weber & Ellram 1993**), Total Cost of Ownership (**Ellram 1995**), Statistical Analysis (**Mummalaneni et al.1996, Petroni and Braglia 2000**), Interpretative Structural Modeling (**Mandal and Deshmukh 1994**), Discrete Choice Analysis experiment (**Verma and Pullman 1998**) and Neural Networks (**Siying et al. 1997**).

Fuzzy set theory can provide a valuable tool to cope with three major problematic areas of vendor selection: imprecision, randomness and ambiguity. As far as imprecision is concerned it provides a powerful tool to weigh selection criteria importance. As far as randomness is concerned, it is more effective than probabilistic approaches in that the selection problems should not use prediction based on previous vents, since each selection case is not repeatable. As far as ambiguity is concerned it copes better than other methods with the treatment of linguistic variables. Fuzzy logic enables us to emulate the human reasoning process and make decisions based on vague or imprecise data. **Albino et al 1998** used fuzzy logic system to support vendor rating and compared it to a neural network in order to evaluate the different system performances. **Nassimbeni and Battain 2003** developed a vendor-rating tool based on fuzzy logic, a neural application and ordinary least squares (OLS) regression. **Kumar, Vrat and Shankar 2004, 2006** used a fuzzy programming approach for vendor selection problem in a supply chain considering a fuzzy Multi-objective Integer Programming formulation and a fuzzy mixed integer goal programming formulation. **Chou et al 2006** used a fuzzy factor rating system to evaluate potential vendors based on a modified re-buy situation.

Although each of these methodologies offers advantages under particular conditions, they do not provide a general workable methodology for combining multiple criteria into a single measure of supplier performance. We are extremely limited in making direct comparisons in terms of raw scores; we need a common scale before comparisons. Standard scores furnish one such common scale. In short fuzzy logic based approach seems to be particularly effective in decisions where fuzzy expressions are more natural for many human judgmental rules and statements than mathematical equations. Our approach is thus based on uncertainty reduction using a fuzzy logic.

2.0 METHODOLOGY

2.1 Z-Score

Suppose we are interested in determining which Vendor is more consistent in his abilities and which one has the greater variability. Would a comparison of the standard deviations of the two sets of raw scores give us the answer? The reply to most of these questions is in the negative. We are extremely limited in making direct comparisons in terms of raw scores because raw score scales are arbitrary and unique. We need a common scale before comparisons such as we have called for can be made. Standard scores furnish one such common scale.

A standard score scale has a mean of zero and a standard deviation of 1.0. Standard score z corresponding to a raw score X and to a deviation x

$$z = \frac{X - M}{\sigma} \qquad \textbf{… (1)}$$

where deviation from mean M is X-M and σ is the standard deviation. See also [25], [26].

One shortcoming of the standard score is that half the scores will be negative in sign, which makes computation awkward. We overcome this shortcoming by adding a constant to all the scores to make them all positive.

3.0 The Linear Model

Notations:
R_i final ratings of i^{th} Vendor (Here total of Z scores for i^{th} Vendor)
X_i Order quantity for i^{th} Vendor
Vi Capacity of i^{th} Vendor
D Demand for the period
q_i Defect percent of ith Vendor
Q Buyer's maximum acceptable defect rate

The Objective function

The objective here is to maximize the total value of purchasing (TVP).

$$\text{Max (TVP)} = \sum_{i=1}^{n} R_i X_i \qquad \textbf{… (2)}$$

Constraints

1. Capacity constraints

As vendor i can provide up to V_i units of the product and its order quantity (X_i) should be equal or less than its capacity, these constraints are:

$$X_i \le V_i \ , i=1, 2 \ldots n. \qquad \textbf{… (3)}$$

On the other hand, aggregate Vendors' capacity should be equal or greater than demand, therefore,

$$\sum_{i=1}^{n} V_i \geq D \qquad \text{... (4)}$$

2. Demand constraint: As sum of the assigned order quantities to n vendors should meet the buyer's demand, it can be stated that

$$\sum_{i=1}^{n} X_i = D \qquad \text{... (5)}$$

3. Quality constraint: Since Q is the buyer's maximum acceptable defect rate and q_i is the defect rate of the ith vendor, the quality constraint can be shown as

$$\sum_{i=1}^{n} X_i q_i \leq QD \qquad \text{... (6)}$$

Final model

The final integrated linear programming model can be shown as

$$\text{Max (TVP)} = \sum_{i=1}^{n} R_i X_i \qquad \text{... (7)}$$

Subject to:

$$\left.\begin{array}{ll} \sum_{i=1}^{n} X_i = D & \text{(demand constraint),} \\ \sum_{i=1}^{n} X_i q_i \leq QD & \text{(aggregate quality constraint),} \\ X_i \leq V_i \quad i = 1,2, \ldots, n & \text{(Vendor's capacity constraints),} \\ X_i \geq 0, \ i = 1,2, \ldots, n & \text{(nonnegativity constraint)} \end{array}\right\} \qquad \text{...(8)}$$

4.0 Euclidean Distance

The Euclidean distance between two vendors is given by

$$d_{ij} = \sqrt{\sum_{k=1}^{3} (Z_{ik} - Z_{jk})^2} \qquad \text{... (9)}$$

Where d_{ij} is the Euclidean distance between ith and jth vendor, $i \neq j=1,2,3,4$.

Z_{ik} = Z score on k-th characteristic of i-th vendor , k=1,2,3.

Z_{jk}= Z score on k-th characteristic of j-th vendor.

5.0 Numerical example

Assume that the management of a JIT manufacturer decides to choose their best Vendors and assign their optimum order quantities to maximize the total value of purchasing. The main criteria for vendor selection are Cost, Quality and Service. According to the corporate strategies the Quality includes Defects and Process capability while Service involves On-time delivery, Response to changes. Four suppliers are included in the evaluation process and their Cost.

Suppose the buyer wishes to find the best suppliers and their optimum order quantities, if the demand is 1000 units and the maximum acceptable defect rate is 0.02.

In order to solve this problem two types of calculations should be carried out: Z-score and linear programming (LP) optimization for both fuzzy and crisp case. The algorithm of the steps is defined as follows: -

Step 1: Consider the information regarding quantitative information of vendors (**Table 1**). Quality refers to percentage defectives.

Step 2: We calculate the Z-score for each vendor using data from **Table 1** and **Table 4** (both fuzzy and crisp). **Tables 2,3,5** show the various results of Z scores. The Z scores are calculated using equation (1). As far as fuzzy Z-scores are concerned we use the same formula as for crisp case, except that we defuzzify it using equation (10). **Tables 6 and 7** show the defuzzified values of the Z-scores.

Step 3: Once the Z-score have been calculated we use these Z-scores as coefficients of the objective function (i.e R_{i}). Using equations (7) to (8) as LPP formulations we calculate the order quantities to be allocated to the vendors.

Step 4: In order to find the best order quantities the TVP is shown in the following programming, maximized as:

Max TVP=$4.621x_1 + 6.196x_2 + 7.235x_3 + 5.536x_4$ (Fuzzy case)

Subject to:

$0.03x_1+0.05x_2+0.01x_3+0.06x_4 \leq 20$

$x_1+x_2+x_3+x_4=1000$

$x_1 \leq 400$

$x_2 \leq 700$

$x_3 \leq 600$

$x_4 \leq 500$

$x_i \geq 0$, I=1,2,3,4.

This LPP has been solved using LINDO 6.1 optimization software. The optimal solution for the above formulation is

TVP_{crisp} =6347.08 where x_1=300,x_2=100,x_3=600 and x_4=0 .

TVP_{fuzzy} = 6318.85 where x_1=333.33,x_2=0,x_3=600 and x_4=66.67.

Table 1:Vendors' quantitative information

	Cost	Quality	On-time del.	Capacity
Supplier 1	30	0.03	.95	400
Supplier 2	40	0.05	.98	700
Supplier 3	50	0.01	.85	600
Supplier 4	45	0.06	.92	500

6.0 Andrews Plot

It is always a good idea to plot data in whatever way seems appropriate. It helps the analyst get a feel of the data and may suggest relationships between variables. The analyst can also spot subjectively any outliers, detect natural clustering of observations and check on distributional assumptions.

With more than three variables pictorial representation becomes impractical and alternative ways must be found (e.g. parallel co-ordinates). **Andrews (1972)** suggested a completely different approach for representing each p-variate observation by a function f (t) plotted over the range (-π, π) of t. For the r-th observation this function is defined as

$$f_r(t) = \frac{x_{r_1}}{\sqrt{2}} + x_{r_2} \sin t + x_{r_3} \cos t + x_{r_4} \cos t + x_{r_4} \sin 2t + x_{r_5} \cos 2t + ...$$

Apart from the first term, this function is a combination of sine and cosine waves and will produce some sort of wave pattern depending on the observed values of the p variables. Observations that are close together in p-dimensional space should give wave patterns, which are somewhat similar. Andrews (1972) showed that if the distance between two functions is defined in the obvious way by $\int_{-\pi}^{\pi} [f_r(t) - f_s(t)]^2 dt$ then this is proportional to the squared Euclidean distance between X_r and X_s.

Remarks 1:The Andrews curve will not identify relationships between variables but do appear to be useful for finding clusters and outliers.

Remarks 2:One drawback of the method is that it depends on the order in which the variables are labeled. The remedy lies in labeling the variables in decreasing order of importance i.e. x_1 representing the most important variable etc.

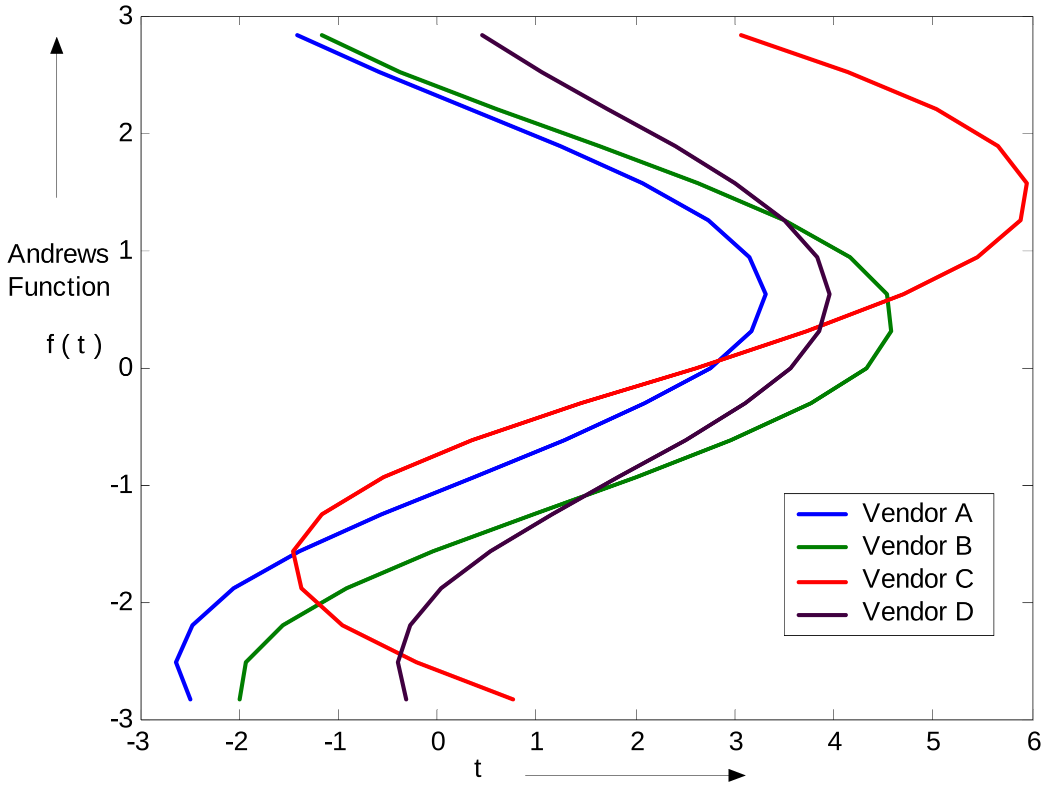

Figure 1: Andrews function f (t) representing four vendors A, B, C, D.

Table 2: Crisp standard scores for the four vendors

Vendor	Cost Scores	Quality Scores	On-time delivery scores	Overall scores
Vendor A	-1.52	-.2714	.4124	-1.379
Vendor B	-.1689	-.6661	1.031	.196
Vendor C	1.182	1.7023	-1.649	1.2353
Vendor D	.5068	-.7648	-.20619	1.0654

Table 3: Crisp standard scores for the four vendors after addition of a constant

Vendor	Cost Scores	Quality Scores	On-time delivery scores	Overall scores
Vendor A	.48	1.7286	2.4124	4.621
Vendor B	1.8311	1.3339	3.031	6.196
Vendor C	3.182	3.7023	.351	7.235
Vendor D	2.5068	1.2352	1.7938	5.536

Table 4:Fuzzy data for the four vendors

Vendors	Cost	Quality	On-time delivery
Vendor A	(25,30,40)	(0.02,0.03,0.04)	(.94,.95,.97)
Vendor B	(30,40,45)	(0.04,0.05,0.06)	(.97,.98,.99)
Vendor C	(45,50,55)	(0,0.01,0.03)	(.80,.85,.90)
Vendor D	(40,45,50)	(0.03,0.06,0.09)	(.90,.92,.95)

Table 5: Fuzzy standard scores for the four vendors

Vendors	Cost	Quality	On-time delivery
Vendor A	(-1.27, -1.52, -1.34)	(-.2027, -.4167, -.6550)	(.5763, .4124,.5075)
Vendor B	(-.633, -.169, -.446)	(1.1486, .625, .2183)	(1.044,1.031,1.104)
Vendor C	(1.27,1.18,1.34)	(-1.55, -1.458,-1.092)	(-1.604, -1.649, -1.582)
Vendor D	(.633, .507, .446)	(.4730,1.1458,1.5284)	(-.0467, -.2062,-.0896)

Table 6: Defuzzified standard scores for the four vendors

Vendors	Cost	Quality	On-time delivery	Overall scores
Vendor A	-1.38	-.4248	-.2758	-2.0806
Vendor B	-.416	.6640	-.6755	-.4247
Vendor C	1.26	-1.3680	1.698	1.59
Vendor D	.529	1.0491	-.7470	.8311

Table 7: Defuzzified standard scores for the four vendors after addition of a suitable constant.

Vendors	Cost	Quality	On-time delivery	Overall scores
Vendor A	.6249	1.575	1.724	3.924
Vendor B	1.584	2.664	1.325	5.573
Vendor C	3.263	.632	3.698	7.593
Vendor D	2.529	3.049	1.253	6.831

7.0 Conclusions and suggestions for future work

We consider a flexible method, which can reflect the corporate strategy in the vendor selection process. A dynamic TVP model is suggested to establish good linkage between vendor selection and buyer's company's policy. This model assigns order quantities to vendors such that the total value of purchasing becomes maximum using z-scores and linear programming. This model also enables the management to make a trade off between several tangible and intangible factors with different priorities.

It is shown that the use of Z scores as a mark of quality is useful in a Vendor Selection Problem. Statistical and fuzzy approaches are both equally good as verified through LPP (**Table.9**). Below is given a table of Euclidean distance between vendor pairs (see also Andrew's plot **fig.1.)**

Table .8: Euclidean distance between the vendors

(Vendor A, Vendor B)	(Vendor A, Vendor C)	(Vendor A, Vendor D)	(Vendor B, Vendor C)	(Vendor B, Vendor D)	(Vendor C, Vendor D)
1.538	3.93	2.176	3.823	1.41	2.937

Table .9: Quota allocations for vendors (fuzzy and crisp case)

Vendor Number	Fuzzy Case	Crisp Case
1	X_1= 333.33	X_1= 300
2	X_2 = 0	X_2 = 100
3	X_3 = 600	X_3 = 600
4	X_4 =66.67	X_4 =0

An interesting finding is that $TVP_{crisp} = 6347.08 > TVP_{fuzzy} = 6318.85$. Vendor C getting the highest allocation (=600) inspite of having the largest total Z score (a large Z score is undesirable). Perhaps this is because he/she is the one who has the lowest quality 0.01 (% defective).

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APPENDIX

FUZZY CONCEPTS

In 1965, **Prof.L. A. Zadeh** laid the foundation of fuzzy sets [24]. Let U be the universe of discourse U = $\{u_1, u_2, \ldots, u_n\}$. A fuzzy set $\tilde{A}$ of U is a set of ordered pairs

$$\{(u_1, f_{\tilde{A}}(u_1)), ((u_2, f_{\tilde{A}}(u_2)), \ \ldots \ ,(u_n, f_{\tilde{A}}(u_n))\}$$

Where $f_{\tilde{A}}, f_{\tilde{A}} : U -> [0,1]$, is the membership of $\tilde{A}$, and $f_{\tilde{A}}(u_i)$ indicates the grade of membership of u_i in $\tilde{A}$.

Definition 1: Fuzzy number is a fuzzy subset in the universe of discourse U that is both convex and normal.

Definition 2: The α-cut $\tilde{A}_\alpha$ of the fuzzy set $\tilde{A}$ in the universe of discourse U is defined by

$$\tilde{A}_\alpha = \{u_i \,/\, f_A(u_i) \ge \alpha, u_i \in U\} \text{ where } \alpha \in [0,1].$$

Definition 3: According to [23], a fuzzy number $\tilde{A}$ of the universe of discourse U may be characterized by a triangular distribution function parameterised by a triplet (a, b, c) shown in **Fig.2.** The membership function of the fuzzy number $\tilde{A}$ is defined as

$$f_{\tilde{A}}(u) = \begin{cases} 0, & u < a \\ \dfrac{u-a}{b-a}, & a \le u \le b, \\ \dfrac{c-u}{c-b}, & b \le u \le c, \\ 0, & u > c. \end{cases}$$

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers (TFN) parameterised by the triplet say (a_1,a_2, a_3) and (b_1,b_2,b_3), respectively.

Then the operations of fuzzy numbers are expressed as:

$$\tilde{A}\,(+)\,\tilde{B} = (a_1, a_2, a_3)(+)(b_1,b_2,b_3) = (a_1+b_1,a_2+b_2,a_3+b_3),$$

$$\tilde{A}\,(-)\,\tilde{B} = (a_1,a_2,a_3)(-)(b_1,b_2,b_3) = (a_1-b_3,a_2-b_2,a_3-b_1),$$

$$\tilde{A}\,(*)\,\tilde{B} = (a_1, a_2,a_3)(x)(b_1,b_2,b_3) = (a_1b_1,a_2b_2,a_3b_3),$$

$$\tilde{A}\,(\div)\,\tilde{B} = (a_1, a_2,a_3)(\div)(b_1,b_2,b_3) = (a_1/b_3, a_2/b_2, a_3/b_1).$$

Fig.3. Shows operations addition and multiplication on two TFNs.

Definition 4: Defuzzification of a triangular fuzzy number (a, b, c) is equal to

$$e = \frac{a + 2 * b + c}{4} \qquad \textbf{...(10)}$$

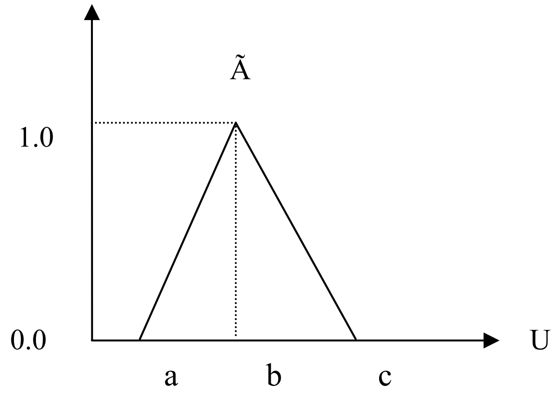

Fig.2. Triangular Fuzzy number

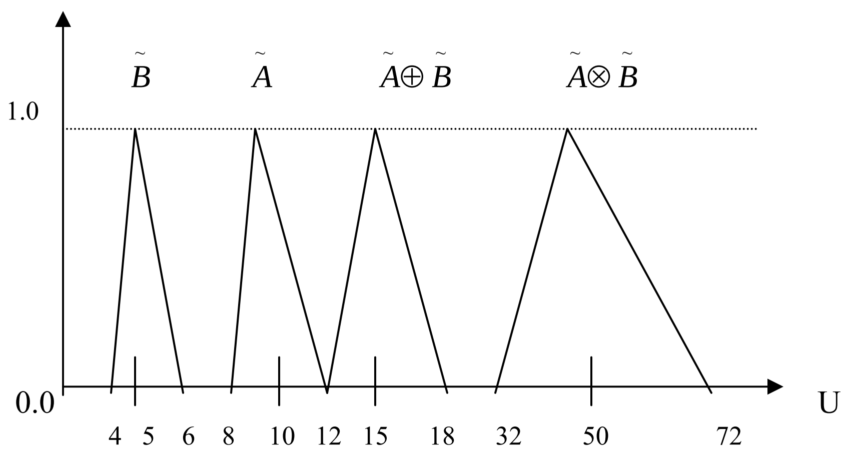

Fig.3. Fuzzy Number Operations.