On the Stochastic Analysis of a Three-Units Ventilation Sytstem for Promoting the Safety of Mines

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Abstract: This paper aims at constructing a stochastic system of ventilation in the underground mines. Thus, a complete "probabilistic structure" of the system has been presented, upon which the stochastic behavior of a three-similar units (fans) ventilation system is statistically modeled.

Having established the model, a statistical analysis of its characteristics is performed to explore the stochastic features of the proposed system. In this context, an explicit probabilistic expressions are derived for the mean time to system failure (MTSF) of the ventilation system. Also, a fifth order polynomial formula for the (MTSF) is provided under the exponential assumption for the failure time and replacement time distribution. A numerical example is presented to demonstrate the dependence of the (MTSF) on repair and replacement rate, as well as its importance, as a measure of system reliability, in elevating the safety levels in mines.

Keywords : Stochastic system, stochastic behavior, probabilistic structure, system reliability, exponential assumptions, mean time to system failure,

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1 - Introduction

One of the main health hazards to mine workers is the exposure to airborne radioactive, especially radon gases and their short-lived decay products. Considerable evidence has been presented, as early as forties, to prove that excessive exposure to such damaging gases is strongly associated with a high incidence rate of lung cancer [5].

Thus, the construction of a "reliable" ventilation system has been considered the most effective safety-promoting measure of controlling radon-air contamination for providing a pure healthful working atmosphere.

This paper introduces a "statistical model" for analyzing a proposed redundant ventilation system. The model provides a "statistical investigation" of the stochastic behavior of a three -units (fans) ventilation system. This is done under the assumption that each unit works in three different modes: normal, partial failure and total failure.

A probabilistic structure has been set up for the model using a set of realistic assumptions. This has been used to assess the transition probabilities between states of the system, as well as, the mean time to system failure (MTSF).

This paper is organized as follows. Section 2, gives an abstract description of the ventilation system, whose stochastic behavior is going to be statistically modeled. The system is assumed to operate in three different modes: normal, partial failure and total failure. A set of nine assumptions is proposed by which the system will be structured and its stochastic behavior is going to be modeled.

In section 3, a graphical representation (transition Diagram) of the possible states of the system is given, in which we realize nine up states and one down state.

In section 4, the above mentioned transition diagram will be used to calculate the transition probabilities between the different states of the system. Also, the "mean sojourn times" of the states of the system are derived.

In section 5, the mean time to system failure (MTSF) is computed as a measure of reliability of the system.

In section 6, special case of MTSF is investigated under the assumption that the failure time and replacement time distributions are exponential. In this case, the derived formula for the (MTSF) is shown to be a polynomial of the fifth order “dependence” on replacement or repair rate.

In section 7, an illustrative example is given by which the dependence of (MTSF) on its parameters is numerically analyzed. This is done for the sake of exploring possible ways of raising the value of (MTSF). Also, the numerical results demonstrate its role in decision-making situations of accepting or rejecting a new unidentified fan. This reflects its importance in promoting the safety standards for mine workers.

In section 8, we suggest a "new dimension" for using the proposed statistical model in analyzing complex systems for the ultimate goal of evaluating and promoting the "Safety" of nuclear installations.

2 - Model Description and Assumptions

2.1. Main assumptions:

In this paper we introduce a probabilistic analysis of three similar-unit standby redundant systems of fans with two types of failure and single replacement facility.

The system is programmed to work automatically at predetermined periods. The system has been investigated under the assumption that each unit works in three different modes: normal, partial failure and total failure. A first unit is assumed to be operative half the time, a second unit is operative the other halftime and a third unit is kept as a cold standby. The following symbols are used for the modes; O-operation, S-standby, P- partial failure and T- total failure.

There are nine assumptions by which the system will be structured:

1. The system consists of three similar units, the first unit is operative half time, the second unit is operative another halftime and the third unit is stand- by.
2. The standby is switched to operative state in negligible time.
3. The operative units have three modes of operation. These are: normal, O- mode which means the functioning of the units with full capacity, partial failure, P- mode which means the functioning of the units with reduced capacity at specified level and the total failure, T- mode which means the capacity goes below a specified level.
4. No unit of the system can attain the total failure mode without passing through the partial failure mode.
5. Only one replacement or repair service to attend a totally failed or partially failed unit.
6. It is assumed that the unit, for which the replacement or repair has been completed at partial failure mode. Enters the normal modes; otherwise; it may go to total failure mode.
7. This assumption has been set up for the unit, which is replaced or repaired in total failure mode. This assumed to go directly to the normal mode without passing through the partial failure mode.
8. The system is down when all units are non- operative.

9. The failure time and replacement time have arbitrary general distributions which are assumed to be:

   $f_1(t), F_1(t)$: pdf and cdf of failure time of an operative unit in O - mode.

   $f_2(t), F_2(t)$: pdf and cdf of failure time of an operative unit in P-mode.

   g(t), G(t): pdf and cdf of replacement or repair time in O and P-mode.

2.2. Probabilistic Notations and operators:

E_0 : States the system at time t = 0 (initial state).

E: set of states $[S_i]$; i = 0, 1, 2, 3, 4, 5, 6.

$q_{ij}(t), Q_{ij}(t)$: pdf and cdf of time for transition from state S_i to state S_j.

P_{ij}: transition probability from S_i to S_j.

$P_{(ij)_{kl}}$: transition probability from $S_{(i)_k}$ to $S_{(j)_l}$.

$P_{(ij)_k}$: transition probability from $S_{(i)_k}$ to $S_{(j)}$.

$P_{(ij)_l}$: transition probability from $S_{(i)}$ to $S_{(j)_l}$.

μ_{ij} : is the mean stay time in state S_i given that transition is to state S_j.

$\mu_{(ij)_{kl}}$: is the mean stay time in state $S_{(i)_k}$ given that transition is to state $S_{(j)_l}$.

$\mu_{(ij)_k}$: is the mean stay time in state $S_{(i)_k}$ given that transition is to state $S_{(j)}$.

$\mu_{(ij)_l}$: is the mean stay time in state $S_{(i)}$ given that transition is to state $S_{(j)_l}$.

μ_i : is the mean Sojourn time in state S_i , $\mu_i = \sum_j \mu_{ij}$.

$\phi_i(t)$: cdf of the time to system failure / $E_0 = S_i$.

*: symbol of convolution,

e.g. $a(t)*b(t) = \int_0^t a(u)b(t-u)du$.

3 - Possible States of the System

Our proposed system has two types of states, namely: up states and a down state.

Up States:

$$S_0 = (O,O,S),\ S_{(1)_1} = (P,O,S),\ S_{(1)_2} = (O,P,S),\ S_{(2)_1} = (T,O,S),\ S_{(2)_2} = (O,T,S),$$

$$S_{(3)_1} = (T,P,S),\ S_{(3)_2} = (P,T,S),\ S_{(4)} = (T,T,O),\ S_{(5)} = (T,T,P).$$

Down State: $S_{(6)} = (T, T, T)$.

Possible states and transitions are shown in Fig. 1.

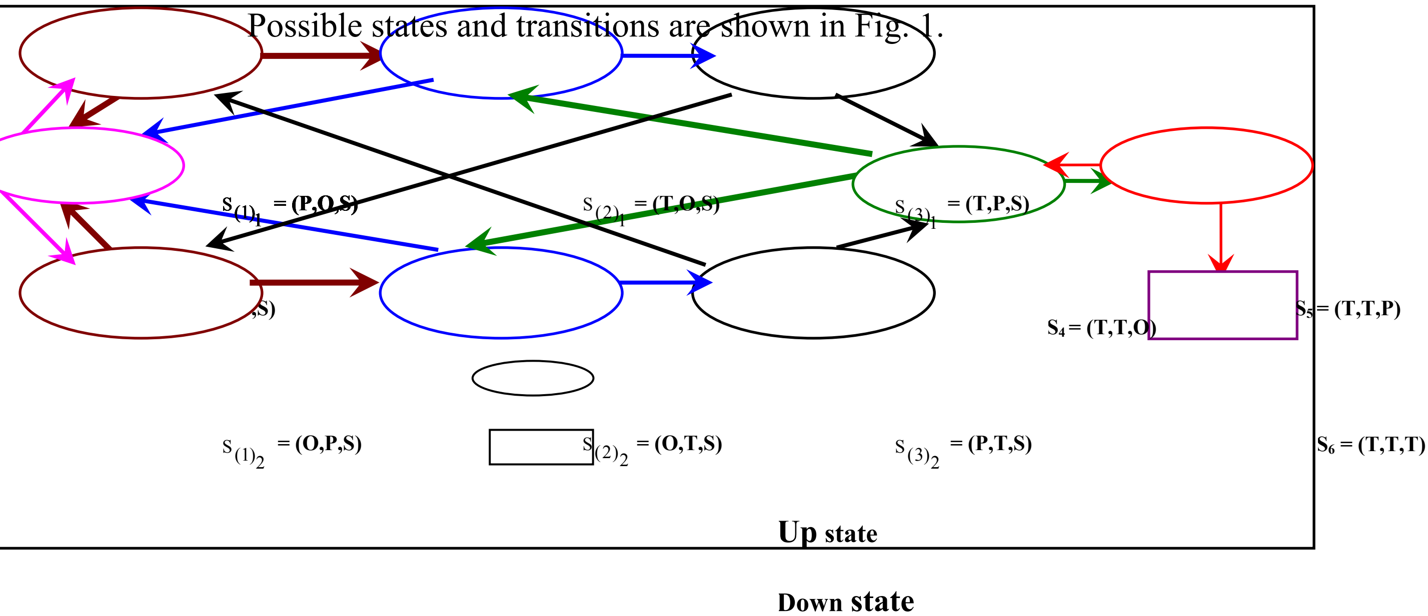

Fig. 1. The transition diagram

4 - Transition Probabilities and Sojourn Times

Let T_0, T_1, …denote the time points at which the system enters the state i ($i \in E$). Let $T_0 \equiv 0$ and X_n denote the state visited at T_n. Then $\{X_n, T_n\}$ constitutes a Markov renewal process with state space E [9], [14] and

$$Q_{ij}(t) = \Pr[X_{n+1} = j,\ T_{n+1} - T_n \le t \,/\, X_n = i] \tag{4.1}$$

is the semi-Markov kernel over E.

Then the transition probability matrix ($Q_{ij}(\infty)$) of the imbedded Markov chain is $P = (p_{ij})$ with non zero elements. [19], [6] given below:

$$P_{(01)_1} = \tfrac{1}{2}\int_0^\infty dF_1(t) = \frac{1}{2} \qquad P_{(01)_2} = \tfrac{1}{2}\int_0^\infty dF_1(t) = \frac{1}{2} \qquad P_{01} = P_{(01)_1} + P_{(01)_2} = \tfrac{1}{2} + \tfrac{1}{2} = 1$$

$$P_{(10)_1} = \tfrac{1}{2}\int_0^\infty \overline{F}_2(t)g(t)dt \qquad P_{(10)_2} = \tfrac{1}{2}\int_0^\infty \overline{F}_2(t)g(t)dt \qquad P_{10} = P_{(10)_1} + P_{(10)_2}$$

$$P_{(12)_{22}} = \tfrac{1}{2}\int_0^\infty \overline{G}(t)f_2(t)dt \qquad P_{(12)_{22}} = \tfrac{1}{2}\int_0^\infty \overline{G}(t)f_2(t)dt \qquad P_{12} = P_{(12)_{11}} + P_{(12)_{22}}$$

$$\therefore\ P_{10} + P_{12} = 1$$

$$P_{(20)_1} = \tfrac{1}{2}\int_0^\infty \overline{F}_1(t)g(t)dt \qquad P_{(20)_2} = \tfrac{1}{2}\int_0^\infty \overline{F}_1(t)g(t)dt \qquad P_{20} = P_{(20)_1} + P_{(20)_2}$$

$$P_{(23)_{11}} = \tfrac{1}{2}\int_0^\infty \overline{G}(t)f_1(t)dt \qquad P_{(23)_{22}} = \tfrac{1}{2}\int_0^\infty \overline{G}(t)f_1(t)dt \qquad P_{23} = P_{(23)_{11}} + P_{(23)_{22}}$$

$$\therefore\ P_{20} + P_{23} = 1$$

$$P_{(31)_{12}} = \tfrac{1}{2}\int_0^\infty \overline{F}_2(t)g(t)dt \qquad P_{(31)_{21}} = \tfrac{1}{2}\int_0^\infty \overline{F}_2(t)g(t)dt \qquad P_{31} = P_{(31)_{12}} + P_{(31)_{21}}$$

$$P_{(34)_1} = \tfrac{1}{2}\int_0^\infty \overline{G}(t)f_2(t)dt \qquad P_{(34)_2} = \tfrac{1}{2}\int_0^\infty \overline{G}(t)f_2(t)dt \qquad P_{34} = P_{(34)_1} + P_{(34)_2}$$

$$\therefore P_{31} + P_{34} = 1$$

$$P_{(42)_1} = \tfrac{1}{2}\int_0^\infty \overline{F}_1(t)g(t)dt \qquad P_{(42)_2} = \tfrac{1}{2}\int_0^\infty \overline{F}_1(t)g(t)dt \qquad P_{42} = P_{(42)_1} + P_{(42)_2}$$

$$P_{45} = \int_0^\infty \overline{G}(t)f_1(t)dt \qquad \therefore\ P_{42} + P_{45} = 1$$

$$P_{54} = \int_0^\infty \overline{F}_2(t)g(t)dt \qquad P_{56} = \int_0^\infty \overline{G}(t)f_2(t)dt \qquad \therefore\ P_{54} + P_{56} = 1 \tag{4.2}$$

5 - Mean Time To System Failure

The integral representation of the system requires the knowledge of the different states of the system. [10]

From the above-mentioned transition diagram we can derive the equations of our system. Thus, to determine the distribution of first passage time to state S_6 when the system completely fails, we use some probabilistic concepts [16], [18] that lead to the following equations.

$$\tilde{\phi}_0(t) = \tilde{Q}_{01}(t) * \tilde{\phi}_1(t), \qquad \tilde{\phi}_1(t) = \tilde{Q}_{10}(t) * \tilde{\phi}_o(t) + \tilde{Q}_{12}(t) * \tilde{\phi}_2(t),$$

$$\tilde{\phi}_2(t) = \tilde{Q}_{20}(t) * \tilde{\phi}_o(t) + \tilde{Q}_{23}(t) * \tilde{\phi}_3(t), \qquad \tilde{\phi}_3(t) = \tilde{Q}_{31}(t) * \tilde{\phi}_1(t) + \tilde{Q}_{34}(t) * \tilde{\phi}_4(t),$$

$$\tilde{\phi}_4(t) = \tilde{Q}_{42}(t) * \tilde{\phi}_2(t) + \tilde{Q}_{45}(t) * \tilde{\phi}_5(t), \qquad \tilde{\phi}_5(t) = \tilde{Q}_{54}(t) * \tilde{\phi}_4(t) + \tilde{Q}_{56}(t). \qquad (5.1)(3.32)$$

Taking Laplace transform of above equation and on solution, one can get Laplace transform of distribution function of the first passage time to S_0.

$$\tilde{\phi}_0(s) = \frac{\tilde{Q}_{01}(s)\tilde{Q}_{12}(s)\tilde{Q}_{23}(s)\tilde{Q}_{34}(s)\tilde{Q}_{45}(s)\tilde{Q}_{56}(s)}{\begin{array}{c}1-\tilde{Q}_{01}(s)\tilde{Q}_{10}(s)-\tilde{Q}_{45}(s)\tilde{Q}_{54}(s)-\tilde{Q}_{01}(s)\tilde{Q}_{12}(s)\tilde{Q}_{20}(s)-\tilde{Q}_{12}(s)\tilde{Q}_{23}(s)\tilde{Q}_{31}(s)\\ -\tilde{Q}_{23}(s)\tilde{Q}_{34}(s)\tilde{Q}_{42}(s)+\tilde{Q}_{01}(s)\tilde{Q}_{10}(s)\tilde{Q}_{45}(s)\tilde{Q}_{54}(s)+\tilde{Q}_{01}(s)\tilde{Q}_{12}(s)\tilde{Q}_{20}(s)\tilde{Q}_{23}(s)\\ \tilde{Q}_{45}(s)\tilde{Q}_{54}(s)+\tilde{Q}_{12}(s)\tilde{Q}_{23}(s)\tilde{Q}_{31}(s)\tilde{Q}_{45}(s)\tilde{Q}_{54}(s)+\tilde{Q}_{23}(s)\tilde{Q}_{34}(s)\tilde{Q}_{42}(s)\tilde{Q}_{01}(s)\tilde{Q}_{10}(s)\end{array}} \qquad (5.2)$$

where, for brevity, we have omitted the arguments from $\tilde{\phi}_{ij}(s)$. Hence

$$\mathrm{MTSF} = \left.\frac{d\,\tilde{\phi}_0(s)}{d(s)}\right|_{s=0}$$

$$\mathrm{MTSF} = \frac{\begin{array}{c}(1-P_{45}P_{54})[\mu_0 - \mu_0 P_{12}P_{23}P_{31} + \mu_1 + \mu_2 P_{12} + \mu_3 P_{12}P_{23}] - \\ P_{23}P_{34}[\mu_0 P_{42} + \mu_1 P_{42} - \mu_4 P_{12} - \mu_5 P_{12}P_{45}]\end{array}}{P_{12}P_{23}P_{34}P_{45}P_{56}} \qquad (5.3)$$

6 - Special Cases of the MTSF

The special case of the MTSF is based on the assumption that the failure time and replacement time distributions are exponential. Justifications of this assumption have been based upon the work of Mokaddis and El-said [12], [13]

Thus, we assume that:

$$f_1(t) = \alpha_1 e^{-\alpha_1 t}, \quad f_2(t) = \alpha_2 e^{-\alpha_2 t} \quad \text{and} \quad g(t) = \beta e^{-\beta t}, \quad \text{where} \tag{6.1}$$

α_1 is failure rate of an operative unit, α_2 is failure rate of partial failure unit and β is a replacement or repair rate. After some statistical manipulations [18] the transition probabilities and the Sojourn times will take the form:

$$P_{01} = 1$$

$$P_{10} = \frac{\beta}{\alpha_2 + \beta} \quad P_{12} = \frac{\alpha_2}{\alpha_2 + \beta} \quad P_{20} = \frac{\beta}{\alpha_1 + \beta} \quad P_{23} = \frac{\alpha_1}{\alpha_1 + \beta}$$

$$P_{31} = \frac{\beta}{\alpha_2 + \beta} \quad P_{34} = \frac{\alpha_2}{\alpha_2 + \beta} \quad P_{42} = \frac{\beta}{\alpha_1 + \beta} \quad P_{45} = \frac{\alpha_1}{\alpha_1 + \beta}$$

$$P_{54} = \frac{\beta}{\alpha_2 + \beta} \quad P_{56} = \frac{\alpha_2}{\alpha_2 + \beta} \tag{6.2}$$

$$\mu_0 = \frac{1}{\alpha_1} \quad \mu_1 = \frac{1}{\alpha_2 + \beta} \quad \mu_2 = \frac{1}{\alpha_1 + \beta}$$

$$\mu_3 = \frac{1}{\alpha_2 + \beta} \quad \mu_4 = \frac{1}{\alpha_1 + \beta} \quad \mu_5 = \frac{1}{\alpha_2 + \beta} \tag{6.3}$$

The MTSF takes the form:

$$MTSF = \frac{3[L_1] + \beta[L_2] + \beta^2[L_3] + \beta^3[L_4] + \beta^4[L_5] + \beta^5}{\alpha_1^3 \alpha_2^3} \tag{6.4}$$

where,

$$L_1 = [\alpha_1^2\alpha_2^3 + \alpha_1^3\alpha_2^2], \qquad L_2 = [\alpha_1^3\alpha_2 + 2\alpha_1\alpha_2^3 + 4\alpha_1^2\alpha_2^2],$$

$$L_3 = [\alpha_2^3 + 4\alpha_1^2\alpha_2 + 5\alpha_1\alpha_2^2], \qquad L_4 = [\alpha_1^2 + 3\alpha_2^2 + 5\alpha_1\alpha_2],$$

$$L_5 = [2\alpha_1 + 3\alpha_2]. \tag{6.5}$$

Then, the mean time to system failure will take the concise polynomial form.

$$\mathbf{MTSF = a\beta^5 + b\beta^4 + c\beta^3 + d\beta^2 + e\beta + f} \tag{6.6}$$

where,

a is the coefficient of β^5 and its value $1/\alpha_1^3\alpha_2^3$.

b is the coefficient of β^4 and its value $L_5/\alpha_1^3\alpha_2^3$.

c is the coefficient of β^3 and its value $L_4/\alpha_1^3\alpha_2^3$.

d is the coefficient of β^2 and its value $L_3/\alpha_1^3\alpha_2^3$.

e is the coefficient of β and its value $L_2/\alpha_1^3\alpha_2^3$.

f is the intercept and its value $3L_1/\alpha_1^3\alpha_2^3$.

The equation of the MTSF gives a polynomial of the fifth order dependence on replacement or repair rate.

7- A numerical example:

This section is devoted to evaluate the effectiveness of the ventilation system as measured by the (MTSF). Thus, we give a numerical example, that aims at exploring possible ways of “raising” the value of the (MTSF) by changing its parameters. The (MTSF) will be numerically analyzed, using a sample data of radiation readings measured from an Egyptian Phosphate Mine. The calculations of the (MTSF) will be obtained from the general equation (6.6).

The numerical example evaluate the (MTSF) at different values of β for equal (constant) values of ($\alpha_1 = \alpha_2 = \alpha$) having the values α = 1, 5, 9, 15, 30 and 40.

The numerical results can be shown by table (7.1).

Table 7.1

The MTSF at different values of β and equal and constant values of α1 & α2

β	MTSF at	MTSF at	MTSF at	MTSF at	MTSF at	MTSF at
	α = 1	α = 5	α = 9	α = 15	α = 30	α = 40
0.01	6.07101	1.20281	0.66753	0.40031	0.20008	0.15004
0.05	6.37616	1.2142	0.67102	0.40156	0.20039	0.15022
0.08	6.62882	1.22292	0.67367	0.40251	0.20062	0.15035
0.1	6.80951	1.22881	0.67545	0.40314	0.20078	0.15044
0.5	13.4688	1.3619	0.71348	0.41632	0.20398	0.15223
0.8	24.9837	1.48325	0.74532	0.42688	0.20647	0.1536
1	38	1.57606	0.76826	0.43426	0.20816	0.15453
2	244	2.22285	0.90676	0.4756	0.21713	0.1594
4	3074	4.99674	1.34322	0.58501	0.2378	0.17024
7	32444	16.9364	2.6492	0.84123	0.27692	0.18961
10	160076	48.8	5.30927	1.25981	0.32812	0.2134
15	1045236	201.6	15.3946	2.53333	0.44896	0.2653

20	4076146	614.8	38.4254	4.96406	0.6299	0.33672
30	2.9E+07	3321.6	166.954	16.2667	1.26667	0.56228

The numerical results, shown by the above table (7.1), confirm the "dependence of the (MTSF) on "β" for the given values of ($\alpha_1 = \alpha_2 = \alpha$). This conclusion is assured also by the following figure (7.1).

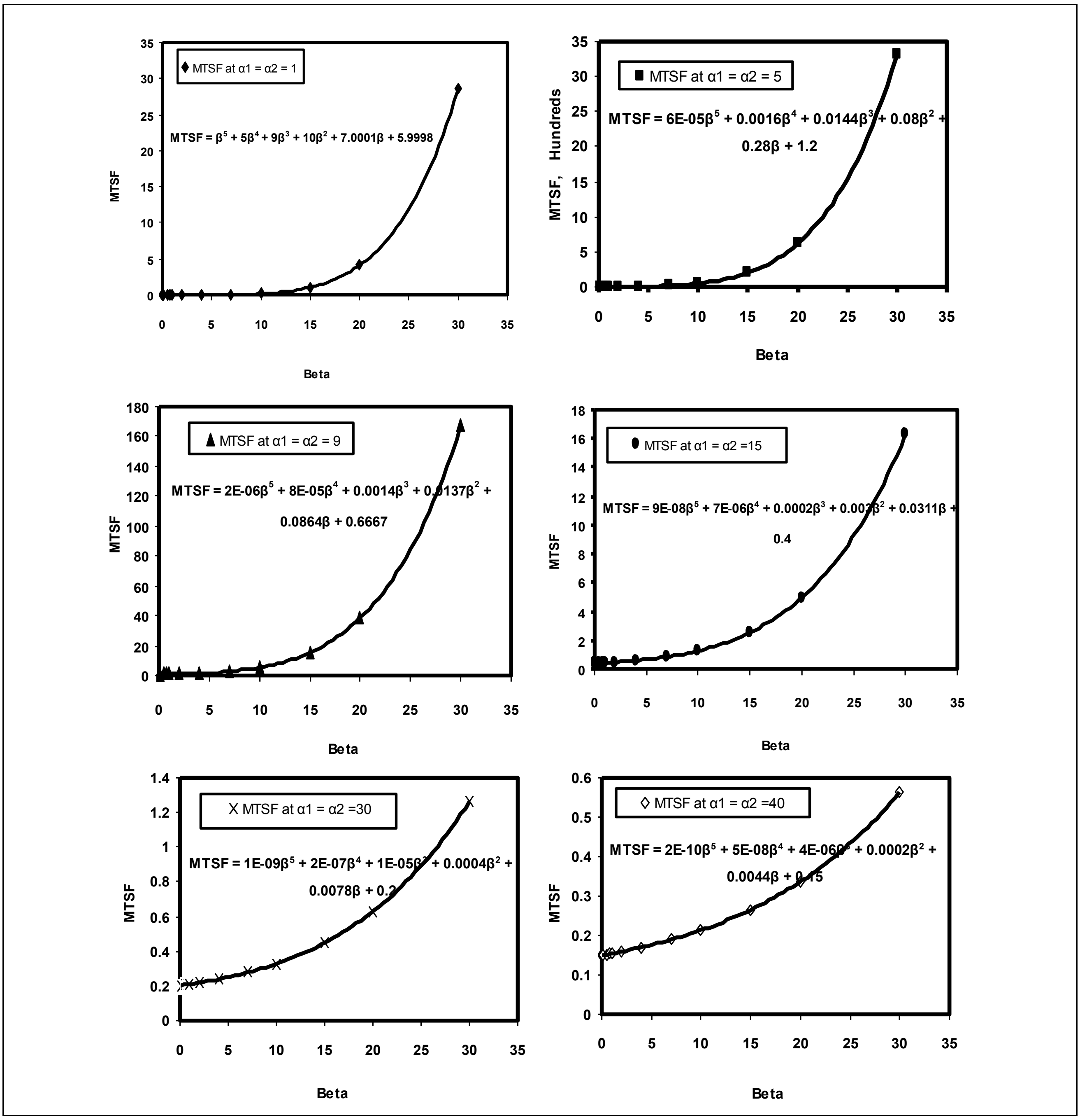

Fig. 7.1. Dependence of MTSF on β at constant values of α of 1, 5, 9, 15, 30 and 40.

The importance of figure (7.1.) is that, it shows an additional features for the relation between "β" and the (MTSF). That is, the polynomial increase in the (MTSF) is significant for values of $\beta > 10$.

Also, figure (7.1) shows that equation (6.6) takes the polynomial form of the fifth order for "small" values of "α" and high values of "β".

In conclusion, the above numerical results confirm that the (MTSF) of the system is "related directly" to β, meaning that the higher the value of "β" – at low values of α-, the "higher" the value of the (MTSF). Consequently, the higher the value of the (MTSF), the "lower" the possibility of the ventilation system to "fail". Such highly "reliable" system will minimize the exposure of the workers to radon hazards and hence elevate the mine's "safety level".

Finally, knowing such numerical results about the (MTSF), makes it easier for the decision-maker to "reject" any fan with specifications that gives "low" value for the (MTSF) of the ventilation system.

8- Suggestions for Future Researches

The model presented in this paper is, in fact, a "generalization" of a previous work [5], because the components of our system (fans) work in "three" different modes not just up and down states.

Thus, our assumptions could be reflecting the "real states" of complex systems like thermal and nuclear power plant. For instance, a steam generator in a thermal power plant can operate in "Partial Failure" mode with a part of tubing having failed, while the important part of the system is isolated and repaired. This "suggests" a new "dimension" for using our statistical model in analyzing such systems. Thus, we can get more realistic measures for assessing reliability as a practical measure of the effectiveness of the system, for the ultimate goal of evaluating the "safety" of nuclear installations.

From a comparative viewpoint, having used a more general assumptions in building our model, a logical perspective confirms that the resulting statistical analysis will "outweigh", the previously given one in [4].

Secondly, according to the derived equations for the (MTSF), which are showing that (MTSF) is related directly to the replacement or repair rate β, i.e. as β increases the (MTSF) increases. We propose that further investigation of our previous equations can be a useful device to predict "the rate" of increase in the (MTSF) with the increase in its parameters.

The final suggestion is concerned with the method of refining a real set of data. It recommends using the power transformation method [1] to refine the given set of data, along with the one we have used in our research. This may constitute a reasonable incentive for conducting a "Comparative study" to choose the best method of data-refinement that gives the highest value of the (MTSF) as a measure of "a reliable performances" [15] of the proposed ventilation system.

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