BAYESIAN INTERIM ANALYSIS OF THE MIXTURE OF LOMAX DISTRIBUTIONS

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1 INTRODUCTION:

Finite mixtures of distributions have proved to be of considerable interest and importance in recent years. They are used as models in a variety of important situations where a non homogeneous population, when it is not possible to distinguish between units of each type, may be considered as comprising two or more components subpopulations mixed in varying proportions. Mixtures of distributions arise frequently in life testing, reliability, biological and physical sciences. Some of the most important references that discussed different types of mixtures of distributions are a monograph by Everitt and Hand (1981) and two survey books by Tittrington, Smith and Makov (1985) and McLachlan and Basford (1988). Review papers are presented by Nigm and AlHussaini (1997) and AlHussaini and Sultan (2001). Often failure can occur for more than one reason and the failure distribution for each reason can be adequately approximated by a simple density function. The overall failure distribution is then a mixture. Several attempts have been made to fit some mixtures to the failure distribution [see for example Mendenhall and Hader (1958) and Kao (1959)].

A random variable X is said to have a mixture distribution $f(x)$ with v components if:

$$f(x) = \sum_{j=1}^{v} p_j f_j(x) \;;$$

where for $j=1, 2, 3, \ldots, v$, p_j are nonnegative real numbers, known as the mixing proportions, such that $\sum_{j=1}^{v} p_j = 1$ and $f_j(x)$ is the density function of the j^{th} component of the mixture.

Several authors have treated the Bayesian estimation of parameters of mixtures of various distributions (the mixed exponential distribution, the mixed Weibull exponential distribution, the mixed Burr type XII and the mixed Weibull distribution). The Bayesian predictive distribution of the mixed Rayleigh distribution is considered by Nigm and El Sayyad (1999).

Nigm and AlHussaini (1997) derived the Bayesian two sample prediction bounds for future observables from a non homogeneous population following a mixture of two Lomax components. AlHussaini (2003) considered the Bayesian two sample predictive density function of a finite mixture of general components. Such components include the Weibull, compound Weibull, Pareto, beta, Gompertz and compound Gompertz distributions.

The Lomax distribution is also known as Pareto distribution of the second kind or Pearson Type VI distribution [see Johnson et al (1994)]. The Pareto distribution is reverse J shaped and positively skewed with a decreasing hazard rate. Several socio- economic and other natural occurring quantities are distributed according to certain statistical distributions with a long tail. Examples of some of these empirical phenomena are distributions of city population sizes, occurrence of natural resources, stock price, fluctuations, size of firms, personal incomes and error clustering in communication circuits. The Pareto distribution has been applied by astronomers to model the brightness distribution of comets. The Pareto family has potential for modeling reliability and life data. Many authors were interested in studying the Pareto distribution [see Nigm and AlHussaini (1997)].

In the present paper attention is directed to a predictive application of the hypothesis testing for the mixed Lomax distribution (interim analysis). Interim analysis discusses a useful procedure in deciding whether or not an experiment should be continued or curtailed. A Bayesian predictive approach is used to determine the probability that if one continued the trial with a further sample of size m, one would come to a particular decision regarding a parameter. As described in Geisser (1992), (1993) and Ismail (2001), what is required is that after a minimum of s observations a posterior probability of at least w is necessary to accept a null hypothesis. However an investigator would like to curtail the experiment if the results at a certain time do not appear sufficiently promising. Geisser (1992) applied the interim analysis on a number of random sampling distributions, among them the Bernoulli, Poisson, exponential and normal with variance equal one and with unknown variance. In Geisser (1993) the same analysis was applied to censored exponential observations. Papandonatos and Geisser (1999) considered the problem of Bayesian interim analysis of lifetime data that are Weibull distributed according to an accelerated-failure time model. Ismail (2001) used the interim analysis regarding a future observation from two parameters Pareto and exponential distributions.

Assuming that a parameter or set of parameters θ belongs to Θ is indicative of the effectiveness of the treatment or agent if θ belongs to Θ_1, a subset of Θ. Therefore if the posterior probability is such that:

$$\Pr[\theta \in \Theta_1 \mid \underline{x}^s] \geq w;$$

where s is the minimum sample size, we decide that the treatment is effective. Otherwise we decide that it is ineffective or not sufficiently effective or even withhold the decision.

For $n+m \geq s$, denote $\underline{X}^{n+m} = (\underline{X}^n, \underline{X}^m)$, where $\underline{X}^n = (X_1, \ldots\ldots, X_n)$, $\underline{X}^m = (X_{n+1}, \ldots\ldots\ldots, X_{n+m})$ and $\underline{X}^{n+m} = (X_1, \ldots\ldots, X_n, X_{n+1}, \ldots\ldots\ldots, X_{n+m})$.

Assuming that $\underline{X}^n$ has been observed, the predictive distribution function $F(\underline{X}^m \mid \underline{x}^n)$ can be calculated to test the hypothesis:

$$H_0 : \theta \leq q \qquad \text{versus} \qquad H_1 : \theta > q \,;$$

where H_0 is accepted if :

$$\Pr(\theta \leq q \mid \underline{x}^{n+m}) \geq w$$

After the first n observations the predictive distribution $F(\underline{X}^m \mid \underline{x}^n)$ is used to obtain the predictive probability, that if testing continued, a decision will be reached.

Besides this introductory section, this paper includes two other sections. The second section presents the Bayesian interim analysis of the mixture of Lomax distributions using the informative prior. Numerical illustration is presented in the third section.

2 Bayesian Interim Analysis of the Mixture of Lomax Distributions Using the Informative Prior:

The density function of the mixture of two Lomax distributions takes the following form:

$$f(x)=\sum_{i=1}^{2} p_i \frac{a_i}{c_i}\left(1+\frac{x}{c_i}\right)^{-(a_i+1)} \quad ; x>0,\ 0<p_1<1,\ \ p_2=1-p_1,\ \ a_i, c_i\ >0\ ,\ i=1,2, \qquad (2.1)$$

where p_1 is the mixing proportion, c_1 , c_2 and a_1 , a_2 are the scale and the shape parameters of the two components, respectively. We shall assume that both c_1 and c_2 are known. Nigm and AlHussaini (1997) supposed that n units from a population with density function (2.1) are subjected to life testing experiment and that the test is terminated after predetermined time t_0 (type I censoring). It is assumed that an item can be attributed to the appropriate subpopulation after it had failed. Suppose that r units have failed during the interval $(0,t_0)$: r_1 from the first subpopulation and r_2 from the second subpopulation, such that $r_1+r_2=r$ and n-r units which cannot be identified with respect to subpopulation are still functioning. Let x_{ij} denote the failure time of the j^{th} unit belonging to the i^{th} subpopulation and that $x_{ij}<t_0$, $j=1,\ldots, r_i$, $i=1, 2$. Such scheme of sampling was first suggested by Mendenhall and Hader (1958). The likelihood function takes the following form:

$$L(\theta,\underline{x}) \propto \prod_{i=1}^{2} \prod_{j=1}^{r_i} \left[p_i f_i(x_{ij})\right] [R(t_0)]^{n-r}.$$

Nigm and AlHussaini (1997) supposed that p_1, a_1 and a_2 are independent random variables such that p_1~beta (b,d) and $a_i \sim$ gamma (β_i,γ_i), for i=1,2 and mentioned that the posterior density function of the parameters (θ) , $\pi^*(\theta\,|\,\underline{X})$, is given by:

$$\pi^*(\theta\,|\,\underline{x}^n) \propto \sum_{j_1=0}^{n-r}\binom{n-r}{j_1}\prod_{i=1}^{2} p_i^{\delta_i} a_i^{\eta_i-1}\exp[-\xi_i a_i] \ ;$$

where

$$\delta_1=r_1+j_1+b-1,\ \delta_2=n-r_1-j_1+d-1 \ ; \quad \text{for } i=1,2,\ \eta_i=r_i+\beta_i,\ \xi_i=A_{ij_i}+\gamma_i$$

$$\text{and}\quad A_{ij_i}=\sum_{j=1}^{r_i}\ell n\left(1+\frac{x_{ij}}{c_i}\right)+j_i\ell n\left(1+\frac{t_0}{c_i}\right).$$

Regarding the mixed Lomax distribution and assuming type I censoring for the informative and the future samples, the Likelihood function of the m future observations is as follows:

$$L(\underline{X^m}\,|\,\theta)=\prod_{i=1}^{2}\prod_{j=1}^{k_i}\left[\frac{p_i a_i}{c_i}\exp[-(a+1)_i\Im_{ij}]\right]\ [R(t_0)]^{m-k};$$

$$\text{where}\quad \Im_{ij}=\ell n(1+\frac{x_{ij}}{c_i}) \qquad R(t_0)=\sum_{i=1}^{2}p_i\exp[-a_i\ \ell n\left(1+\frac{t_0}{c_i}\right)]$$

where k units of the m future observations have failed during the interval $(0, t_0)$: k_1 from the first subpopulation and k_2 from the second subpopulation, such that $k_1+k_2=k$ and $m-k$ units which cannot be identified as to subpopulation are still functioning.

Using the previous likelihood $L(\underline{X^m}|\,\theta)$ and the posterior distribution and by multiplying the two components and integrating out the three parameters, the predictive density of $\underline{X^m}$ takes the following form:

$$f\left(\underline{X^m}\,|\,\underline{x^n}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} \binom{\tau_2+j_1}{j_1} \left[\prod_{i=1}^{2} \prod_{j=n+1}^{k_i} \exp[\Im_{ij}]\right]^{-1} [\prod_{i=1}^{2} \; G(\eta_i^*, \xi_i^*)] \; \beta(\tau_3, \tau_4);$$

where $\tau_1 = j_1,\ \tau_2 = (\mathrm{m}+n-r-k-j_1),\ \tau_3 = j_1+k_1+b+r_1\ ,\ \tau_4 = n+\mathrm{m}-r-k+k_2+d+r_2-j_1\ ,$

$$\xi_i^* = \tau_i \ell n(1+t_0/c_i) + \sum_{j=n+1}^{k_i} \Im_{ij} + \gamma_i + \sum_{j=1}^{r_i} \Im_{ij} \;\text{ and }\; \eta_i^* = k_i + \beta_i + r_i \;. \qquad (2.2)$$

2.1 *Hypothesis Testing on p_1:*

The joint posterior distribution $\pi^*(\,\theta\,|\underline{x^{n+m}}\,)$takes the following form:

$$\pi^*\left(\theta\,|\,\underline{x^{n+\mathrm{m}}}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} \binom{\tau_2+j_1}{j_1} p_1^{\tau_3-1} p_2^{\tau_4-1} \{\prod_{i=1}^{2} a_i^{\eta^*_i-1} \exp[-\xi^*_i\, a_i]\} \;. \qquad (2.3)$$

To obtain the posterior probability of accepting the null hypothesis that $p_1 \le q_0$ using the $n+m$ observations, we derive the marginal posterior distribution of p_1 which takes the following form:

$$f\left(p_1\,|\,\underline{x^{n+\mathrm{m}}}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} [\prod_{i=1}^{2} \; G(\eta_i^*, \xi_i^*)] \binom{\tau_2+j_1}{j_1} p_1^{\tau_3-1} p_2^{\tau_4-1}.$$

The posterior probability of accepting the null hypothesis is then given by:

$$\Pr\left(p_1 \le q_0\,|\,\underline{x^{n+m}}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} \{\prod_{i=1}^{2} \; G(\eta_i^*, \xi_i^*)\} \binom{\tau_2+j_1}{j_1} in\,\beta_{q_0}(\tau_3, \tau_4) \;.$$

Due to some mathematical intractability the predictive probability that the previous probability is greater than a fixed value (Pr {Pr [$p_1 \le q_0\,|\,\underline{x^{n+m}}$] $\ge w$ }) could not be obtained. But as an approximation the predictive expectation of accepting the null hypothesis of the mixing proportion ($E\{\Pr\left(p_1 \le q_0\,|\,\underline{x^{n+m}}\right)|\,\underline{x^n}\}$) is calculated. The following predictive expectation is calculated with respect to the predictive distribution of $\underline{X^m}$, $f(\underline{X^m}|\underline{x^n})$, which mentioned in (2.2).

$$E\{\Pr\left(p_1 \le q_0 \mid \underline{x^{n+m}}\right) \mid \underline{x^n}\} \propto \int_{\underline{X^m}} \Pr\left(p_1 \le q_0 \mid \underline{x^{n+m}}\right) f\left(\underline{X^m} \mid \underline{x^n}\right) d\underline{x^m}$$

$$= \int_{\underline{X^m}} \{\sum_{j_1=0}^{\tau_2+j_1} [\prod_{i=1}^{2} G(\eta_i^*, \xi_i^*)] \binom{\tau_2+j_1}{j_1} in\beta_{q_0}(\tau_3,\tau_4)\} \{ \sum_{j_1=0}^{\tau_2+j_1} \prod_{i=1}^{2} \binom{\tau_2+j_1}{j_1}$$

$$\times \left[\prod_{j=n+1}^{k_i} \exp\{\Im_{ij}\} \right]^{-1} G(\eta_i^*, \xi_i^*)\, \beta(\tau_3,\tau_4)\} d\underline{x^m}. \qquad (2.4)$$

2.2 *Hypothesis Testing on a_1:*

Integrating the joint posterior distribution mentioned in (2.3) out of p_1 and a_2, the marginal posterior distribution of a_1 takes the following form:

$$\pi^*\left(a_1 \mid \underline{x^{n+m}}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} \binom{\tau_2+j_1}{j_1} a_1^{\eta^*_1-1} \exp[-\xi^*_1\, a_1] \beta(\tau_3,\tau_4)\ \ G(\eta_2^*, \xi_2^*)\ .$$

Using the previous posterior distribution, the posterior probability of accepting the null hypothesis of a_1 is given by:

$$\Pr\left(a_1 \le q_1 \mid \underline{x^{n+m}}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} \binom{\tau_2+j_1}{j_1} in\, G_{q_1}(\eta_1^*, \xi^*_1)\ \beta(\tau_3,\tau_4)\ \ G(\eta_2^*, \xi^*_2).$$

As an approximation, the predictive expectation of the previous probability is obtained in the following form using the predictive distribution mentioned in (2.2):

$$E\{\Pr\left(a_1 \le q_1 \mid \underline{x^{n+m}}\right) \mid \underline{x^n}\} \propto \int_{\underline{X^m}} \Pr\left(a_1 \le q_1 \mid \underline{x^{n+m}}\right) f\left(\underline{X^m} \mid \underline{x^n}\right) d\underline{x^m}$$

$$= \int_{\underline{X^m}} \{\sum_{j_1=0}^{\tau_2+j_1} \binom{\tau_2+j_1}{j_1} inG_{q_1}(\eta_1^*, \xi_1^*)\ \beta(\tau_3,\tau_4)\ G(\eta_2^*, \xi_2^*)\}$$

$$\times \{\prod_{i=1}^{2} \sum_{j_1=0}^{\tau_2+j_1} \binom{\tau_2+j_1}{j_1} \left[\prod_{j=n+1}^{k_i} \exp[\Im_{ij}] \right]^{-1} G(\eta_i^*, \xi_i^*)\ \beta(\tau_3,\tau_4)\} d\underline{x^m}. \qquad (2.5)$$

2.3 *Hypothesis Testing on a_2:*

Integrating the joint posterior distribution mentioned in (2.3) out of p_1 and a_1, the marginal posterior distribution of a_2 is given by:

$$\pi^*\left(a_2 \mid \underline{x^{n+m}}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} \binom{\tau_2+j_1}{j_1} G(\eta^*_1, \xi^*_1) \beta(\tau_3,\tau_4)\ a_2^{\eta^*_2-1} \exp[-\xi^*_2\, a_2]\ .$$

Therefore, the posterior probability of accepting the null hypothesis takes the following form:

$$\Pr\left(a_2 \le q_2 \mid \underline{x}^{n+m}\right) \propto \sum_{j_1=0}^{\tau_2+j_1} \begin{pmatrix} \tau_2 + j_1 \\ j_1 \end{pmatrix} in\, G_{q_2}(\eta_2^*, \xi_2^*) G(\eta_1^*, \xi_1^*)\, \beta(\tau_3, \tau_4) \ .$$

Similarly, using the predictive distribution mentioned in (2.2) the predictive expectation of the previous probability is derived as follows:

$$E\{\Pr\left(a_2 \le q_2 \mid \underline{x}^{n+m}\right) \mid \underline{x}^n\} \propto \int_{\underline{X}^m} \Pr\left(a_2 \le q_2 \mid \underline{x}^{n+m}\right) f\left(\underline{X}^m \mid \underline{x}^n\right) d\underline{x}^m$$

$$= \int_{\underline{X}^m} \{ \sum_{j_1=0}^{\tau_2+j_1} \begin{pmatrix} \tau_2 + j_1 \\ j_1 \end{pmatrix} inG_{q_2}(\eta_2^*, \xi_2^*) \beta(\tau_3, \tau_4)\, G(\eta_1^*, \xi_1^*) \}$$

$$\{ \prod_{i=1}^{2} \sum_{j_1=0}^{\tau_2+j_1} \begin{pmatrix} \tau_2 + j_1 \\ j_1 \end{pmatrix} \left[\prod_{j=n+1}^{k_i} \exp[\Im_{ij}] \right]^{-1} G(\eta_i^*, \xi_i^*)\, \beta(\tau_3, \tau_4) \} d\underline{x}^m \ . \tag{2.6}$$

Using the Simpson's rule, the integrations in (2.4), (2.5) and (2.6) are numerically calculated to have the predictive expectations of the probability of accepting the null hypotheses on p_1, a_1, a_2.

It can be noted that substituting by $\{ \beta_i = \gamma_i = b = d = 0 \ , i = 1,2 \}$ in the results of the informative prior, we get the results of the non informative prior.

3 The Numerical Illustration:

Using an informative and a non informative priors, the following results are calculated. Applying the programs of Mathematica, the predictive expectations of accepting the null hypotheses are computed.

a. Sample one

Nigm and Al-Hussaini (1997) assumed that c_1=0.6 and c_2=0.8. They considered that b=4, d=2, γ_1 =0.7 , γ_2=0.9, β_1=3.5 and β_2=4. The prior information about p_1, a_1 and a_2 suggests that p_1=0.644, a_1=2.431 and a_2 =4.25. For n=20 and t_0=0.5, a sample is generated from the mixture of Lomax components in such a way that $X_{ij} \le t_0$, j=1,…r_i, i=1,2. The observations are .009997, .037338, .062746, .095554, .157269, .183814, .184866, .188411, .278438, .385052, .408002, .470246, .484809, (r_1 =13), .026219, .028740, .234755, .393827, .486525, (r_2 =5).

b. Sample two

A simulated sample is generated with the same specifications of the sample generated by Nigm and Al-Hussaini (1997). The observations are .327706, .133189, .335364, .402836,

.302448, .094759, .026533, .043157, .027158, .065419, (r_1 =10), .040592, .059651, .162686, .124618, .251748, .006457, (r_2 =6).

c. Sample three

A random type I censored sample of size n=40 is generated from the mixture of two Lomax distributions with a_1=2.5, a_2=4.5, c_1 =.55, c_2 = .85, p_1=.75 and t_0=0.3. The observations are .155449, .036772, .1302, .081721, .13324, .088452, .256831, .068431, .051312, .046717, .013405, .002042, .071606, .147396, .158058, .213121, .084003, .086357, .199691, (r_1 =19), .286071, .033854, .01679, .02885, .077981, .069582, .200291, (r_2 =7).

d. Sample four

A random type I censored sample of size n=35 is generated from the mixture of two Lomax distributions with a_1=2.3, a_2=4.0, c_1 =.65, c_2 = .75, p_1=.7143 and t_0=0.45. The observations are .087881, .11817, .022716, .106213, .135772, .400004, .373365, .243551, .243352, .029022, .076288, .316557, .04905, .150919, .170466, .071076, .230336, (r_1 =17), .120867, .055419, .431498, .164816, .122368, .395678, .176396, .033223, .141842, .077045, (r_2 =10).

e. Sample five

A random type I censored sample of size n=40 is generated from the mixture of two Lomax distributions with a_1=2.1, a_2=4.1, c_1 =.69, c_2 = .77, p_1=.75 and t_0=0.40. The observations are .07851, .1523, .299931, .021792, .005823, .005916, .313086, .153878, .064404, .248117, .250234, .108705, .017851, .045161, .218616, .162121, .012511, .204438, .159048, .070247, (r_1 =20), .059972, .020639, .178397, .232956, .083893, .025722, .018481, .055046, .21773, (r_2 =9).

Tables (1), (2) and (3) present the predictive expectations of accepting the null hypotheses with respect to p_1, a_1 and a_2 respectively.

Table (1) The predictive expectation of the probability of accepting the null hypothesis of p_1

The prior		The informative prior			The non informative prior		
Sample	t_0 q_0	.04	.2	.4	.04	.2	.4
(1)	.1	1E-17	2E-14	8E-12	4E-18	7E-15	2E-12
	.2	1E-13	7E-9	3E-6	1E-12	1E-13	1E-6
	.3	6E-9	1E-5	0.005	2E-9	4E-6	0.002
	.4	1E-6	0.002	0.73	3E-7	0.001	0.264
	.5	4E-5	0.55	0.750	1E-5	0.03	0.527
(2)	.1	4E-15	1E-11	9 E-9	4E-16	3E-12	1E-9
	.2	1E-10	7E-7	0.0003	1E-11	1E-07	5E-5
	.3	6E-8	0.02	0.138	7E-9	5E-5	0.023
	.4	4E-6	3E-4	0.35	5E-7	0.004	0.581
	.5	9E-5	0.37	0.836	1E-5	0.093	0.829
(3)	.1	1E-21	1E-18	3E-16	2E-22	4E-19	8E-17
	.2	9E-15	1E-11	2E-9	1E-15	3 E-12	6E-10
	.3	4E-11	6E-8	1E-5	7E-12	1E-8	2E-06
	.4	1E-8	2E-5	0.004	1E-9	3E-6	0.001
	.5	6E-7	7E-4	0.191	8E-8	0.002	0.180
(4)	.1	1E-18	1E-9	1E-13	8E-19	2E-16	5E-14
	.2	3E-12	5E-6	4E-7	2E-12	7E-10	1E-7
	.3	1E-8	6E-6	0.002	9 E-9	2E-6	0.0005
	.4	2E-6	0.0010	0.356	2 E-6	5E-4	0.115
	.5	1E-4	0.050	0.548	8E-5	0.022	0.484
(5)	.1	3E-17	9E-10	1E-13	3E-20	1E-19	3E-16
	.2	1E-12	2E-6	2E-7	8E-14	2E-13	8E-10
	.3	1E-6	4E-6	0.001	2E-10	9 E-10	2E-06
	.4	3E-6	4E-4	0.133	4 E-8	1E-7	0.0003
	.5	0.01	.02	0.485	3E-5	6E-6	0.011

Table (2) The predictive expectation of the probability of accepting the null hypothesis of a_1

The prior		The informative prior			The non informative prior		
Sample	t_0 q_1	.04	.2	.4	.04	.2	.4
(1)	5	0.018	0.096	0. 352	0.008	0.200	0.414
	10	0.018	0.116	0.353	0.008	0.204	0.414
	15	0.018	0.117	0.353	0.008	0.204	0.414
	20	0.018	0.118	0.353	0.008	0.204	0.414
(2)	5	0.016	0.014	2E-6	0.004	0.016	0.559
	10	0.016	0.015	2E-6	0.004	0.016	0.565
	15	0.016	0.015	2E-6	0.004	0.016	0.565
	20	0.016	0.015	2E-6	0.004	0.016	0.565
(3)	5	0.003	0.186	0.848	0.0008	0.045	0.093
	10	0.003	0.186	0.851	0.0008	0.045	0.093
	15	0.003	0.186	0.851	0.0008	0.045	0.093
	20	0.003	0.186	0.851	0.0008	0.045	0.093
(4)	5	0.025	0.379	0.446	1E-5	0.006	0.014
	10	0.026	0.419	0.449	1E-5	0.007	0.015
	15	0.026	0.419	0.449	1E-5	0.007	0.015
	20	0.026	0.419	0.449	1E-5	0.007	0.015
(5)	5	0.014	0.256	0.313	0.009	0.232	0.260
	10	0.014	0.256	0.314	0.009	0.232	0.260
	15	0.014	0.256	0.314	0.009	0.240	0.260
	20	0.014	0.257	0.314	0.009	0.240	0.260

Table (3) The predictive expectation of the probability of accepting the null hypothesis of a_2

The prior		The informative prior			The non informative prior		
Sample	t_0 / q_2	.04	.2	.4	.04	.2	.4
(1)	5	0.008	0.208	0.591	0.004	0.104	0.172
	10	0.008	0.212	0.599	0.005	0.202	0.271
	15	0.008	0.212	0.599	0.005	0.204	0.271
	20	0.008	0.212	0.599	0.005	0.204	0.271
(2)	5	0.001	0.004	0.015	5E-5	0.002	0.009
	10	0.001	0.011	0.015	2E-4	0.002	0.009
	15	0.001	0.011	0.015	3E-4	0.002	0.009
	20	0.005	0.011	0.0162	3E-4	0.002	0.009
(3)	5	0.001	0.191	0.271	1E-4	0.008	0.096
	10	0.003	0.194	0.828	5E-4	0.036	0.097
	15	0.003	0.194	0.851	7E-4	0.045	0.097
	20	0.003	0.194	0.851	7E-4	0.045	0.097
(4)	5	0.024	0.399	0.449	0.001	0.006	0. 012
	10	0.026	0.439	0.449	0.002	0.007	0. 012
	15	0.026	0.439	0.449	0.002	0.007	0. 012
	20	0.026	0.439	0.449	0.002	0.007	0. 012
(5)	5	0.013	0.305	0.864	0.008	0.156	0.787
	10	0.014	0.314	0.864	0.009	0.159	0.789
	15	0.014	0.314	0.867	0.009	0.159	0.789
	20	0.014	0.314	0.867	0.009	0.159	0.789

The decision of continuing the draw of the next m observations is based on the value of the predictive expectations of the probability of accepting the null hypotheses.

From the previous tables, we get the following:

1- In the cases of the informative prior and the non informative prior, it is noticed that the predictive expectations of accepting the null hypotheses on p_1, a_1 and a_2 increase with the increase of q_0 , q_1 and q_2 which is expected.

2- The values of the predictive expectations of the probability of accepting the null hypotheses are sensitive to the variations in t_0.

3- We notice that the predictive expectations of accepting the null hypotheses in the case of the informative prior are greater than them in the case of the non informative prior in most cases.

Bibliography

Aitchison, J. and Dunsmore, I.R. (1975). *Statistical prediction analysis*, Cambridge University Press.

AlHussaini, E.K. (1999). Bayesian prediction under a mixture of two Exponential components model based on type I censoring, *Journal of applied statistical science*, 8, 173-185.

AlHussaini, E.K. (2003). Bayesian predictive density of order statistics based on finite mixture models, *Journal of statistical planning and inference*, 113, 15-24.

AlHussaini, E.K. and Sultan, K.S. (2001). *Reliability and hazard based on finite mixture models*, Hand book of Statistics, Elsevier Science B.V. 20, 139- 183.

Everitt, B.S. and Hand, D. J. (1981). *Finite mixture distributions,* Chapman and Hall, London.

Geisser, S. (1992).On the curtailment of sampling, *Canadian Journal of Statist*istics, 20, 297-309.

Geisser, S. (1993). Bayesian interim analysis of censored exponential observations, *Statistics and Probability Lett*ers , 18, 163-168.

Ismail, M. A. (2001). Bayesian interim analysis regarding a future observation from two-parameter Pareto and exponential distributions, *The Egyptian Statistical Journal*, 1-26.

Johnson, N. L., S. Kotz and N. Balakrishnan (1994). *Continuous Univariate Distributions*, Vol. 1,2nd Edn. Wiley, New York.

Kao, J. H. K. (1959). A graphical estimation of mixed Weibull parameters in life testing of electron tubes, *Technometrics* , 1, 389- 407.

McLachlan, G.J. and Basford, K.E. (1988). *Mixture Models*, Marcel Dekker, Inc., New York.

Mendenhall, W. and Hader, R. J. (1958). Estimation of the parameter of life mixed exponentially distributed random failure time distributions from censored test data, *Biometrika* ,45, 504 - 520.

Nigm, A.M and Hamdy, H. I. (1987). Bayesian prediction bounds for the Pareto life time model, *Communications in Statistics* –Theory and Methods. 16, 1761-1772.

Nigm, A.M and AlHussaini, E.K.. (1997). *Bayesian Prediction bounds for Pareto lifetime distributions and their finite mixtures*, Technical report, King Abdulaziz University.

Nigm, A. M. and Elsayyad, G. M. (1999). *Bayesian Prediction bounds for Rayleigh lifetime distributions and their finite mixtures*, Technical report, King Abdulaziz University.

Papandonatos, G.D. and Geissser, S. (1999). Bayesian interim analysis of life-time data, *Canadian Journal of Statist*istics, 27, 1- 21.

Titterington, D.M., Smith, A.F.M. and Makov, V.E. (1985). *Statistical analysis of finite mixture distributions,* John Wiley and Sons, Inc., New York.