Parameter estimation for first-order superdiagonal bilinear time series: An algorithm for maximum likelihood procedure

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Abstract
Thanks to their possible application to a wide variety of fields including signal, demography, economics..., the bilinear time series models have acquired a great importance in the statistical literature.
This paper deals with the design of a new algorithm for estimating the parameters of a particular bilinear time series model. This iterative algorithm is based on maximum likelihood method and Kalman filter algorithm.
To demonstrate the efficiency of our algorithm, series of simulations were performed.

Keywords: First-order Superdiagonal Bilinear Time Series Model, Maximum Likelihood, Kalman Filter.
1 Introduction

The last two decades have seen an increasing interest in bilinear time series models, introduced by Granger and Anderson (1978), and extended by Tong (1990), Guegan (1993). This interest is due to the widespread use of bilinear models in various fields: signal, environmental studies, demography, economics... etc.

Regardless of theoretical difficulties, the fundamental properties have been solved for several particular cases. The stationarity, invertibility and Markovian representation, have been solved, for first-order superdiagonal model by Guegan (1981), and for first-order diagonal bilinear model by Pham and Tran (1981).

Testing problems have been treated, widely, for null hypothesis of linear Autoregressive dependence against bilinear alternatives, see for instance Benghabrit and Hallin (1992, 1996, 1998).

Parameter estimation has been studied, for several particular cases by Pham and Tran (1981), Guegan (1984), Pham (1985), Guegan and Pham (1989), Bourin and Bondon (1997), Hristova (2005).

The present work focus on a first-order superdiagonal bilinear time series model, that has been studied by Guegan (1981), who has proposed the method moments for estimating the parameters of such model.

In our contribution, we suggest to estimate parameters of first-order superdiagonal bilinear model, by maximum likelihood method. We construct the log-likelihood function using Kalman filter algorithm (see Hamilton (1994)), which is applicable to non-linear process. The log-likelihood function made in this way, is numerically maximized using the Powell method (1964) and Brent method (1976). To examine the properties of our estimators, series of Monte Carlo simulations experiments have done.

This paper is organized as follows: In section 2 we present the definition and some main properties of first-order superdiagonal bilinear model. In Section 3 we describe the methodology used to establish our estimating algorithm. The performance of this algorithm is examined by Monte Carlo simulations, and compared to method moments, in section 4. Finally, the conclusion is made in section 5.

2 First-order superdiagonal bilinear model

The general definition of bilinear model given by Granger and Anderson (1978) is

$$X_t = \sum_{i=1}^{p} a_i X_{t-i} + \sum_{j=1}^{q} c_j e_{t-j} + \sum_{k=1}^{P} \sum_{j=1}^{Q} b_{kj} X_{t-k} e_{t-j} + e_t,$$

(1)

where \( \{X_t\}_{t \in \mathbb{Z}} \) is a discrete time process, \((a_i, 1 \leq i \leq p), (c_j, 1 \leq j \leq q)\) and \((b_{kj}, 1 \leq k \leq P, 1 \leq j \leq Q)\) are the coefficients of the model, and where \( e_t \) belongs to iid Gaussian process with zero-mean and finite variance \( \sigma^2 \). In statistical literature, the above model is often denoted as the BL(p,q,P,Q) model. We distinguish three subclass of these models: the
diagonal models, for which \( b_{kj} = 0 \) if \( j \neq k \), subdiagonal models, for which \( b_{kj} = 0 \) if \( j < k \), and the superdiagonal models, for which \( b_{kj} = 0 \) if \( j > k \).

Herein, we restrict our study to investigate the first-order univariate superdiagonal bilinear model BL(0,0,2,1) defined by

\[
X_t = e_t + bX_{t-2}e_{t-1}, \quad (2)
\]

with \( b = b_{21} \) and \( e_t \sim NID(0, \sigma^2) \).

Guegan (1981) has established the necessary and sufficient conditions for the existence of a stationary solution in (2) and given the conditions for invertibility. We state these results below:

**Theorem 1 (Guegan (1981))**

If \( b^2 \sigma^2 < 1 \) then there exists a unique strictly stationary process \( X_t \), satisfying (2), given by

\[
X_t = e_t + \sum_{j \geq 1} \left[ b^j e_{t-2j} \right] \prod_{k=1}^{j} \{ e_{t-2k+1} \}, \quad (3)
\]

which converges on quadratic average.

**Theorem 2 (Guegan (1981))**

If \( b^2 \sigma^2 < \frac{1}{2} \), then the model (2) is invertible on \( b \).

In this case

\[
e_t^b = X_t - \sum_{j=1}^{t-2} \left( -b \right)^j bX_{t-j-2}X_{t-j-1} \prod_{l=1}^{j} X_{t-l-1} - bX_{t-2}X_{t-1}. \quad (4)
\]

In the next section, we will illustrate the important role of the expression (4) in our estimating algorithm.

### 3 Estimating the parameters of the first-order superdiagonal bilinear model: Algorithm description

Let \( \theta = (b, \sigma^2) \) be the vector of unknown parameters and let \( X = (x_1, \ldots, x_n) \) be the observed data. In this study we propose to estimate \( \theta \) by means of the Maximum Likelihood method. We assume that the process \( \{X_t\}_{t=1}^n \) is Gaussian.

The likelihood function of the process \( \{X_t\}_{t=1}^n \) can be expressed by:

\[
f(X; \theta) = f(x_1; \theta) \prod_{t=2}^{n} f(x_t|X_{t-1}; \theta), \quad (5)
\]
where \( X_t = (x_1, \ldots, x_t) \) and \( f(x_t|X_{t-1}; \theta) \) denoted the recursively expressed probability density function of \( X_t \) given \( X_{t-1} \). The log likelihood function (denoted \( L(X; \theta) \)) can be found by taking logs of (5):

\[
L(X; \theta) = \log f(x_1; \theta) + \sum_{t=2}^{n} \log f(x_t|X_{t-1}; \theta).
\]

The distribution of \( X_t \) conditional on \((X_1, X_2, \ldots, X_{t-1})\) is Gaussian with mean \( \hat{X}_{t|t-1} \) and variance \( \hat{M}_{t|t-1} = E[(X_t - \hat{X}_{t|t-1})^2] \) (see Hamilton (1994)). Hence, the log likelihood function of \( \theta \) is:

\[
L(X; \theta) = -\frac{n}{2} \log(2\pi) - \sum_{t=1}^{n} \log(\hat{M}_{t|t-1}) - \sum_{t=1}^{n} \frac{(x_t - \hat{X}_{t|t-1})^2}{\hat{M}_{t|t-1}}.
\]

Note that \( \hat{X}_{t|t-1} \) and \( \hat{M}_{t|t-1} \) can be computed by Kalman filter algorithm presented in section (3.1).

### 3.1 State representation and Kalman filter algorithm

To calculate the log-likelihood function, we have to construct a convenient space-state representation of our BL(0,0,2,1) model (2). Our construction leads to the state form (8), with time-varying parameters, given by

\[
\begin{align*}
\xi_{t+1} &= F_t \xi_t + v_{t+1} \\
X_t &= H \xi_t
\end{align*}
\]

where \( t = 1, \ldots, n, \xi_t = [X_t, X_{t-1}, X_{t-2}]' \), \( H = [1, 0, 0], v_t = [e_t, 0, 0]' \) and \( F_t = \begin{pmatrix} 0 & b e_t & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \)

is time-varying parameter (Hamilton (1994)).

It’s worthwhile to mention that the state representation (8) is derived from polynomial state affine representation given by Sontag (1979), and utilized by Guegan (1987) to prove some properties of bilinear time series models.

The matrix \( F_t \) depends on parameter \( b \) and \( e_t \), so, we have also mentioned that if \( b \) and \( \sigma^2 \) fulfilled the conditions of stationarity and invertibility, \( e_t \) is expressed (see equation (4)) as function of \( b \) and the observations \( x_k \) for \( k = 1, \ldots, t \). It implies that \( F_t \) is a function of \( x_k \) for \( k = 1, \ldots, t \). Thus, under the condition, \( b^2 \sigma^2 < \frac{1}{2} \), the Kalman filter can also be adapted to our state representation (8), which is not necessarily unique.

The Kalman filter recursively generates an optimal forecast \( \hat{\xi}_{t+1|t} \) of the state vector \( \xi_{t+1} \), \( t = 1, \ldots, n \). The recursion begins with \( \hat{\xi}_{1|0} \), which denotes a forecast of \( \xi_1 \), so, we have

\[
\hat{\xi}_{1|0} = E[\xi_1] = [0, 0, 0]',
\]
with associated mean square error

\[
P_{t|0} = E\{[ξ_1 - E[ξ_1]]|ξ_1 - E[ξ_1]|'\} = \begin{pmatrix} \frac{\sigma^2}{1-b_2}\delta_2 & 0 & 0 \\ 0 & \frac{\sigma^2}{1-b_2}\delta_2 & 0 \\ 0 & 0 & \frac{\sigma^2}{1-b_2}\delta_2 \end{pmatrix}.
\]

These results are obtained from the work of Guegan (1984), where the properties of BL(0,0,2,1) model are studied.

Given starting values \(\hat{ξ}_{1|0}\) and \(P_{1|0}\), the next step in Kalman filter algorithm is to calculate \(\hat{ξ}_{2|1}\) and \(P_{2|1}\). The calculations for \(t = 1, \ldots, n\) have the same basic form, so we will describe them in general.

Step 1: Compute the forecasting \(\hat{X}_{t|t-1}\) of the observation \(X_t\), and the error \(\hat{M}_{t|t-1}\) of this forecast, given, respectively, by the equations \(\hat{X}_{t|t-1} = H\hat{ξ}_{t|t-1}, \hat{M}_{t|t-1} = HP_{t|t-1}H'\).

Step 2: Updating the state vector \(\hat{ξ}_{t|t}\), given by the equation

\[
\hat{ξ}_{t|t} = \hat{ξ}_{t|t-1} + P_{t|t-1}H'[HP_{t|t-1}H']^{-1}[X_t - H\hat{ξ}_{t|t-1}].
\]

Compute \(P_{t|t}\) the MSE of this updated projection, given by the equation \(P_{t|t} = P_{t|t-1} - P_{t|t-1}H'[HP_{t|t-1}H']^{-1}HP_{t|t-1}\).

Step 3: Compute the forecasting \(\hat{ξ}_{t+1|t}\) given by the equation \(\hat{ξ}_{t+1|t} = F_t\hat{ξ}_{t|t}\), and the MSE \(P_{t+1|t} = F_tP_{t|t}A(t)F_t' + Q\) of this forecast.

where

* \(\hat{ξ}_{t|t-1} = E[ξ_t/X_1, ..., X_{t-1}]\).
* \(P_{t|t-1} = E[(ξ_t - \hat{ξ}_{t|t-1})(ξ_t - \hat{ξ}_{t|t-1})'] = V(ξ_t - \hat{ξ}_{t|t-1})\).
* \(\hat{X}_{t|t-1} = E[X_t/X_1, ..., X_{t-1}]\).
* \(\hat{M}_{t|t-1} = V(X_t - X_{t|t-1})\).
* \(\hat{ξ}_{t|t} = E[ξ_t/X_1, ..., X_t]\).
* \(P_{t|t} = E[(ξ_t - \hat{ξ}_{t|t})(ξ_t - \hat{ξ}_{t|t})'] = V(ξ_t - \hat{ξ}_{t|t})\).
* \(I_d\) is 3 \times 3 identity matrix.
* \(Q = \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\).

Our aim is to estimate \(b\) and \(\sigma^2\) by applying the maximum likelihood method. It’s straightforward to show that it is difficult to find the partial derivatives of \(L(X; θ)\), so, it is more natural to use a method which does not need derivatives. Therefore, we used the method of Powell (see Himmelblau (1972)). Powell’s method is a modification of a quadratically converge method proposed by Smith (1962). It ensures convergence in a finite number of steps for positive quadratic function, by using a convenient conjugate directions. For non-quadratic function the method still valid.
3.2 Algorithm description

Before describing the algorithm, it is worthwhile to provide a sub algorithm which tests if parameters fulfill the conditions of stationarity and invertibility, we will denote it by Test. The second sub algorithm, which we must provide, concerns the compute of \( L(X; \theta) \) by kalman filter (KF). These two sub algorithm will be implemented in our global estimating algorithm. Herein, we are interesting in minimizing \( l(X, \theta) = -L(X, \theta) \), which is clearly equivalent to maximizing \( L(X, \theta) \).

**Sub algorithm Test(\( \theta \))**

If \( \theta_1^2 \times \theta_2 < \frac{1}{2} \)

Then go to next.

Else return to the previous step and take the previous point as starting point. "Here we force our algorithm to chose another value of vector parameters”.

End Sub

\( \theta_1 \) and \( \theta_2 \) are the values of parameters, \( b \) and \( \sigma^2 \) respectively. In this test, we just considered the invertibility condition, because \( \theta_1^2 \times \theta_2 < \frac{1}{2} \) implies \( \theta_1^2 \times \theta_2 < 1 \), and then the stationarity condition is verified.

**Sub algorithm KF(\( \theta \))**

Step 1 Given the starting condition \( \hat{\xi}_{1|0} \) and \( P_{1|0} \).

Compute \( \hat{X}_{1|0} = H\hat{\xi}_{1|0} \), \( \hat{M}_{1|0} = HP_{1|0}H' \).

Step 2 For \( t=1 \) to \( n \) do

Compute \( K_t, \hat{\xi}_{t|t}, P_{t|t}, F_t, \hat{\xi}_{t+1|t}, P_{t+1|t} \).

Compute \( \hat{X}_{t+1|t}, \hat{M}_{t+1|t} \).

End For

Step 3 \( \text{som}=\frac{n}{2} \log(2\pi) \)

For \( t=1 \) to \( n \) do

\[ \text{som} = \text{som} + \log(M_{t|t-1}) + \frac{(x_t-\hat{X}_{t|t-1})^2}{M_{t|t-1}} \]

End For

\( l(X; \theta) = \text{som} \).

end Sub.

Now, we propose the global algorithm for parameter estimation, where we integrate all the sub algorithms described above. We use the values of estimators obtained by moments method
for BL(0,0,2,1) (see Guegan (1984)) as initial values of the parameters $b$ and $\sigma^2$ for our algorithm. These estimators are defined by:

$$\hat{b} = \frac{\hat{c}(0)^2\hat{c}(0, 1, 4)}{2\hat{c}(1, 2)^3}, \quad \hat{\sigma}^2 = \frac{\hat{c}(1, 1, 3)^2}{2\hat{c}(0)\hat{c}(0, 1, 4)},$$

where

$$\hat{c}(k) = \frac{1}{n} \sum_{t=1}^{n} X_t X_{t-k},$$

$$\hat{c}(i, j) = \frac{1}{n} \sum_{t=1}^{n} X_t X_{t-i} X_{t-j},$$

$$\hat{c}(i, j, k) = \frac{1}{n} \sum_{t=1}^{n} X_t X_{t-i} X_{t-j} X_{t-k}.$$

$\hat{c}(k), \hat{c}(i, j), \hat{c}(i, j, k)$ are respectively the estimators of $c(k), c(i, j), c(i, j, k)$ defined by

$$c(k) = E[X_t X_{t-k}],$$
$$c(i, j) = E[X_t X_{t-i} X_{t-j}],$$
$$c(i, j, k) = E[X_t X_{t-i} X_{t-j} X_{t-k}].$$

The above considerations are summarized in the following parameter estimation algorithm denoted MLKF.

**MLKF algorithm**

**Step 1**: Let $\theta^{(0)}$ be an initial point (fulfill the test of invertibility) and let $d_1, \ldots, d_m$ the basis vectors (initial set of directions).

**Step 2**: For $k = 1$ to $m$

(m is a number of parameters)

Call sub algorithm KF($\theta^{(k)}$);

**Step 3**: Solve $\min_{\lambda} \ell(X; \theta^{(k-1)} + \lambda * d_k) = \ell(X; \theta^{(k-1)} + \lambda_k * d_k)$;

**Step 4**: call Test($\theta^{(k-1)} + \lambda_k * d_k$);

**Step 5**: Set $\theta^{(k)} \leftarrow \theta^{(k-1)} + \lambda_k * d_k$;

**End For**

**Step 6**: Compute:

- $l_0 = \ell(X; \theta^{(0)})$;
- $l_m = \ell(X; \theta^{(m)})$;
- $l_E = \ell(X; 2 * \theta^{(m)} - \theta^{(0)})$;
- $\Delta l = \max_{i=1, \ldots, m}\{l(X; \theta^{(i-1)}) - l(X; \theta^{(i)})\}$; let $s$ be the index for which the maximum is attained (the high diminution of $l$ obtained at step 3 in the direction $d_s$). Then we have two cases:

Step 7: If \((l_E \geq l_0)\text{ and/or } (l_0 - 2 \cdot l_m + l_E)(l_0 - l_m - \Delta L)^2 \geq \frac{1}{2}\Delta l(l_0 - l_E)^2\)

Then keep the old set of directions for the next iteration and keep $\theta^{(m)}$ as the starting point.

Else If \((l_E < l_0)\text{ and/or } (l_0 - 2 \cdot l_m + l_E)(l_0 - l_m - \Delta l)^2 < \frac{1}{2}\Delta l(l_0 - l_E)^2\)

Then go to step 1 and take $\theta^{(m)} - \theta^{(0)}$ as new direction and the obtained point as a new starting point in the next iteration. In the other hand, the direction $d_s$ is substituted by $\theta^{(m)} - \theta^{(0)} = d$. The new set of directions which we shall use through the next iterations was $(d_1, d_2, \ldots, d_{s-1}, d_{s+1}, \ldots, d_m, d)$.

Step 8: the steps 1 to 7 are repeated until, the minimum is obtained, or, the maximum iteration are exceeded.

The step 3 of our algorithm consists on minimizing a function $\varphi(\lambda) = l(X; \theta + \lambda \cdot d)$ with one variable. To overcome this problem we chose the Brent’s method which works faster and guarantees convergence in a reasonable number of steps (see Brent (1973)). In the next section, we illustrate the parameter estimation technique experimentally, and we verify that our method improves the estimations obtained by the moments method.

4 Simulations results

To assess the performance of our algorithm, we use two different methods. In the first, we use the estimates obtained by moments method as initial values and to be compared with estimates obtained by our algorithm.

In the second, we deduce that our algorithm still have a good performance even though the initial values are chosen randomly.

In the first study, we conducted a series of Monte Carlo simulation experiments from the models

- $X_t = e_t + 0.5e_{t-1}X_{t-2}, \quad e_t \sim NID(0,1)$
- $X_t = e_t + 0.85e_{t-1}X_{t-2}, \quad e_t \sim NID(0,0.5)$

In each experiment, we generate series of length $n = 50, 100, 150$. We apply MLKF algorithm to obtain the estimators of $b$ and $\sigma^2$, this operation is repeated 300 times to compute mean, bias, MSE and T-statistic of our estimators.

The results of this first experience are presented in table 1, 2 and 3, in which the following notation is used, MM is the Moments Method estimation, and MLKF is the Maximum Likelihood and Kalman Filter estimation.
The values of T-statistic reveal insignificance of the bias of MLKF estimators, the MLKF estimates are closer to true values of parameters and it outperforms the estimates obtained by moments method.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$b$</th>
<th>$\sigma^2$</th>
<th>$b$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>0.5</td>
<td>1</td>
<td>0.85</td>
<td>0.5</td>
</tr>
<tr>
<td>Mean of MLKF</td>
<td>0.5112</td>
<td>0.9886</td>
<td>0.842</td>
<td>0.5649</td>
</tr>
<tr>
<td>Bias of MLKF</td>
<td>-0.0112</td>
<td>0.0114</td>
<td>0.008</td>
<td>-0.064</td>
</tr>
<tr>
<td>MSE of MLKF</td>
<td>0.0491</td>
<td>0.225</td>
<td>0.0931</td>
<td>0.1396</td>
</tr>
<tr>
<td>T-statistic</td>
<td>-0.05</td>
<td>0.024</td>
<td>0.0262</td>
<td>-0.173</td>
</tr>
<tr>
<td>Mean of MM</td>
<td>0.4001</td>
<td>0.6551</td>
<td>0.6574</td>
<td>0.3469</td>
</tr>
<tr>
<td>MSE of MM</td>
<td>0.0501</td>
<td>0.272</td>
<td>0.0922</td>
<td>0.1396</td>
</tr>
</tbody>
</table>

Table 1: Mean, Bias, MSE and T-statistic of parameters estimation for $n=50$

<table>
<thead>
<tr>
<th>Parameters</th>
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<th>$\sigma^2$</th>
<th>$b$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>0.5</td>
<td>1</td>
<td>0.85</td>
<td>0.5</td>
</tr>
<tr>
<td>Mean of MLKF</td>
<td>0.4999</td>
<td>0.9986</td>
<td>0.8175</td>
<td>0.5220</td>
</tr>
<tr>
<td>Bias of MLKF</td>
<td>0.001</td>
<td>0.0014</td>
<td>0.0325</td>
<td>-0.022</td>
</tr>
<tr>
<td>MSE of MLKF</td>
<td>0.0273</td>
<td>0.1341</td>
<td>0.1024</td>
<td>0.0371</td>
</tr>
<tr>
<td>T-statistic</td>
<td>0.0006</td>
<td>0.0038</td>
<td>0.101</td>
<td>-0.114</td>
</tr>
<tr>
<td>Mean of MM</td>
<td>0.395</td>
<td>0.643</td>
<td>0.6376</td>
<td>0.2686</td>
</tr>
<tr>
<td>MSE of MM</td>
<td>0.0325</td>
<td>0.1767</td>
<td>0.0958</td>
<td>0.0471</td>
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</table>

Table 2: Mean, Bias, MSE and T-statistic of parameters estimation for $n=100$

<table>
<thead>
<tr>
<th>Parameters</th>
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<th>$\sigma^2$</th>
<th>$b$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>True values</td>
<td>0.5</td>
<td>1</td>
<td>0.85</td>
<td>0.5</td>
</tr>
<tr>
<td>Mean of MLKF</td>
<td>0.4987</td>
<td>1.0412</td>
<td>0.8419</td>
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<td>Bias of MLKF</td>
<td>0.0013</td>
<td>-0.0412</td>
<td>0.0081</td>
<td>0.001</td>
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<td>MSE of MLKF</td>
<td>0.017</td>
<td>0.1603</td>
<td>0.1065</td>
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<td>T-statistic</td>
<td>0.0099</td>
<td>-0.1029</td>
<td>0.024</td>
<td>0.0004</td>
</tr>
<tr>
<td>Mean of MM</td>
<td>0.380</td>
<td>0.5633</td>
<td>0.6774</td>
<td>0.2908</td>
</tr>
<tr>
<td>Variance of MM</td>
<td>0.0243</td>
<td>0.1356</td>
<td>0.1172</td>
<td>0.0679</td>
</tr>
</tbody>
</table>

Table 3: Mean, Bias, MSE and T-statistic of parameters estimation for $n=150$

In the second study, some Monte Carlo simulations are performed to show that our algorithm still has a good performance even though the initial values of parameters are chosen randomly. The series of length $n = 100$ are generated from the models

- $X_t = e_t + 0.9e_{t-1}X_{t-2}, \quad e_t \sim NID(0, 0.5)$
- $X_t = e(t) + 0.99e_{t-1}X_{t-2}, \quad e_t \sim NID(0, 0.4)$
where the values of $b$ are chosen close, progressively, to 1. The initial values are randomly chosen from uniform distribution between 0 and 1. The results illustrated in table 4 show what was expected.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>True value</th>
<th>Mean</th>
<th>MSE</th>
<th>Bias</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.9</td>
<td>0.9185</td>
<td>0.025</td>
<td>-0.0185</td>
<td>-0.1170</td>
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<tr>
<td>$\sigma^2$</td>
<td>0.5</td>
<td>0.4977</td>
<td>0.0061</td>
<td>0.023</td>
<td>0.0294</td>
</tr>
<tr>
<td>$b$</td>
<td>0.99</td>
<td>1.0047</td>
<td>0.0041</td>
<td>-0.0147</td>
<td>-0.2295</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.4</td>
<td>0.4096</td>
<td>0.0033</td>
<td>-0.0096</td>
<td>-0.1671</td>
</tr>
</tbody>
</table>

Table 4: Mean, Bias, MSE and T-statistic of parameters, the values of initial parameters are chosen randomly

From the results, it can be concluded that our new approach performs better, it estimates parameters fairly accurately and it improves the values of estimators obtained by moments method.

5 Conclusion

A new estimate algorithm for the first-order superdiagonal bilinear time series model is presented. The parameters estimations calculations are straightforward, and the large number of simulations show that the algorithm works well and leads to efficient results. Furthermore, this algorithm offers a possibility to be extended to all bilinear models which are invertible.

References


