Bayesian and Non-Bayesian Estimation of Pr{Y<X} in Two Parameters Lomax Distribution

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Abstract

In this paper, the Bayesian and non Bayesian estimation problem of reliability R= P(Y<X) will be considered when X and Y are two independent but not identically distributed belonging to Lomax distribution with two unknown parameters. The maximum likelihood estimator (MLE) of the two unknown parameters of Lomax distribution and the asymptotic variance-covariance matrix of them will be obtained. The MLE of the reliability R will be presented. Also the sampling distribution of the maximum likelihood estimators will be obtained. Posterior function of R and Bayes estimator of R will be discussed. Finally, numerical examples are given to illustrate some of theoretical results.

Keywords: Reliability; Stress- Strength model; Maximum likelihood estimator; Bayes estimator; Lomax distribution, Pearson system.

1. Introduction

Lomax (1954) introduced the Lomax random variable X whose probability density function is

$$f(x) = \frac{\alpha}{\lambda}(1+\frac{x}{\lambda})^{-(\alpha+1)}, x > 0, \alpha, \lambda > 0$$

where α and λ are the shape and scale parameters, respectively. This distribution has a position of importance in a field of life testing because of its uses to fit business failure data.

In stress-strength model, the stress (Y) and the strength (X) are treated as random variables and the reliability of a component during a given period is taken to be the probability that its strength exceeds the stress during the entire interval, i.e. the reliability R of a component is R=P(Y<X). Due to the practical point of view of reliability stress-strength model, the estimation problem of R=P(Y<X) has attracted

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the attention of many authors. This model, first considered by Birnbaum (1956), and is used in many applications such as civil, mechanical, and aerospace engineering. A good overview on stress-strength model in reliability is presented by Johnson (1988).

Awad and Charraf (1986) studied the case when X and Y are independent Burr random variables of type XII, they obtained maximum likelihood, uniformly minimum unbiased (MVUE) and Bayesian estimates of R. Mahmoud (1996) considered the case when X and Y are independent Weibull random variables, and obtained maximum likelihood, minimum variance unbiased estimator (MVUE) and Bayesian estimators of R. Ahmed, et al (1997) dealt with the same estimation of problem when X and Y are two independent but not identically distributed Burr-type X random variables, they obtained the maximum likelihood and the Bayes estimates for reliability R. Surles and Padgett (1998) discussed inference for R=P(Y<X) when X and Y are independently distributed Burr type X random variables. Abd-Elfattah and Mandouh (2004) studied the case when X and Y are independent Lomax random variables with known scale parameter. They obtained maximum likelihood, MVUE and Bayesian estimates for R= P(Y<X).

In this paper, we study the Bayesian and non Bayesian estimation of reliability R= P(Y<X) when X and Y are two independent but not identically random variables belonging to Lomax distribution with two unknown parameters, i.e. we shall discuss the maximum likelihood estimator (MLE) of the two unknown parameters of Lomax distribution, the asymptotic variance-covariance matrix of them and the MLE of the reliability R will be presented in section (2). Also the sampling distribution of the MLE of the two unknown parameters of Lomax distribution and the sampling distribution of the MLE of reliability R will be obtained in section (3). Finally, Posterior function of R and Bayes estimator of R will be discussed in section (4).

2. Maximum Likelihood Estimator

Let X be the strength of the component which is subjected to stress Y where X and Y are random variables distributed as Lomax distribution with parameters (λ, α_1) and (λ, α_2), respectively. The probability density function of X and Y are given by

$$f(x) = \frac{\alpha_1}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha_1+1)}, x > 0, \alpha_1, \lambda > 0 \qquad (1)$$

and

$$f(y)=\frac{\alpha_2}{\lambda}(1+\frac{y}{\lambda})^{-(\alpha_2+1)}\ ,y>0,\ \alpha_2,\lambda>0 \tag{2}$$

using equations (1) and (2), the reliability function R will be defined as

$$R=P(Y<X)=\int_0^\infty\int_0^x f(x)f(y)dydx$$

$$=\int_0^\infty f(x)F_Y(x)dx$$

$$=\int_0^\infty[1-(1+\frac{x}{\lambda})^{-\alpha_2}].\frac{\alpha_1}{\lambda}(1+\frac{x}{\lambda})^{-(\alpha_1+1)}dx$$

$$\frac{\alpha_2}{\alpha_1+\alpha_2} \tag{3}$$

Clearly $\hat{R}$ is independent of λ and depends on α_1 *and* α_2.

To obtain the MLE $\hat{R}$ of R we must first obtain the MLE`s $\hat{\alpha}_1$ and $\hat{\alpha}_2$ of α_1 and α_2, respectively, i.e. then the MLE of R is $\hat{R}=\frac{\hat{\alpha}_2}{\hat{\alpha}_1+\hat{\alpha}_2}$

Let $X_1,\ldots\ldots\ldots,X_n$ be a random sample drawn from Lomax distribution with parameters (λ,α_1), the likelihood function is

$$L(x,\lambda,\alpha_1)=(\frac{\alpha_1}{\lambda})^n\prod_{i=1}^{n}(1+\frac{x_i}{\lambda})^{-(\alpha_1+1)} \tag{4}$$

taking logarithms of both sides, we have

$$\ln L(x,\lambda,\alpha_1)=n\ln\alpha_1-n\ln\lambda-(\alpha_1+1)\sum_{i=1}^{n}\ln(1+\frac{x_i}{\lambda})$$

Differentiating partially with respect to the unknown parameters (λ,α_1), setting the results equal to zero and solve for $\hat{\alpha}_1$ and $\hat{\lambda}$ we get

$$\hat{\alpha}_1=\frac{n}{\sum_{i=1}^{n}\ln(1+\frac{x_i}{\hat{\lambda}})} \tag{5}$$

and

$$\frac{n}{\hat{\lambda}\left(\sum_{i=1}^{n}\frac{x_i}{(\hat{\lambda}^2+\hat{\lambda}x_i)}\right)}-1=\frac{n}{\sum_{i=1}^{n}\ln(1+\frac{x_i}{\hat{\lambda}})} \tag{6}$$

An iteration procedure may be used to solve (6) for $\hat{\lambda}$, and then substitute in (5) to obtain $\hat{\alpha}_1$.

Similarly, if $Y_1,\ldots\ldots,Y_m$ is a random sample drawn from Lomax (λ,α_2), similar procedure will give $\hat{\lambda}$ and $\hat{\alpha}_2$, hence $\hat{R}$ the MLE for R will be

$$\hat{R} = \frac{\hat{\alpha}_2}{\hat{\alpha}_1 + \hat{\alpha}_2} \tag{7}$$

The exact variance-covariance matrix of $(\hat{\lambda},\hat{\alpha}_1)$ is obtained by inverting the information matrix with elements that are negatives of expected values of the second order derivatives of logarithms of the likelihood functions, and an approximate asymptotic variance-covariance matrix of $(\hat{\lambda},\hat{\alpha}_1)$ may be obtained by replacing expected values by their maximum likelihood estimators, that is, the approximate asymptotic variance-covariance matrix will be

$$I^{-1}(\lambda,\alpha_1) = \begin{bmatrix} -\frac{n}{\hat{\lambda}^2} + (\hat{\alpha}_1 + 1)\sum_{i=1}^{n} \frac{(2\hat{\lambda} + x_i)x_i}{(\hat{\lambda}^2 + \hat{\lambda}x_i)^2} & -\sum_{i=1}^{n} \frac{x_i}{(\hat{\lambda}^2 + \hat{\lambda}x_i)} \\ -\sum_{i=1}^{n} \frac{x_i}{(\hat{\lambda}^2 + \hat{\lambda}x_i)} & -\frac{-n}{\hat{\alpha}_1^2} \end{bmatrix}^{-1}$$

Using software package Mathcad (2001), an extensive numerical investigation will be carried out to obtain and study the properties of the MLE for the unknown parameters on R. The following steps will be considered to obtain these numerical results:

Step (1): Generate 1000random samples $X_1, X_2,\ldots..X_n$ from Lomax distribution with sample sizes n=10, 20 and 30, scale parameter $\lambda = 1$ and with shape parameter $\alpha_1 =$ 2.1, 2.5 and 2.7.

Step (2): Using equations (5) and (6), obtain 1000 estimates for α_1 *and* λ .

Step (3): Similarly, generate 1000 random sample from Lomax distribution we can obtain the maximum likelihood estimator for Lomax with parameters $\lambda = 1$ and $\alpha_{2 = 2.2}$.

Step (4): Using the results of the MLE of the two unknown parameters of Lomax distribution and using equation (6), the MLE of R will be obtained.

From table (1) we can note that the mean square error of $\hat{R}$ decreasing by increasing sample size n with sample size m is constant and the mean square error of $\hat{R}$ decreasing by increasing sample size m with sample size n is constant and also it decreasing by increasing the both of them. The changes in mean square error of $\hat{R}$ due to change in α_1 and α_2 can be ignored.

Table (1)
The MLE of Reliability R when the Scale parameter unknown

λ=1 , α_2=2.2, R=0.5116					
	n	m	$\hat{R}$	Bias	MSE
α_1=2.1	10	10	0.5058	-0.00586	0.013
		20	0.5175	-0.00583	0.0102
		30	0.5084	-0.0032	0.00884
	20	10	0.4995	-0.0121	0.0109
		20	0.5152	0.00353	0.00748
		30	0.5009	-0.0107	0.00626
	30	10	0.5056	-0.0061	0.00912
		20	0.5174	0.00573	0.0064
		30	0.5141	0.00251	0.00534
λ =1 , α_2=2.2, R=0.4681					
α_1=2.5	10	10	0.4876	0.0195	0.0139
		20	0.5055	0.0374	0.0132
		30	0.4936	0.0255	0.0108
	20	10	0.4594	-0.00866	0.0112
		20	0.4708	0.00267	0.00848
		30	0.4656	-0.00248	0.00702
	30	10	0.463	-0.0051	0.00959
		20	0.4711	0.00296	0.00733
		30	0.4696	0.00151	0.00538
λ =1 , α_2=2.2, R=0.449					
α_1=2.7	10	10	0.4903	0.0413	0.0148
		20	0.4993	0.0503	0.0139
		30	0.4926	0.0436	0.0124
	20	10	0.452	0.00298	0.011
		20	0.4549	0.00596	0.00867
		30	0.4481	-0.00065	0.00785
	30	10	0.4481	-0.0009	0.00927
		20	0.4505	0.0015	0.00707
		30	0.4465	-0.0026	0.00592

3. The Sampling Distribution of Maximum Likelihood Estimators

In this section, we will obtain the sampling distributions of the maximum likelihood estimators of the two unknown parameters of Lomax distribution and maximum likelihood estimator of reliability R. Pearson's technique will be applied for each of the three estimators to obtain the sampling distributions for each one. These distributions will be used to study the properties of the three estimators. Some of these properties are the mean, variance, skewness, kurtosis and Pearson's coefficient.

Pearson's criterion for fixing the distribution family in a particular case is

$$\mathrm{K} = \frac{\beta_1(\beta_2+3)^2}{4(4\beta_2-3\beta_1)(2\beta_2-3\beta_1-6)}$$

where β_1 and β_2 are the measures of skewness and kurtosis, respectively. This value of K differs among the types of Pearson curves:

Type I when $\mathrm{K}<0$

Type II when $\mathrm{K}=0$, $\beta_1=0$ and $\beta_2<3$

Type III when $2\beta_2=3\beta_1-6=0$ and $\mathrm{K}=\infty$

Type IV when $0<\mathrm{K}<1$

Type V when $\mathrm{K}=1$

Type VI when $\mathrm{K}>1$

Type VII when $\mathrm{K}=0$, $\beta_1=0$ and $\beta_2>3$

Now, using Mathcad (2001) package, we can obtain the sampling distribution of the estimators numerically as follow:

Step (1): Generate 1000 random samples $X_1, X_2, \ldots X_n$ from Lomax distribution with sample sizes n =10, 20 and 30, scale parameter λ = 1 and with shape parameter α_1 = 2.1, 2.5 and 2.7. Using these data we obtain 1000 estimate of the MLE of λ and α_1. Similarly, we can obtain the maximum likelihood estimator for Lomax with parameters λ = 1 and α_2 = 2.2 then we can obtain 1000 estimate of R.

Step (2): Calculate Pearson's coefficient K and then determine the types of Pearson curves.

Each type of Pearson's system has a specific probability density function, the following probability density functions defining a specific of Pearson curves that appear in the tables (2). The type I Pearson curves has the density function

$$f(x) = \kappa\left(1+\frac{x}{a_1}\right)^{L_1}\left(1-\frac{x}{a_2}\right)^{L_2}, \; -a_1 < x < a_2;$$

$L_1 > -1$, where a_1, a_2, L_1 and L_2 are the parameters of the family of distributions and κ is constant. This type is Beta distribution. The type IV of Pearson curves has the following density function

$$f(x) = \kappa\left[1+\frac{x^2}{a^2}\right]^{-\lambda} \exp\left[-\nu \tan^{-1}\frac{x}{a}\right], \; -\infty < x < \infty$$

Table (2)
The Sampling Distribution for MLE of parameters λ, α_1 and of Reliability R

The values of λ, α_1 and n			For $\hat{\lambda}$		For $\hat{\alpha}_1$		For $\hat{R}$	
λ	α_1	n	K	type	K	type	K	type
1	2.1	10	0.101	IV	-0.303	I	0.036	IV
		20	0.095	IV	0.0769	IV	-0.0067	I
		30	0.063	IV	0.351	IV	0.178	IV
	2.5	10	0.067	IV	-0.234	I	-0.018	I
		20	0.051	IV	-0.78	I	0.061	IV
		30	0.096	IV	0.473	IV	0.003	IV
	2.7	10	-5.939	I	-0.153	I	-0.048	I
		20	-0.035	I	-0.325	I	-0.067	I
		30	0.094	IV	0.533	IV	-0.029	I

K :is the criterion for fixing the distribution family
type: the type of Pearson curves

where κ is constant and a, λ and ν are the parameters of the distribution of type IV Pearson curve. No common statistical distributions are of the form of type IV.

From table (2), it can note that the sampling distributions of the MLE of the scale parameter (λ) is often type IV. The sampling distributions of the MLE of the shape parameter (α) are of types I and IV. Finally, the sampling distributions of the MLE of the reliability function R of Lomax distribution has types I and IV.

4. Posterior Distribution and Bayes Estimator of the Reliability R

In this section we shall obtain a posterior distribution of R, let $X_1,\ldots,X_n$ be a random sample drawn from Lomax distribution with parameters (λ, α_1). Assume that the prior distributions of λ and α_1 are, respectively

$$\pi_{01}(\lambda) = \frac{1}{\lambda}, \quad \lambda > 0$$

and

$$\pi_{01}(\alpha_1) = \frac{\beta_1^{k_1}}{\Gamma(k_1)} \alpha_1^{k_1-1} e^{-\beta_1\alpha_1} \quad , \alpha_1 > 0, \beta_1, k_1 > 0$$

Let the parameters λ and α_1 are independent, then the joint prior of λ and α_1 will be

$$\pi_1(\lambda, \alpha_1) \propto \lambda^{-1} \alpha_1^{k_1-1} e^{-\beta_1\alpha_1}, \qquad \lambda > 0,\ \alpha_1 > 0 \tag{8}$$

combine the likelihood function (4) and the prior density (8) to obtain the posterior as follows:

$$\pi(\lambda,\alpha_1 \mid x) \propto \lambda^{-(n+1)} e^{-(z\ (n,\lambda,x))} \alpha_1^{\delta_1-1} e^{-\alpha_1 S(\lambda)}\ ,\ \alpha_1 > 0,\ \lambda > 0 \tag{9}$$

where $\delta_1 = n + k_1$, $z(n,\lambda,x) = \sum_{i=1}^{n} \ln(1+\frac{x_i}{\lambda})$ and $S(\lambda) = \beta_1 + z(n,\lambda,x)$

Similarly, let $Y_1,\ldots,Y_m$ be a random sample from a Lomax distribution with parameters (λ,α_2), assume that the joint prior distribution of λ and α_2 are given by

$\pi_2(\lambda,\alpha_2) \propto \lambda^{-1} \alpha_2^{k_2-1} e^{-\beta_2 \alpha_2}, \qquad \lambda > 0,\ \alpha_2 > 0$

hence the posterior density of λ and α_2 will be

$$\pi(\lambda,\alpha_2 \mid x) \propto \lambda^{-(m+1)} e^{-(z(m,\lambda,y))} \alpha_1^{\delta_2-1} e^{-\alpha_2 V(\lambda)},\quad \alpha_2 > 0,\ \lambda > 0 \tag{10}$$

where $\delta_2 = m + k_2$, $z(m,\lambda,y) = \sum_{i=1}^{m} \ln(1+\frac{y_i}{\lambda})$ and $V(\lambda) = \beta_2 + z(m,\lambda,y)$

Assume that α_1 and α_2 are independent, from (9) and (10), the joint bivariate posterior density of α_1, α_2 and λ will be

$$\pi(\alpha_1,\alpha_2,\lambda \mid x,y) \propto \lambda^{-(n+m+2)} \alpha_1^{\delta_1-1} \alpha_2^{\delta_2-1} e^{-w(\lambda)} e^{-\alpha_1[S(\lambda)]-\alpha_2[V(\lambda)]}, \alpha_1 > 0, \alpha_2 > 0, \lambda > 0 \tag{11}$$

where $w(\lambda) = z(n,\lambda,x) + z(m,\lambda,y)$

Applying the transformations technique of random variables, let

$$r = \frac{\alpha_2}{\alpha_1+\alpha_2}, and \quad u = \alpha_1 + \alpha_2\ , u > 0,\ 0 < r < 1$$

Then

$$\pi(u,r,\lambda \mid x,y) \propto \lambda^{-(n+m+2)} e^{-w(\lambda)} (1-r)^{\delta_1-1} (r)^{\delta_2-1} u^{\delta_1+\delta_2-1} e^{-ru[V(\lambda)]-u(1-r)[S(\lambda)]}$$

integrate out *u*

$$\pi(r,\lambda \mid x,y) = c\lambda^{-(n+m+2)} e^{-[w(\lambda_1)]} \frac{r^{\delta_2-1}(1-r)^{\delta_1-1}}{[(1-r)(S(\lambda))+r(V(\lambda))]^{\delta_1+\delta_2}},\ 0<r<1 \tag{12}$$

where c is a normalized constant and it can be obtained by integrate out λ and r as follow

$$c = 1/[\int_0^\infty\int_0^1 \lambda^{-(n+m+2)} e^{-[w(\lambda)]} \frac{r^{\delta_2-1}(1-r)^{\delta_1-1}}{(S(\lambda))^{\delta_1+\delta_2}[1+r(\frac{V(\lambda)}{S(\lambda)}-1)]^{\delta_1+\delta_2}} dr d\lambda]$$

From equation (12) the posterior distribution of reliability function R may be obtained by integrate out λ. The J[th] moment about zero of the posterior distribution will be

$$\mu_J^{'} = E(R^J \mid x, y, \lambda) = c\int_0^\infty\int_0^1 \lambda^{-(n+m+2)} e^{-w(\lambda)} \frac{r^{\delta_2+J-1}(1-r)^{\delta_1-1}}{(S(\lambda))^{\delta_1+\delta_2}[1+r(\frac{V(\lambda)}{S(\lambda)}-1)]^{\delta_1+\delta_2}} dr d\lambda \qquad (13)$$

when J = 0 the normalized constant will be obtained, if J =1 we will obtain the posterior mean of R

Using equation (12), with respect to the mean square error, Bayes estimator, R^* of R will be the posterior mean, i.e.

$$R^* = E(R \mid x, y, \lambda) = c\int_0^\infty\int_0^1 \lambda^{-(n+m+2)} e^{-w(\lambda)} \frac{r^{\delta_2}(1-r)^{\delta_1-1}}{(S(\lambda))^{\delta_1+\delta_2}[1+r(\frac{V(\lambda)}{S(\lambda)}-1)]^{\delta_1+\delta_2}} dr d\lambda \cdot$$

$$= c\int_0^\infty \frac{\lambda^{-(n+m+2)} e^{-w(\lambda)}}{[S(\lambda)]^{\delta_1+\delta_2}} \int_0^1 r^{\delta_2}(1-r)^{\delta_1-1}(1-r(1-p))^{-(\delta_1+\delta_2)} dr d\lambda$$

where $p = \dfrac{V(\lambda)}{S(\lambda)}$

$$= c\int_0^\infty \frac{\lambda^{-(n+m+2)} e^{-w(\lambda)}}{[S(\lambda)]^{\delta_1+\delta_2}} \left[\frac{\Gamma(\delta_2+1)\Gamma(\delta_1)}{\Gamma(\delta_1+\delta_2+1)} F(\delta_1+\delta_2, \delta_2+1; \delta_1+\delta_2+1; 1-p)\right] d\lambda$$

$$= c\frac{\Gamma(\delta_2+1)\Gamma(\delta_1)}{\Gamma(\delta_1+\delta_2+1)} \int_0^\infty \frac{\lambda^{-(n+m+2)} e^{-w(\lambda)}}{[S(\lambda)]^{\delta_1+\delta_2}} [F(\delta_1+\delta_2, \delta_2+1; \delta_1+\delta_2+1; 1-p)] d\lambda$$

$$= c\frac{\Gamma(\delta_2+1)\Gamma(\delta_1)}{\Gamma(\delta_1+\delta_2+1)} \int_0^\infty \frac{\lambda^{-(n+m+2)} e^{-w(\lambda)}}{[S(\lambda)]^{\delta_1+\delta_2}} [F(\delta_1+\delta_2, \delta_2+1; \delta_2+1; p)] d\lambda, \quad if\ |p| < 1 \quad (14)$$

where *F(., .; . ; .)* is Gauss hypergeomtric function. From equation (14), it is difficult to obtain a closed form of Bayes estimator, R^*, a numerical integration procedure is used to evaluate R^*.

$$E(r \mid \lambda, x, y) = c_1\int_0^\infty\int_0^\infty\int_0^\infty (\frac{\alpha_2}{\alpha_1+\alpha_2}) \lambda^{-(n+m+2)} \alpha_1^{\delta_1-1}\alpha_2^{\delta_2-1} e^{-w(\lambda)} e^{-\alpha_1[S(\lambda)]-\alpha_2[V(\lambda)]} d\alpha_1 d\alpha_2 d\lambda \qquad (15)$$

where c_1 is a normalized constant which represents the marginal density of X, and it can be obtained by integration out α_1, α_2 and λ as follow

$$c = 1/[\Gamma(\delta_1)\Gamma(\delta_2)\int_0^\infty \lambda^{-(n+m+2)} \frac{e^{-w(\lambda)}}{[S(\lambda)]^{n+k}[V(\lambda)]^{m+k_1}} d\lambda]$$

from (15)

$$E(r \mid \lambda, x, y) = c\int_0^\infty\int_0^\infty\int_0^\infty (1+\frac{\alpha_1}{\alpha_2})^{-1} \lambda^{-(n+m+2)} \alpha_1^{\delta_1-1}\alpha_2^{\delta_2-1} e^{-w(\lambda)} e^{-\alpha_1[S(\lambda)]-\alpha_2[V(\lambda)]} d\alpha_1 d\alpha_2 d\lambda$$

using Binomial theorem, we have

$$=c\int_0^\infty\int_0^\infty\int_0^\infty\sum_{s=0}^{\infty}(-1)^s(\frac{\alpha_1}{\alpha_2})^s\lambda^{-(n+m+2)}\alpha_1^{\delta_1-1}\alpha_2^{\delta_2-1}e^{-w(\lambda)}e^{-\alpha_1[S(\lambda)]-\alpha_2[V(\lambda)]}d\alpha_1 d\alpha_2 d\lambda,\ if\left|\frac{\alpha_1}{\alpha_2}\right|<1$$

$$=c\int_0^\infty\sum_{s=0}^{\infty}(-1)^s\Gamma(\delta_1+s)\Gamma(\delta_2-s)\frac{\lambda^{-(n+m+2)}e^{-w(\lambda)}}{[S(\lambda)]^{\delta_1+s}[V(\lambda)]^{\delta_2-s}}d\lambda$$

Mathcad (2001) will be used to obtain a numerical illustration for Bayes estimator of reliability function R as follow:

Step (1): Sample of size 10 was generated from Lomax distribution with parameters $\lambda=1,\alpha_1=2.1$, α_2=2.5 i.e. R=0.5435.

Step (2): Using equation (15) and take the prior parameters with the values $k_1=3,\ k_2=3,\ \beta_1=2$ and $\beta_2=2$.

It can be noted that the results are the constant c=3000.094, R^*=0.554, bias=0.01 and mean square error= 0.000111.

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