Profit analysis of a two unit cold standby system with preventive maintenance and random change in units

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Abstract

This paper discuss the reliability of two units cold standby system with single repair. For achieving high reliability of the system, the operative and the standby units are interchanged at random epochs and additional preventive maintenance of operative and the standby units also. After the repair, the unit is sent for inspection to decide whether the repair is satisfactory . In case the repair is found unsatisfactory the unit is again sent for post repair .Using a regenerative point technique, various measures of system effectiveness are obtained.

Introduction

Most authors [1,2] have studied two unit standby systems assuming that the operating unit works continuously until it fails, the operative and the standby units are interchanged at random epochs which is discussed also. In many engineering systems it is observed that system reliability can be increased by putting the operative and the standby units are interchanged at random epochs and additional preventive maintenance of operative and the standby units .

The purpose of the present paper is to analyses a standby system in which the operative and standby units are interchanged after a random amount of time and additional preventive maintenance of operative and the standby units . A single repair facility is used to repair and post repair the failed unit. The post repair is needed only when the repair of the failed unit is found unsatisfactory on inspection.

Identifying the suitable regenerative points, the following measures of system effectiveness are obtained:

(1) transient and steady state transition probabilities;
(2) mean sojourn time in regenerative states;
(3) distribution of time to system failure and its mean (MTSF);
(4) point wise availability and steady state availability of the system;
(5) expected busy period of the repairman in the interval $(0,t]$;
(6) expected number of visits by the repairman in $(0,t]$;
(7) expected number of preventive maintenance in $(0,t]$;
(8) cost analysis;
(9) comparative study;

Model description and assumptions

(1) The system comprises two non- identical units in cold standby configuration. Each unit has two modes, normal (N) and total failure (F).The units are named as N and F units in their respective modes.
(2) Upon failure of an operative unit, the cold standby unit becomes operative instantaneously.
(3) preventive maintenance (e.g.overhaul,..,etc) is provided to this system in state S_0

(4) After a random amount of time t, the operative unit becomes standby and the standby unit becomes operative if the standby is available.
(5) After the repair , a unit goes for inspection to decide whether the repair is Satisfactory or not. If the repaired unit is found to de unsatisfactory then it is sent for post repair. The probability of having satisfactory repair is fixed.
(6) Failure rate of the operative unit is linearly increasing while the distributions of time to repair, inspection and post repair are general.
(7) The distribution of time for interchanging the operative and standby units is general.
(8) A single repair facility is available for repair, inspection and post repair.

Notation

αt Linearly increasing failure rate of operative unit.

$g(\cdot), G(\cdot)$ pdf and Cdf of time after which operative unit changes.

$f(\cdot), F(\cdot)$ pdf and Cdf of repair time of failed unit.

$h(\cdot), H(\cdot)$ pdf and Cdf of time to complete inspection.

$k(\cdot), K(\cdot)$ pdf and Cdf of time to complete post repair.

$u(t)$ pdf of time for taking a unit into preventive maintenance

i.e $u(t) = \beta t \exp(-\beta t^2/2)$, $\beta, t > 0$.

$v(t)$ pdf preventive maintenance time $v(t) = \lambda t \exp(-\lambda t^2/2)$, $\lambda, t > 0$.

$p = (1-q)$ probability that the repair is satisfactory after the inspection.

$q_{ij}(t), Q_{ij}(t)$ pdf and cdf of transition time from regenerative state S_i to S_j .

E set of regenerative states ($S_0 - S_4, S_6, S_9, S_{11}, S_{14}$).

$\pi(\cdot)$ cdf of time to system failure when $S_i \in E$.

$M_i(t)$ pr[system up initially in state $S_i \in E$ is up at time t without going to any regenerative state].

μ_i mean sojourn time in state $S_i \in E$.

$A_i(t)$ pr[starting from $S_i \in E$,the system is up at time t].

$B_i(t)$ pr[repairman is busy at time t $[E_0 = S_i \in E]$].

$V_i(t)$ expected number of visits by repairman in $(0, t]$.

$N_{ip}(t)$ expected frequency of preventive maintenance in $(0, t], E_0 = S_i \in E$

© symbol of ordinary convolution e.g. $a(t) \copyright\ b(t) = \int_0^t a(u) b(t-u) du$

(s) symbol of Stieltjes convolution e.g. $A(t)(s)B(t) = \int_0^t B(t-u) dA(u)$

The limits of integration are $(0, \infty)$, when not mentioned.
Symbols in the transition diagram are :

N_O normal unit when it is operative

N_S normal unit when it is warm standby

F_{wr} failed unit waiting for the repair

F_r failed unit under repair
F_R repair of failed unit is continued from earlier state
F_I failed unit under inspection
F_{pr} failed unit under post repair
N_{Op} normal unit under preventive maintenance
N_{Sp} standby unit under preventive maintenance

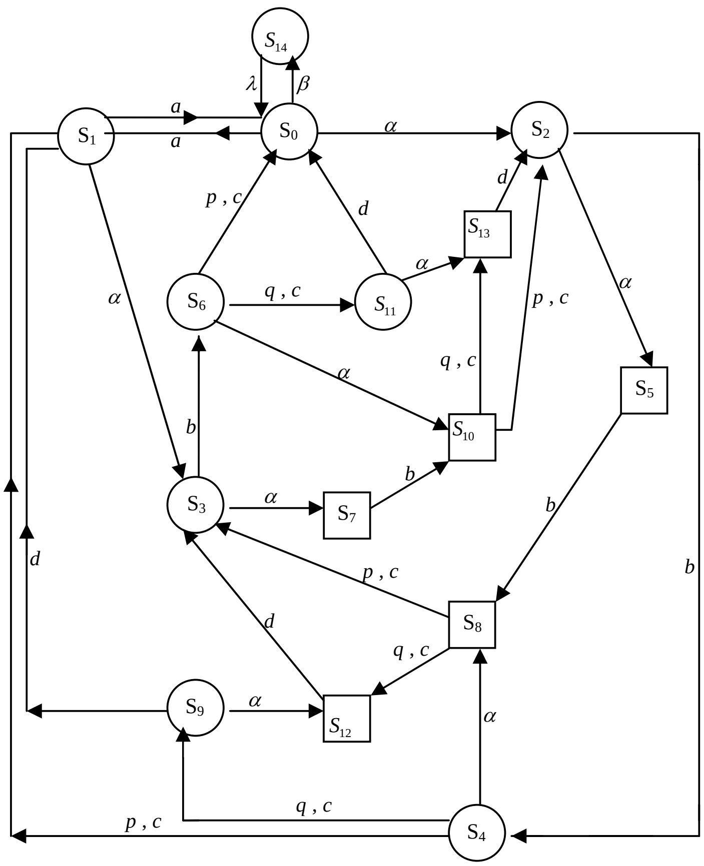

Figure 1. Transition diagram

Possible states of the system are:

Up states:

$S_0(N_O, N_S)$; $S_1(N_S, N_O)$; $S_2(F_r, N_O)$; $S_3(N_O, F_r)$; $S_4(F_I, N_O)$; $S_6(N_O, F_I)$
$S_9(F_{pr}, N_O)$; $S_{11}(N_O, F_{pr})$; $S_{14}(N_{Op}, N_{Sp})$

Down states:

$S_5(F_R, F_{wr})$; $S_7(F_{wr}, F_R)$; $S_8(F_I, F_{wr})$; $S_{10}(F_{wr}, F_I)$; $S_{12}(F_{pr}, F_{wr})$; $S_{13}(F_{wr}, F_{pr})$

Possible states and transitions are shown in Fig. 1.

3. Transition probabilities and sojourn times

Let $T_0(=0)$, T_1 and T_2, be the regenerative epochs and let X_n denote the state visited at epoch T_n^+, i.e. just after the transition at T_n. Then (X_n, T_n) is a Markov renewal process. Let

$$Q_{ij}(t) = \Pr\left[X_{n+1} = j, T_{n+1} - T_n \le t / X_n = i\right]$$

Then the transition probability matrix of the embedded Markov chain is
$P = (p_{ij}) = Q_{ij}(\infty) = Q(\infty)$
with non-zero elements as

$$P_{0,1} = \int_0^\infty \exp(-\alpha t^2/2) g(t)\overline{U}(t)dt \;,\quad P_{0,2} = \int_0^\infty \alpha t \exp(-\alpha t^2/2)\overline{G}(t)\overline{U}(t)dt$$

$$P_{0,14} = \int_0^\infty \exp(-\alpha t^2/2) u(t)\overline{G}(t)dt \,,\quad P_{1,3} = \int_0^\infty \alpha t \exp(-\alpha t^2/2)\overline{G}(t)dt$$

$$P_{1,0} = \int_0^\infty \exp(-\alpha t^2/2) g(t)dt \quad,\quad P_{3,7} = P_{2,5} = \int_0^\infty \alpha t \exp(-\alpha t^2/2)\overline{F}(t)dt$$

$$P_{2,4} = P_{3,6} = \int_0^\infty \exp(-\alpha t^2/2) f(t)dt \quad,\quad P_{4,1} = P_{60} = p\int_0^\infty \exp(-\alpha t^2/2) h(t)dt$$

$$P_{4,8} = P_{6,10} = \int_0^\infty \alpha t \exp(-\alpha t^2/2)\overline{H}(t)dt \;,\quad P_{9,1} = P_{11,0} = \int_0^\infty \exp(-\alpha t^2/2) k(t)dt$$

$$P_{9,12} = P_{11,13} = \int_0^\infty \alpha t \exp(-\alpha t^2/2)\overline{K}(t)dt \;, P_{4,9} = P_{6,11} = q\int_0^\infty \exp(-\alpha t^2/2) h(t)dt$$

$$P_{8,3} = P_{10,2} = p\int_0^\infty h(t)dt \quad,\; P_{8,12} = P_{10,13} = q\int_0^\infty h(t)dt \quad,\; P_{5,8} = P_{7,10} = \int_0^\infty f(t)dt$$

$$P_{13,2} = P_{12,3} = \int_0^\infty k(t)dt \quad,\quad P_{14,0} = \int_0^\infty v(t)dt$$

Mean sojourn time in state S_i is defined as the time of stay in state S_i before transiting to any other state .If T denotes the sojourn time in S_i then
$\mu_i = E(T) = \int \Pr[T > t]dt$
Thus we have

$$\mu_0 = \int_0^\infty \exp(-\alpha t^2/2)\overline{G}(t)\overline{U}(t)dt \quad,\quad \mu_1 = \int_0^\infty \exp(-\alpha t^2/2)\overline{G}(t)dt$$

$$\mu_2 = \mu_3 = \int_0^\infty \exp(-\alpha t^2/2)\overline{F}(t)dt \quad,\quad \mu_4 = \mu_6 = \int_0^\infty \exp(-\alpha t^2/2)\overline{H}(t)dt$$

$$\mu_5 = \mu_7 = \int_0^\infty \overline{F}(t)dt = m_2 \quad,\quad \mu_8 = \mu_{10} = \int_0^\infty \overline{H}(t)dt = m_3$$

$$\mu_{12} = \mu_{13} = \int_0^\infty \overline{K}(t)dt = m_4 \quad,\quad \mu_9 = \mu_{11} = \int_0^\infty \exp(-\alpha t^2/2)\overline{K}(t)dt \;,\quad \mu_{14} = \int_0^\infty \overline{V}(t)dt$$

4.Mean time to system failure

To investigate the distribution function $\pi_i(t)$ of the time to system failure with starting state s_i, the failed states are taken to be absorbing . Using the arguments for a regenerative process , we obtain the following relation for $\pi_i(t)$:

$$\pi_0(t) = Q_{01}(t)(s)\pi_1(t) + Q_{02}(t)(s)\pi_2(t) + Q_{0,14}(t)(s)\pi_{14}(t)$$
$$\pi_1(t) = Q_{10}(t)(s)\pi_0(t) + Q_{13}(t)(s)\pi_3(t)$$
$$\pi_2(t) = Q_{25}(t) + Q_{24}(t)(s)\pi_4(t)$$
$$\pi_3(t) = Q_{37}(t) + Q_{36}(t)(s)\pi_6(t)$$
$$\pi_4(t) = Q_{48}(t) + Q_{41}(t)(s)\pi_1(t) + Q_{49}(t)(s)\pi_9(t)$$
$$\pi_6(t) = Q_{6,10}(t) + Q_{60}(t)(s)\pi_0(t) + Q_{6,11}(t)(s)\pi_{11}(t)$$
$$\pi_9(t) = Q_{9,12}(t) + Q_{91}(t)(s)\pi_1(t)$$
$$\pi_{11}(t) = Q_{11,13}(t) + Q_{11,0}(t)(s)\pi_0(t)$$
$$\pi_{14}(t) = Q_{14,0}(t)(s)\pi_0(t) \qquad (4.1\text{-}4.9)$$

Taking the Laplace- Stieltjes transform of these relation and solve for $\tilde{x}_0(s)$ and omitting the argument s for brevity , we obtain:-

$$MTSF = E(T) = -\left.\frac{d\tilde{\pi}_0(s)}{ds}\right|_{S=0} = \frac{N_1}{D_1} \qquad (4.10)$$

Where

$$N_1 = \mu_0 + P_{02}\{\mu_2 + P_{24}(\mu_4 + P_{49}\mu_9)\}$$
$$+[\mu_1 + P_{13}\{\mu_3 + P_{36}(\mu_6 + P_{6,11}\mu_{11})\}][P_{01} + P_{02}P_{24}(P_{41} + P_{49}P_{91})] + P_{0,14}\mu_{14}$$

and

$$D_1 = 1 - \{P_{01} + P_{02}P_{24}(P_{41} + P_{49}P_{91})\}\{P_{10} + P_{13}P_{36}(P_{6,11}P_{11,0} + P_{60})\} - P_{0,14}$$

5. Availability analysis

As defined , $M_i(t)$ denotes the probability that the system starting in up state $S_i \in E$ is up at time t without passing through any regenerative state .Thus we have

$$M_0(t) = \exp(-\alpha t^2/2)\bar{G}(t)\bar{U}(t), \quad M_1(t) = \exp(-\alpha t^2/2)\bar{G}(t)$$
$$M_2(t) = M_3(t) = \exp(-\alpha t^2/2)\bar{F}(t), \quad M_4(t) = M_6(t) = \exp(-\alpha t^2/2)\bar{H}(t)$$
$$M_9(t) = M_{11}(t) = \exp(-\alpha t^2/2)\bar{K}(t), \quad M_{14}(t) = \bar{V}(t)$$

Using the arguments of the theory of a regenerative process, the point wise availability $A_i(t)$ is seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + Q_{01}(t)© A_1(t) + Q_{02}(t)© A_2(t) + Q_{014}(t)© A_{14}(t)$$
$$A_1(t) = M_1(t) + Q_{10}(t)© A_0(t) + Q_{13}(t)© A_3(t)$$
$$A_2(t) = M_2(t) + Q_{24}(t)© A_4(t) + Q_{25}(t)© A_5(t)$$
$$A_3(t) = M_3(t) + Q_{36}(t)© A_6(t) + Q_{37}(t)© A_7(t)$$
$$A_4(t) = M_4(t) + Q_{41}(t)© A_1(t) + Q_{48}(t)© A_8(t) + Q_{49}(t)© A_9(t)$$
$$A_5(t) = Q_{58}(t)© A_8(t)$$
$$A_6(t) = M_6(t) + Q_{60}(t)© A_0(t) + Q_{610}(t)© A_{10}(t) + Q_{611}(t)© A_{11}(t)$$
$$A_7(t) = Q_{710}(t)© A_{10}(t)$$
$$A_8(t) = Q_{83}(t)© A_3(t) + Q_{812}(t)© A_{12}(t)$$
$$A_9(t) = M_9(t) + Q_{91}(t)© A_1(t) + Q_{912}(t)© A_{12}(t)$$
$$A_{10}(t) = Q_{102}(t)© A_2(t) + Q_{1013}(t)© A_{13}(t)$$

$$A_{11}(t) = M_{11}(t) + Q_{110}(t)© A_0(t) + Q_{1113}(t)© A_{13}(t)$$
$$A_{12}(t) = Q_{123}(t)© A_3(t)$$
$$A_{13}(t) = Q_{132}(t)© A_2(t)$$
$$A_{14}(t) = M_{14}(t) + Q_{140}(t)© A_0(t) \quad (5.1\text{-}5.15)$$

Taking the Laplace transform of above equations and solving for $A^*{}_0(s)$. And omitting the argument (s) for brevity we obtain

$$A^*{}_0(s) = \frac{N_2(s)}{D_2(s)} \quad (5.16)$$

The steady state availability , when the system starts from S_i, is obtained as follows :

$$A_0(\infty) = \lim_{S\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)} = \frac{N_2}{D_2} \quad (5.17)$$

where

$$N_2 = P_{24}\{(1 - P_{36}(P_{60} + P_{6,11}P_{11,0}))(P_{10}(\mu_0 + P_{0,14}\mu_{14}) + P_{01}\mu_1) + P_{02}(\mu_1 - P_{10}(\mu_3 + P_{36}(\mu_6 + P_{6,11}\mu_{11})))\}(P_{41} + P_{49}P_{91}) - P_{36}\{P_{01}P_{13}(\mu_2 + P_{24}\mu_4) - (\mu_0 + P_{0,14}\mu_{14}) - P_{01}\mu_1\}(P_{11,0}P_{6,11} + P_{60}) - \{\mu_3 + \mu_2 + P_{24}\mu_4 + P_{36}(\mu_6 + P_{6,11}\mu_{11})\}(P_{01}P_{10} + P_{0,14} - 1)$$

and

$$D_2 = \{\mu_2 + \mu_3 + P_{24}\mu_4 + P_{25}\mu_5 + P_{36}\mu_6 + P_{37}\mu_7 + (\mu_8 + P_{8,12}\mu_{12})(1 + P_{24}P_{48} - P_{24}) + (\mu_{10} + \mu_{13}P_{10,13})(1 - P_{36} + P_{36}P_{6,10}) + P_{6,11}P_{36}(\mu_{11} + \mu_{13} - P_{11,0}\mu_{13}) + P_{49}P_{24}(\mu_9 + \mu_{12} - \mu_{12}P_{91})\}(1 - P_{01}P_{10} - P_{0,14}) + P_{24}\{\mu_1(1 - P_{0,14}) + P_{10}(\mu_0 + P_{0,14}\mu_{14} + P_{02}(-\mu_3 - \mu_{10} - P_{37}\mu_7 - P_{10,13}\mu_{13} + P_{36}(\mu_{10} - \mu_6 - P_{6,10}\mu_{10} - P_{6,11}\mu_{11} + \mu_{13}(-P_{6,11}P_{11,13} + P_{10,13} - P_{10,13}P_{6,10})))))\}(P_{49}P_{91} + P_{41}) + P_{36}\{\mu_0 + P_{01}\mu_1 + P_{0,14}\mu_{14} - P_{01}P_{13}(P_{24}\mu_4 + P_{25}\mu_5 + P_{8,12}\mu_{12}) - P_{01}P_{13}P_{24}\mu_{12}(P_{49}(1 - P_{91} - P_{8,12}) - P_{8,12}P_{41}) - P_{24}(P_{10}(\mu_0 + P_{0,14}\mu_{14}) + P_{01}\mu_1)(P_{49}P_{91} + P_{41}) - P_{01}P_{13}(\mu_2 + \mu_9P_{24}P_{49} + \mu_8(1 + P_{24}P_{48} - P_{24}))\}(P_{6,11}P_{11,0} + P_{60})$$

α	Availability(1)	Availability(2)	Availability(3)	Availability(4)
0.1	0.884977	0.882547	0.829253	0.766001
0.2	0.68429	0.679744	0.575692	0.552668
0.3	0.52453	0.520785	0.44756	0.437648
0.4	0.41957	0.417024	0.372734	0.367812
0.5	0.351174	0.349497	0.323902	0.321182
0.6	0.304697	0.303575	0.289428	0.287799
0.7	0.271521	0.270748	0.263671	0.262633
0.8	0.246756	0.246209	0.2436	0.242905

Table 1.

6.Busy period analysis

As defined earlier, $B_i(t)$ is the probability that the system having started from regenerative state S_i at $t = 0$ is under repair. By probabilistic arguments, we have

$$B_0(t) = q_{0,1}(t) © B_1(t) + q_{0,2}(t) © B_2(t) + q_{0,14}(t) © B_{14}(t)$$
$$B_1(t) = q_{1,0}(t) © B_0(t) + q_{1,3}(t) © B_3(t)$$
$$B_2(t) = q_{2,4}(t) © B_4(t) + q_{2,5}(t) © B_5(t) + W_2$$

$$B_3(t) = q_{3,6}(t) © B_6(t) + q_{3,7}(t) © B_7(t) + W_3$$
$$B_4(t) = q_{4,1}(t) © B_1(t) + q_{4,8}(t) © B_8(t) + q_{4,9}(t) © B_9(t) + W_4$$
$$B_5(t) = q_{5,8}(t) © B_8(t) + W_5$$
$$B_6(t) = q_{6,0}(t) © B_6(t) + q_{6,10}(t) © B_{10}(t) + q_{6,11}(t) © B_{11}(t) + W_6$$
$$B_7(t) = q_{7,10}(t) © B_{10}(t) + W_7$$
$$B_8(t) = q_{8,3}(t) © B_3(t) + q_{8,12}(t) © B_{12}(t) + W_8$$
$$B_9(t) = q_{9,1}(t) © B_1(t) + q_{9,12}(t) © B_{12}(t) + W_9$$
$$B_{10}(t) = q_{10,2}(t) © B_2(t) + q_{10,13} © B_{13}(t) + W_{10}$$
$$B_{11}(t) = q_{11,0}(t) © B_0(t) + q_{11,13}(t) © B_{13}(t) + W_{11}$$
$$B_{12}(t) = q_{12,3}(t) © B_3(t) + W_{12}$$
$$B_{13}(t) = q_{13,2}(t) © B_2(t) + W_{13}$$
$$B_{14}(t) = q_{14,0}(t) © B_0(t) + W_{14} \qquad (6.1\text{-}6.15)$$

where,

$$W_2(t) = W_3(t) = \exp(-\alpha t^2/2)\bar{F}(t) \ , \ W_4(t) = W_6(t) = \exp(-\alpha t^2/2)\bar{H}(t)$$
$$W_9(t) = W_{11}(t) = \exp(-\alpha t^2/2)\bar{K}(t) \ , \ W_5(t) = W_7(t) = \bar{F}(t)$$
$$W_8(t) = W_{10}(t) = \bar{H}(t) \ , \ W_{12}(t) = W_{13}(t) = \bar{K}(t) \ , \ M_{14}(t) = \bar{V}(t)$$

Taking the Laplace transform of above equations and solving for $B^*{}_0(s)$. And omitting the argument (s) for brevity we obtain

$$B^*{}_0(s) = \frac{N_3(s)}{D_2(s)} \qquad (6.16)$$

The steady state Busy period , when the system starts from S_i, is obtained as follows :

$$B_0(\infty) = \lim_{S\to 0} sB_0^*(s) = \frac{N_3(0)}{D_2'(0)} = \frac{N_3}{D_2} \qquad (6.17)$$

Where,

$$\begin{aligned} N_3 = {} & P_{36}(P_{11,0}P_{6,11} + P_{60})[P_{0,14}(1 - P_{24}P_{10}(P_{41} + P_{49}P_{91}))\mu_{14} - P_{01}P_{13}\{\mu_2 + \mu_8(1 - P_{24}P_{41}) \\ & + P_{24}\mu_4 + P_{25}\mu_5 + (P_{8,12} - P_{24}(P_{8,12}P_{41} + P_{49}(P_{9,12} - P_{8,12})))\mu_{12} - P_{24}P_{49}(\mu_8 - \mu_9)\}] \\ & + P_{24}P_{10}(P_{41} + P_{49}P_{91})[P_{0,14}\mu_{14} - P_{02}\{\mu_3 + P_{36}\mu_6 + P_{37}\mu_7 + (1 - P_{36} + P_{36}P_{6,10})\mu_{10} \\ & + P_{36}P_{6,11}\mu_{11} + (P_{36}(P_{11,13}P_{6,11} - P_{10,13} + P_{10,13}P_{6,10}) + P_{10,13})\mu_{13}\}] \\ & + (1 - P_{0,14} - P_{01}P_{10})[\mu_2 + \mu_3 + \mu_8 + P_{24}\mu_4 + P_{25}\mu_5 + P_{36}\mu_6 + P_{37}\mu_7 + P_{8,12}\mu_{12} \\ & + P_{10,13}\mu_{13} + P_{24}P_{49}(\mu_9 - \mu_8) + P_{6,11}P_{36}(\mu_{11} - \mu_{10}) - P_{24}P_{41}\mu_8 - P_{36}P_{60}\mu_{10} \\ & - P_{24}(P_{41}P_{8,12} + P_{49}(P_{8,12} - P_{9,12}))\mu_{12} + P_{36}(P_{11,13}P_{6,11} - P_{10,13}P_{6,11} - P_{10,13}P_{60})\mu_{13}]. \end{aligned}$$

7.Expected number of visits by the repairman

We defined $V_i(t)$ as the expected number of visits by the repairman in $(0,t]$, given that the system initially starts from regenerative state S_i. By probabilistic arguments we have the following recursive relations:

$$V_0(t) = Q_{0,1}(t)(s)V_1(t) + Q_{0,2}(t)(s)[1+V_2(t)] + Q_{0,14}(t)(s)V_{14}(t)$$
$$V_1(t) = Q_{1,0}(t)(s)V_0(t) + Q_{1,3}(t)(s)[1+V_3(t)]$$
$$V_2(t) = Q_{2,4}(t)(s)V_4(t) + Q_{2,5}(t)(s)V_5(t)$$
$$V_3(t) = Q_{3,6}(t)(s)V_6(t) + Q_{3,7}(t)(s)V_7(t)$$
$$V_4(t) = Q_{4,1}(t)(s)V_1(t) + Q_{4,8}(t)(s)V_8(t) + Q_{4,9}(t)(s)V_9(t)$$
$$V_5(t) = Q_{5,8}(t)(s)V_8(t)$$
$$V_6(t) = Q_{6,0}(t)(s)V_0(t) + Q_{6,10}(t)(s)V_{10}(t) + Q_{6,11}(t)(s)V_{11}(t)$$
$$V_7(t) = Q_{7,10}(t)(s)V_{10}(t)$$
$$V_8(t) = Q_{8,3}(t)(s)V_3(t) + Q_{8,12}(t)(s)V_{12}(t)$$
$$V_9(t) = Q_{9,1}(t)(s)V_1(t) + Q_{9,12}(t)(s)V_{12}(t)$$
$$V_{10}(t) = Q_{10,2}(t)(s)V_2(t) + Q_{10,13}(t)(s)V_{13}(t)$$
$$V_{11}(t) = Q_{11,0}(t)(s)V_0(t) + Q_{11,13}(t)(s)V_{13}(t)$$
$$V_{12}(t) = Q_{12,3}(t)(s)V_3(t)$$
$$V_{13}(t) = Q_{13,2}(t)(s)V_2(t)$$
$$V_{14}(t) = Q_{14,0}(t)(s)V_0(t) \qquad (7.1\text{-}7.15)$$

Taking the Laplace- Stieltjes transform of these relation and solve for $\tilde{V}_0(s)$ and omitting the argument s for brevity , we obtain:-

$$\tilde{V}_0(s) = \frac{N_4(s)}{D_2(s)} \qquad (7.16)$$

In the steady state, the number of visits per unit time is given by

$$V_0 = \lim_{t\to\infty}\left[\frac{V_0(t)}{t}\right] = \lim_{S\to 0} S\tilde{V}_0(S) = \frac{N_4}{D_2} \qquad (7.17)$$

Where,

$$N_4 = P_{36}(P_{11,0}P_{6,11} + P_{60})\{(1 - P_{0,14} - P_{01}P_{10}) + P_{24}(1 - P_{02}P_{10} - P_{01} - P_{0,14})(P_{41} + P_{49}P_{91})$$

8.Expected frequency of preventive maintenance

We defined $N_{ip}(t)$ as the expected frequency of preventive maintenance in $(0,t]$, given that the system initially starts from regenerative state S_i. By probabilistic arguments we have the following recursive relations:

$$N_{0p}(t) = Q_{0,1}(t)(s)N_{1p}(t) + Q_{0,2}(t)(s)N_{2p}(t) + Q_{0,14}(t)(s)N_{14p}(t)$$
$$N_{1p}(t) = Q_{1,0}(t)(s)N_{0p}(t) + Q_{1,3}(t)(s)N_{3p}(t)$$
$$N_{2p}(t) = Q_{2,4}(t)(s)N_{4p}(t) + Q_{2,5}(t)(s)N_{5p}(t)$$
$$N_{3p}(t) = Q_{3,6}(t)(s)N_{6p}(t) + Q_{3,7}(t)(s)N_{7p}(t)$$
$$N_{4p}(t) = Q_{4,1}(t)(s)N_{1p}(t) + Q_{4,8}(t)(s)N_{8p}(t) + Q_{4,9}(t)(s)N_{9p}(t)$$
$$N_{5p}(t) = Q_{5,8}(t)(s)N_{8p}(t)$$
$$N_{6p}(t) = Q_{6,0}(t)(s)N_{0p}(t) + Q_{6,10}(t)(s)N_{10p}(t) + Q_{6,11}(t)(s)N_{11p}(t)$$
$$N_{7p}(t) = Q_{7,10}(t)(s)N_{10p}(t)$$
$$N_{8p}(t) = Q_{8,3}(t)(s)N_{3p}(t) + Q_{8,12}(t)(s)N_{12p}(t)$$

$N_{9p}(t) = Q_{9,1}(t)(s)N_{1p}(t) + Q_{9,12}(t)(s)N_{12p}(t)$

$N_{10p}(t) = Q_{10,2}(t)(s)N_{2p}(t) + Q_{10,13}(t)(s)N_{13p}(t)$

$N_{11p}(t) = Q_{11,0}(t)(s)N_{0p}(t) + Q_{11,13}(t)(s)N_{13p}(t)$

$N_{12p}(t) = Q_{12,3}(t)(s)N_{3p}(t)$

$N_{13p}(t) = Q_{13,2}(t)(s)N_{2p}(t)$

$N_{14p}(t) = Q_{14,0}(t)(s)[1 + N_{0p}(t)]$ (8.1-8.15)

Taking the Laplace- Stieltjes transform of these relation and solve for $\tilde{N}_{0p}(s)$ and omitting the argument s for brevity , we obtain:-

$$\tilde{N}_{0p}(s) = \frac{N_5(s)}{D_2(s)} \tag{8.16}$$

In the steady state, the number of visits per unit time is given by

$$N_{0p} = \lim_{t\to\infty}\left[\frac{N_{0p}(t)}{t}\right] = \lim_{S\to 0} s\tilde{N}_{0p}(s) = \frac{N_5}{D_2} \tag{8.17}$$

Where ,

$$N_5 = P_{0,14}[P_{36}\{1 - P_{24}P_{10}(P_{41} + P_{49}P_{91})\}(P_{11,0}P_{6,11} + P_{60}) + P_{24}P_{10}(P_{41} + P_{49}P_{91})]$$

9.cost analysis

To compute the profit incurred to this system , we have expected total profit in $(0,t]$ equal expected total revenue in $(0,t]$ minus expected total repair in $(0,t]$ minus expected cost of visits by repairman in $(0,t]$ minus expected cost of frequency of preventive maintenance in $(0,t]$.

The expected profit per unit time in steady state is

$$G = \lim_{t\to\infty}\left[\frac{G(t)}{t}\right] = \lim_{S\to 0} S^2 G^*(s) \tag{9.1}$$

Profit = total revenue - total cost (9.2)

$$G(t) = C_0A_0(t) - C_1B_0(t) - C_2V_0(t) - C_3N_{0p}(t) \tag{9.3}$$

α	Profit (1)	Profit (2)	Profit (3)	Profit (4)
0.1	14.76665	14.57549	14.27075	12.62058
0.2	10.77876	10.61689	8.681562	8.095967
0.3	7.599645	7.48833	5.984835	5.733998
0.4	5.51014	5.439107	4.447025	4.322053
0.5	4.14842	4.102896	3.457097	3.387555
0.6	3.223135	3.19309	2.764077	2.722054
0.7	2.562702	2.542171	2.249034	2.221977
0.8	2.069781	2.05529	1.849022	1.830729

Table 2.

Where,

C_0 is revenue per unit up – time.

C_1 is cost per unit time for repairing the unit .

C_2 is cost per visit by the repairman .

C_3 is cost per preventive maintenance.

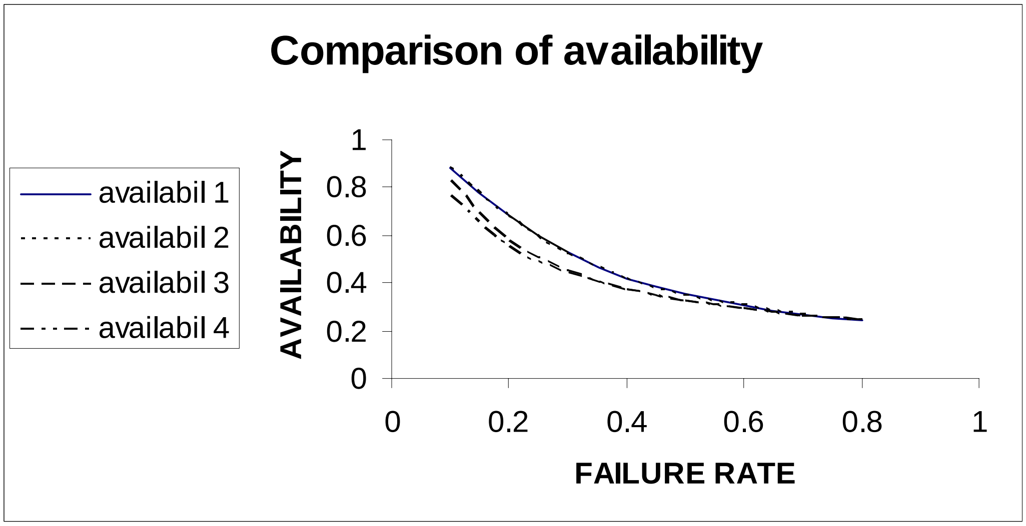

Figure 2. Comparison of availability

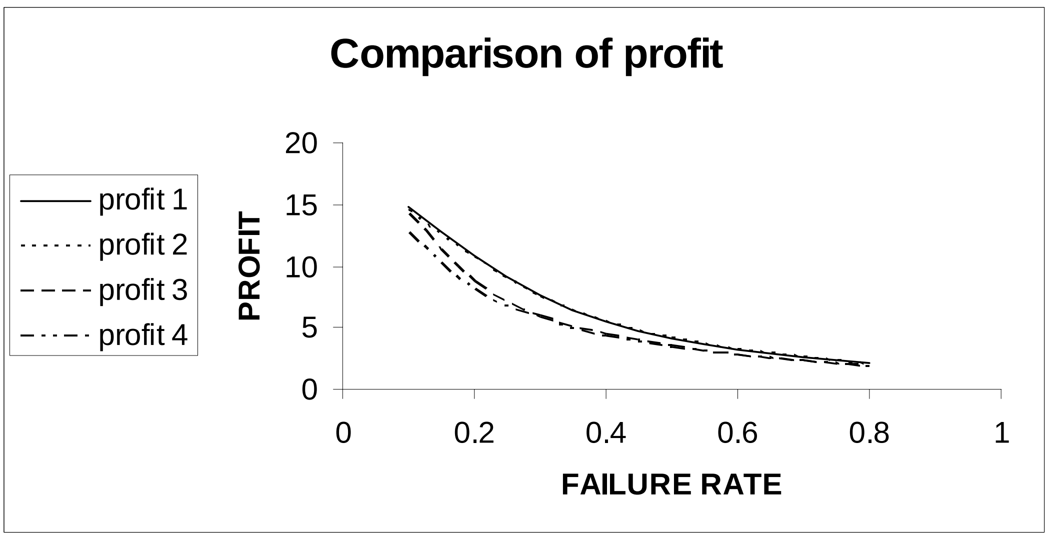

Figure 3. Comparison of profit

10. Comparison of availability and profit due to changeover of units and effect the preventive maintenance.

Suppose that

$$g(t) = at\exp(-at^2/2)\ ,\quad f(t) = bt\exp(-bt^2/2)\ ,\quad h(t) = ct\exp(-ct^2/2)$$

$$k(t) = dt\exp(-dt^2/2)\ ,\quad u(t) = \beta t\exp(-\beta t^2/2)\ ,\quad v(t) = \lambda t\exp(-\lambda t^2/2)$$

where the parameters a , b , c , d , λ, β and some others have been fixed as

$b = .25$, $c = .30$, $d = 0.40$, $p = q = 0.50$, $\lambda = 0.01$, $\beta = 0.45$, $C_0 = 20$, $C_1 = 3$

$C_2 = 2$, $C_3 = 10$

Then the values of availability and profit are obtained in four cases.

Case 1: *when changeover of units and preventive maintenance are allowed*

Setting the changeover parameter at $a = 0.20$, the values of availability (1) and profit (1) for different values of the failure rate $\alpha(t)$ are shown in Table 1.& Table 2.

Case 2: *when preventive maintenance is allowed and changeover of units are not allowed*

Setting the changeover parameter at $a = 0$, the values of availability (2) and profit (2) for different values of the failure rate $\alpha(t)$ are shown in Table 1.& Table 2.

Case 3: *when changeover of units are allowed only* .

Setting the changeover parameter at $a = 0.20, P_{0,14} = 0$, $P_{14,0} = 0, \mu_{14} = 0$, the values of availability (3) and profit (3) for different values of the failure rate $\alpha(t)$ are shown in Table 1.& Table 2.

Case 4: *when changeover of units and preventive maintenance are not allowed*

Setting the changeover parameter at $a = 0, P_{0,14} = 0$, $\mu_{14} = 0$, the values of availability (4) and profit (4) for different values of the failure rate $\alpha(t)$ are shown in Table 1.& Table 2.

The comparisons of availability and profit of the system in the cases are shown in Figs 2 and 3 , respectively.

11. Conclusion

It is observed that the changeover and preventive maintenance of units increases both the availability and profit of the system.

12.References

[1] GOEL,L.R., AGNIHOTRI R.K.and GUPTA,R." profit evaluation of a two unit cold standby system with random change in units" INT.J.SYSTEMS SCI.,1992, VOL. 23,p 367-377.

[2] MOKADDIS,G.S.,TAWFEK,M.L. and ELHSSIA,S.A.M., "Analysis of a two-dissimilar unit cold standby redundant system subject inspection and random change in units" Microelectron. Reliab., 1997, VOL.37,p329-334.

[3] MOKADDIS,G.S.,TAWFEK,M.L. and ELHSSIA,S.A.M., "cost analysis of a two dissimilar- unit cold standby redundant system subject to inspection and two types of repair", Microelectron. Reliab., 1997, VOL.37,p335-340.

[4] Rander,M.C.,Suresh,K. and Ashok,k.," cost analysis of a two dissimilar cold standby system with preventive maintenance and replacement of standby", Microelectron. Reliab., 1994, VOL.34,p171-174.

[5] Tuteja,R.K., and Gulshan, T. "cost – benefit analysis of a two- server-two unit warm standby system with different type of failure", Microelectron. Reliab., 1992, VOL.32,p1353-1359.