FUZZY MEMBERSHIP FUNCTION IN DETERMINING SPC ALLOCATION

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Abstract

Statistical Process Control (SPC) is a technical tool that is used to control and to improve almost any kind of process. However, because of cost consideration, management need to decide which process should apply SPC. In this paper, we propose the use of probability and fuzzy membership function to determine SPC allocation. Conditional probability is used to analyse process failure rate and process repair rate. Then, using Markov Matrix, we calculate the probability of out-of-control process (PO). Nevertheless, in a production line that consists of many parts, the probability value is not adequate to be used as a reference to determine SPC allocation. There are cases for instance, where the value of PO in one part does not mean the same as in other parts since each part may have different sensitivity degree to the final product. For example 0.25 of PO in part 1 may have higher influence to the final product compare to 0.25 of PO in part 2 or part 3. Furthermore, we cannot randomly choose one of those parts to apply SPC or even decide to apply SPC in all parts of the production line. To overcome this problem we propose fuzzy membership function that uses linguistic terms and degree of memberships to analyse PO instead of the probability values. By this mean, the SPC allocation could be determined without ambiguity. For this purpose, the membership function is classified into three categories, namely LOW, MEDIUM and HIGH. Any part with PO fall into the "HIGH" category and high degree of membership is prioritized to apply SPC.

Keywords: Statistical Process Control (SPC), Conditional Probability, Markov Matrix, Fuzzy membership function.

1.0 Introduction

The expression of "quality" is used when excellent product or service is fulfilled or exceed customer's expectations (Besterfield 1998). The issue of quality causes global competition among companies to provide that quality products. Companies are making every effort to reduce product variation before the product is shipped to customer. Failure to fulfil customer's expectation as well as the variations in product's parts and processes can increase production cost, which in turn affecting the product returns, product repair, scarp and rework. Regardless that products and manufacturing system are carefully design, sudden process change that affect quality product stills occur. It is impossible to reduce variation to zero (Thornton 1999, Jang 1999).

Controlling the manufacturing process using Statistical Process Control (SPC) could improve the process and prevent the system from defection. However, because of cost consideration, not all parts of the system should be equipped with SPC. Consequently, there is a need to evaluate where the SPC should be allocated to get good quality product. When the decision is made on which parts should be controlled using SPC, there should also a need to consider system sensitivity and manufacturing sensitivity (Jang 1999). In case of system characteristic sensitivity, the sensitivity degree of one part compare to final product could be analysed using key characteristic. However, system sensitivity alone does not provide enough information for determining optimal allocation of SPC in a production line. In addition to regarding system sensitivity, we also need to consider material used in manufacturing process that could affect final product (Jang 1999).

The aim of this research is to determine optimal allocation of SPC in a production line based on manufacturing sensitivity and fuzzy membership function. Manufacturing sensitivity is used to obtain any variations in a manufacturing process that will cause mean shift to the process. The mean shift is then analyzed using transition probability to determine the failure rate (departing process from a stable state to out-of-control condition) and the repair rate (returning process from out-of-control condition back to stable state). In other word, transition probability will determine future trend of the process, conditional to the entire past history. Since the application of SPC is related to out-of-control condition of the process, we then use Markov Matrix to get the state probability of the out-of-control process (PO) and the state probability of in-control process (PI) for each part in the production line. In this step we provide simulation by using computer generated random data, to examine such transition; it is applied to production line that consist of two parts (Nababan et.al. 2003).

In order to achieve the aforementioned aim, we proposed an intelligent tool to determine the allocation of SPC by integrating the transition probability and fuzzy logic technique. As a case study we would like to apply the tool on a production line that consists of three parts. The three parts indicates three measurable dimension of a beverage, which are Carbonate (Gas Volume), Brix, and Filler Height. Data was taken from a beverage manufacturing company and will be analysed using transition probability to get the state probability of out-of-control process (PO). Then, we construct an algorithm to generate membership function of the PO from a set of observation data as the basis for determining whether the process is in the degree of "HIGH" so that it is considered to apply SPC.

2.0 Ambiguities in Deciding Out-of-control Condition

From the state probability that we have got, we discovered that there is a certain condition that will cause ambiguity in deciding which part should apply SPC. To illustrate this ambiguity, consider the following crisp definition of the out-of-control condition:

If the probability of out-of-control process (PO) is 0.25 (or 25%), with a tolerance of 0.05, then the process is considered "high", otherwise it is considered "low".

Now, suppose the value of PO is 0.23 (or 23%), with the same value of tolerance, could we definitely say it as "low"? Let us consider another case, in which we need to consider the out-of-control conditions that occur in several parts of a production line and we should choose one part that has a "high" out-of-control condition. In this case, we cannot put our judgement of the high out-of-control condition based on the probability value of each part since we cannot be sure that the same number of the probability value has the same meaning in each part. For example, we cannot be sure whether the 25% out-of-control in part 1 is the same as 25% out-of-control in part 2, as well as 25% out-of control in part 3, because each part has its own characteristics that is different the other parts.

The flexibility of fuzzy set could overcome this ambiguity and calibrate the vagueness (El-Shal & Morris (1999); Negnevitsky (2002)). Fuzzy logic provides a simple way to arrive at a definite conclusion by incorporating linguistic approach to solve control problem rather than attempting to model the system mathematically (Kaehler 1998). It also provides a tool for modelling human-centred system that pervades most human perception and thinking processes (Dubois & Prade 1980). In our case, we can make fuzzy assertion on probability, since probability is also a variable, like pressure, temperature, etc. (Hajek and Godo 1995).

Fuzzy membership function could be applied to classify the out-of-control conditions using linguistic values, such as "LOW", "MEDIUM", and "HIGH", which is defined on a 0 to 100 percent unit interval of the PO probability. By this mean, we could choose which part will apply SPC by looking at the linguistic value of each part, instead of the numerical value of the PO, for instance part that have "HIGH" out-of-control conditions. Since every part of production line has different sensitivity degree to the final product, then every part of a production line has different membership function. By using membership function, the process that is considered "HIGH" could be decided to apply SPC.

3.0 Methodology

The methodology to determine the SPC allocation consists of four stages: modelling the production line, analysing and calculating PI/PO, constructing membership function, and deciding the SPC allocation. The four stages are described as in the following.

3.1 Modelling the Production Line

In this step, to simplify the problem, we modelled production line that consists of three parts in the form of block diagram, as shown below. Each part represents one dimension of the beverage, i.e. part 1: Gas Volume (GV/Carbonate), part 2: Brix and part 3: Filler Height. There are three variables considered as manufacturing properties: expected mean shift, process failure rate and process repair rate (Jang 1999). Sudden process change cause mean shift to the process and this state of process considered as process failure. Process failure indicates the process departs from the stable state, it quantifies the frequency of mean shift from the target value, while process repair indicate the process that have shifted from the target, back to stable or normal state (Jang 1999 and Nembhard 2001).

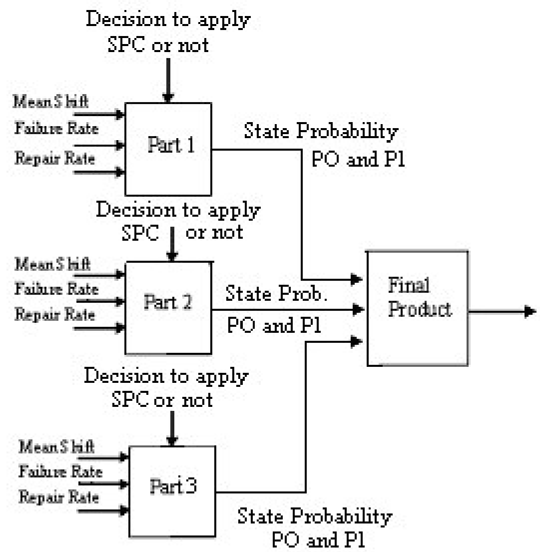


Figure 1 Model of 3 parts production line

3.2 Analysing and Calculating PI/PO

Since this research is focussed on the application of SPC, failure rate and repair rate become the main observation. These two states (process failure and process repair) will be analysed using transition probability and Markov matrix to get the state probability of out-of control process and in-control process.

Transition Probability

A major objective of probability modelling is to determine how likely it is that an event A will occur when a certain experiment is performed (Bain & Engelhardt 1992). However, in numerous cases the probability assigned to A will be affected by facts of the occurrence or non-occurrence of another event B. The terminology conditional probability of A given B is notated as P(A|B).

$$P(A/B) \quad = \quad \frac{P(A \cap B)}{P(B)} \qquad [1]$$

Let A(n) is the event that the system in state A immediately after the n^{th} trial , the probability of this event is P(A(n)). Transition probability is define as the conditional probability (Jang

1999) that the system will be in state A_j right after the next trial, given that the present state of the process is A_i.

$$p_{ij} = P[A_j(n) / A_i(n-1)] \qquad [2]$$

Process control has two states to be considered: in-control and out-of-control states. Using transition probability to the process we can figure out the probability of next state. The transition probabilities of process failure P_f and probability of repair process P_r are parameters of a geometric distribution (Jang 1999). If X denotes the number of the trial at which this first success occurs and F and S represent failure and success, the sequence starts with F,F,F,S….where X = 4. The probability of such sequence is P(x=4)=(1-p)(1-p)(1-p) p = $(1-p)^3$ p., then the probability distribution function of X is represent as:

$$f(x) = (1-p)^{x-1}\, p \qquad [3]$$

In probability, x has a geometric distribution. And solving for p, we get

$$p = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{\bar{x}} \qquad [4]$$

Using eq.4,

$$P_f = \frac{1}{\bar{x}_f} \quad \text{and} \quad P_r = \frac{1}{\bar{x}_r} \qquad [5]$$

Markov Matrix

If the system experiences multiple trials then a transition probability can be defined as a set of transition probability and it is define as Markov process (Kofke 2004) and the calculation of PI and PO from P_f and P_r can be done using Markov Matrix. The general equation for state probabilities is:

$$\begin{bmatrix} P_{11}-1 & P_{21} & \cdots & \cdots & P_{m1} \\ P_{12} & P_{22}-1 & P_{23} & \cdots & P_{m2} \\ \vdots & & & \ddots & \vdots \\ P_{1(m-1)} & \cdots & & & P_{m(m-1)} \\ 1 & 1 & \cdots & \cdots & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{m-1} \\ P_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad [6]$$

where p_{ij} is a transient probability from state i to j and P_1 is a probability that the process is in state i. Applying the above equation to the model in Fig.1 we get transition probability matrix for calculating PI and PO of each part in the production line as:

$$\begin{bmatrix} -P_f & P_r \\ 1 & 1 \end{bmatrix} \begin{bmatrix} PI \\ PO \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad [7]$$

$$\begin{bmatrix} PI \\ PO \end{bmatrix} = \begin{bmatrix} \dfrac{P_r}{P_f + P_r} \\ \dfrac{P_f}{P_f + P_r} \end{bmatrix} \qquad [8\ ]$$

3.3 Fuzzy Membership Function

In generating fuzzy membership function we refer to the papers of Lucero & Nava (1999), Bilgic & Turksen (1999), Shi & Sen (2000), Zhao & Bose (2002), and Negnevitsky (2002). The papers give an overview of how to generate membership function from the observation data. In this paper we use the piecewise linier function since this is the simplest type of MFs, yet powerful enough to classify various linguistic values. The piecewise linier function may be generally either triangular or trapezoidal type, where the trapezoidal MF has the shape of a truncated triangle. After obtaining the membership functions we then implement the function to a set of running data and calculate its degree of membership. By considering the degree of memberships, part that has a HIGH degree of membership is decided to apply SPC.

The membership functions are constructed from a set of observation data and from the membership function obtained, the exact class of the actual PO could be determined. For this purpose, we divide the observation data into two groups: training data and running data. The training data are those used to construct the membership function, while the running data are the actual data that needs to be evaluated.

Constructing PO membership functions

The steps needed to build the membership functions are as follows:

1. Divide the observation data into n groups with each group contains m number of samples and p sample size.

   The structure of the observation data in terms of the number of groups, the number of data and the sample size is represented in the form of three-dimensional matrix as shown in Figure 2.

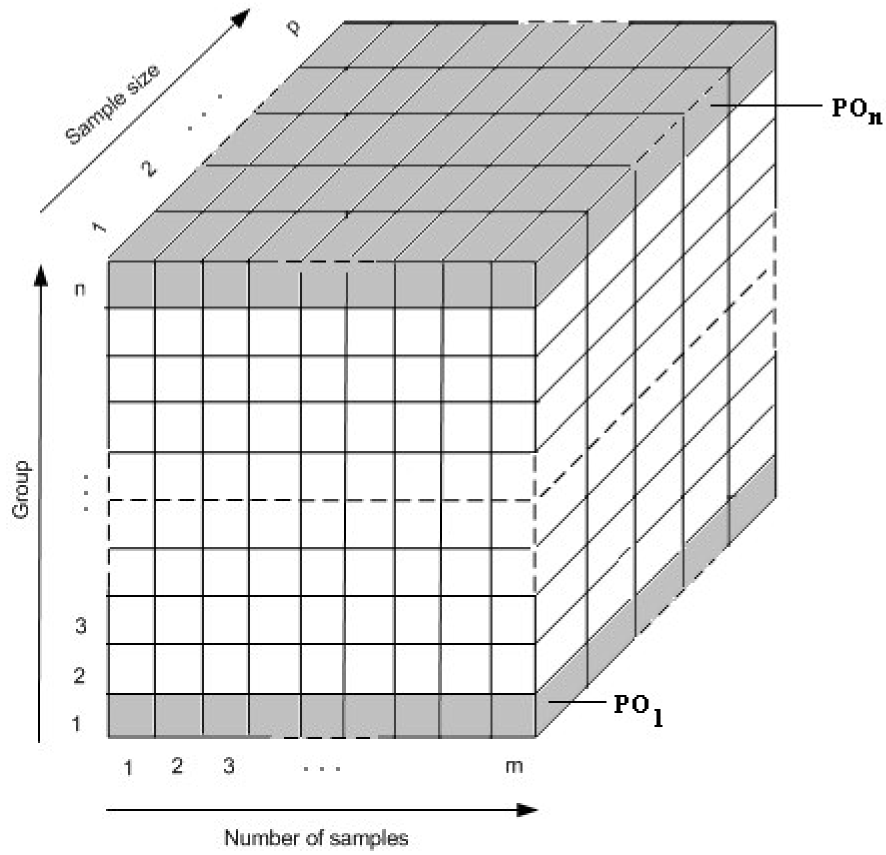


Figure 2 Three-dimensional matrix representing groups, number of sample and sample size

To calculate how big is n, let the number of output data (X) = 2600, k = 150 and p = 5. Then the number of group:

$$n = Int\ (\tfrac{x}{k}) = Int(\tfrac{2600}{150}) = 17$$

2. Calculate the PO for each group i (i = 1, 2, 3, …, n), that is for each slice of the three-dimensional matrix. Put the POs obtained into a one-dimensional matrix P:

$$P = \{PO_1, PO_2, PO_{3,} \ldots, PO_n\}$$

3. Construct the membership function of PO from the set *P*.
    a. Arrange the value of PO into an ascending order.
    b. Determine the end points of membership function based on the distribution of PO by referring to the graph shown in Figure 3.

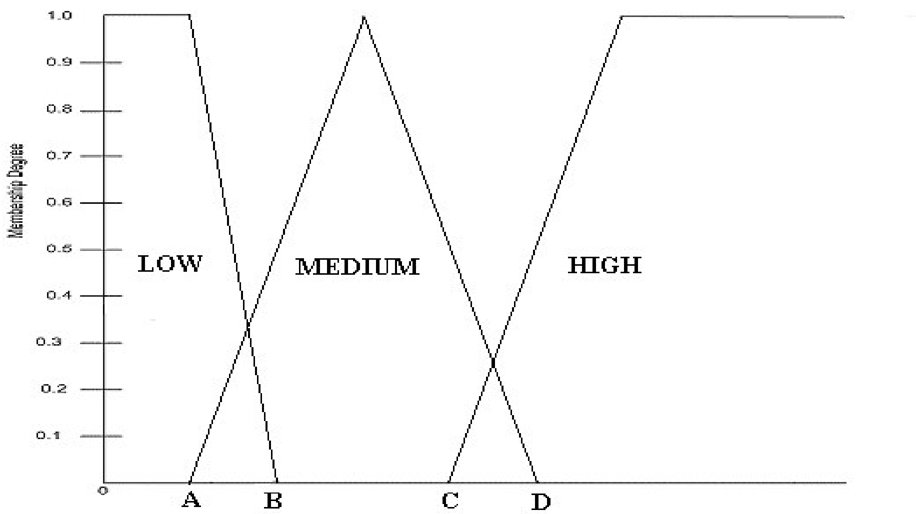


Figure 3 End points of membership function (A, B, C and D)

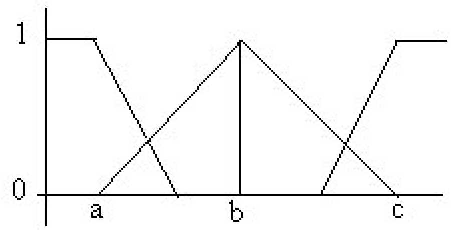
Calculating the degree of membership

The following steps are performed in order to calculate the degree of membership for each running data from each part of the production line:

1. Read the file containing the observation data.
2. Organize the data into a (m x p) matrix, where m is the number of samples and p = the sample size.
3. Calculate PO.
4. Determine the membership degree of PO using the formula below (adapted from Zhao & Bose (2002) and Shi & Sen (2000)). The membership degree could be calculated using these formulas:

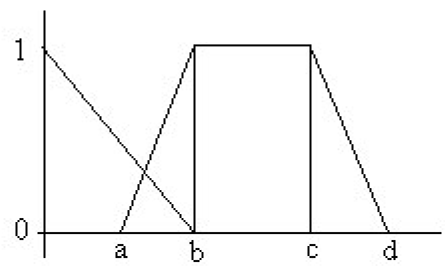
Triangular formula:

$$f(x;a,b,c)=\begin{cases} 0, x < a \\ \dfrac{x-a}{b-a}, a \le x \le b \\ \dfrac{c-x}{c-b}, b < x \le c \\ 0, x > c \end{cases} \quad [9]$$



Trapezoidal formula:

$$f(x;a,b,c,d)=\begin{cases} \dfrac{x-a}{b-a},\ a \le x < b \\ 1, b \le x \le c \\ \dfrac{d-x}{d-c}, c < x \le d \\ 0, x > d \end{cases} \quad [10]$$



To illustrate the above process, suppose that from the training data we got the end points of the membership function of part 1 are 0.10, 0.20, 0.36 and 0.50 (see Figure 4). Let the PO of the calculated running data is 0.40. Then the membership degree is calculated as in the following.

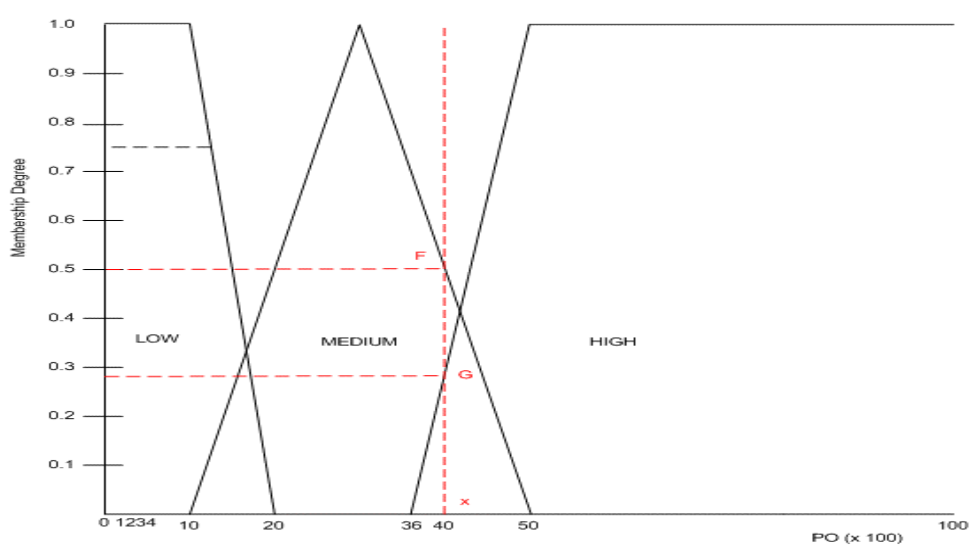


Figure 4 Degree of membership of PO 0.40

The PO of 0.40 has an intersection with the triangle area at point F, so the degree of membership could be calculated using triangular formula:

$$f(x) = \frac{c - x}{c - b}$$

So,

$$F = \frac{50 - 40}{50 - 30} = \frac{10}{20} = 0.5$$

Point G is the intersection of the PO with the trapezoidal area, so the degree of membership is calculated using the following formula:

$$f(x) = \frac{x - a}{b - a}$$

Thus,

$$G = \frac{40 - 36}{50 - 36} = \frac{4}{14} = 0.29$$

The above calculation is repeated for each part of the production line.

3.4 Deciding the SPC Allocation

After obtaining the degree of membership of PO from each part of the production line, we are ready to decide which part should apply SPC based on the value PO of HIGH and highest degree of membership. The determination of the SPC allocation is based on the following rules:

IF PO_1 is HIGH (x) and PO_2 is HIGH (y) and PO_3 is HIGH (z)
AND $x > y$ AND $x > z$ THEN Apply SPC at Part-1
IF PO_1 is HIGH (x) and PO_2 is HIGH (y) and PO_3 is HIGH (z)
AND $x > y$ AND $x < z$ THEN Apply SPC at Part-3
IF PO_1 is HIGH (x) and PO_2 is HIGH (y) and PO_3 is HIGH (z)
AND $x < y$ AND $y > z$ THEN Apply SPC at Part-2
IF PO_1 is HIGH (x) and PO_2 is HIGH (y) and PO_3 is HIGH (z)
AND $x < y$ AND $y < z$ THEN Apply SPC at Part-3

IF PO_1 is HIGH (x) and PO_2 is HIGH (y) and PO_3 is MEDIUM (z)
AND $x > y$ THEN Apply SPC at Part-1
IF PO_1 is HIGH (x) and PO_2 is HIGH (y) and PO_3 is MEDIUM (z)
AND $x < y$ THEN Apply SPC at Part-2
IF PO_1 is HIGH (x) and PO_2 is MEDIUM (y) and PO_3 is HIGH (z)
AND $x > z$ THEN Apply SPC at Part-1

IF PO_1 is HIGH (x) and PO_2 is MEDIUM (y) and PO_3 is HIGH (z)
AND $x < z$ THEN Apply SPC at Part-3

IF PO_1 is MEDIUM (x) and PO_2 is HIGH (y) and PO_3 is HIGH (z)
AND $y > z$ THEN Apply SPC at Part-2

IF PO_1 is MEDIUM (x) and PO_2 is HIGH (y) and PO_3 is HIGH (z)
AND $y < z$ THEN Apply SPC at Part-3

IF PO_1 is HIGH (x) and PO_2 is MEDIUM (y) and PO_3 is MEDIUM (z)
THEN Apply SPC at Part-1

IF PO_1 is MEDIUM (x) and PO_2 is HIGH (y) and PO_3 is MEDIUM (z)
THEN Apply SPC at Part-2

IF PO_1 is MEDIUM (x) and PO_2 is MEDIUM (y) and PO_3 is HIGH (z)
THEN Apply SPC at Part-3

IF PO_1 is HIGH (x) and PO_2 is MEDIUM (y) and PO_3 is LOW (z)
THEN Apply SPC at Part-1

IF PO_1 is MEDIUM (x) and PO_2 is HIGH (y) and PO_3 is LOW (z)
THEN Apply SPC at Part-2

IF PO_1 is LOW (x) and PO_2 is MEDIUM (y) and PO_3 is HIGH (z)
THEN Apply SPC at Part-3

The above rules considers the combination of the value of PO that is high in all parts, high in two parts and medium in the third part, and high only in one part. The x, y, and z in the above rules denote the degree of membership of part 1, part 2 and part 3, respectively.

4.0 Results

The use of membership function could help operator to make a decision on determining SPC application in a production line easier. It is more flexible on decision-making compare to probability. Degree of membership show how large is the degree of process's out-of-control of each part of production line. In case of PO is within the shaded area, the membership degree could be calculated use fuzzy intersection formula [11] (Dubois & Prade 1980).

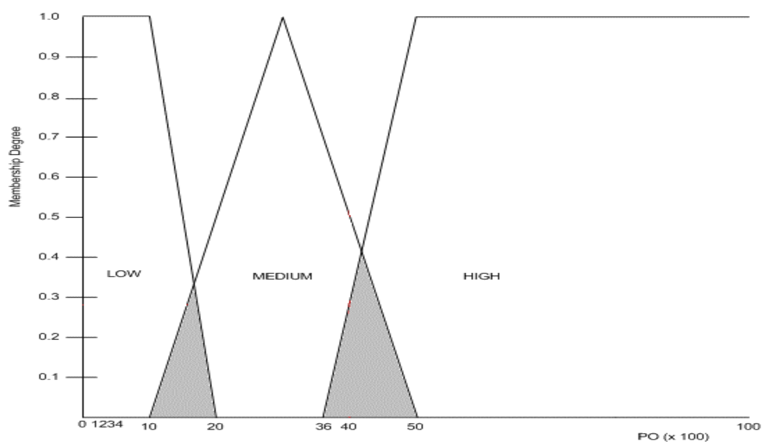


Figure 5 Membership intersection

$$f_{(A\cap B)}(x) = \min(\mu_A(x), \mu_B(x)) \quad [11]$$

As we see from Fig.4. PO 0.40 has an intersection at the point F and G. Membership degree of point F is 0.50 and point G is 0.29. By using formula [11], we got the membership degree of PO 0.40 is 0.29. We used this formula to determine membership degree of GV (Gas Volume), Brix and Filler Height.

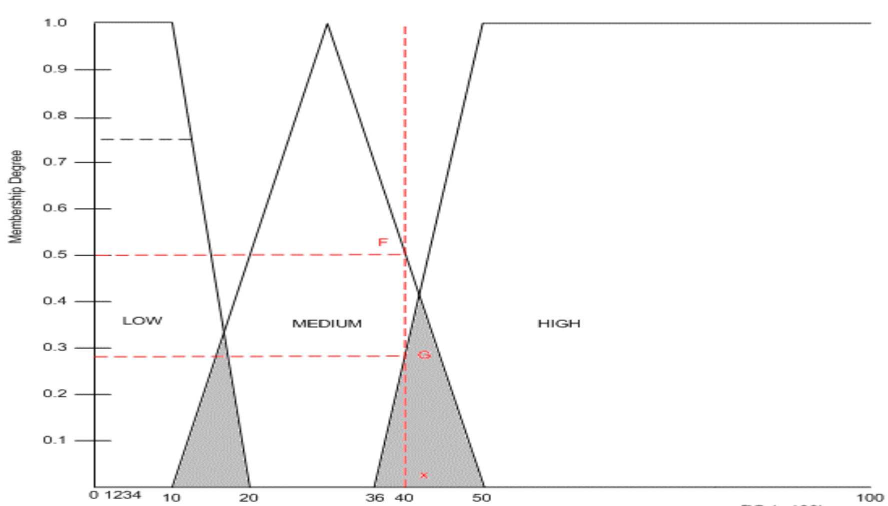


Figure 6 Membership degree of GV

The membership degree of GV is 0.29 HIGH.

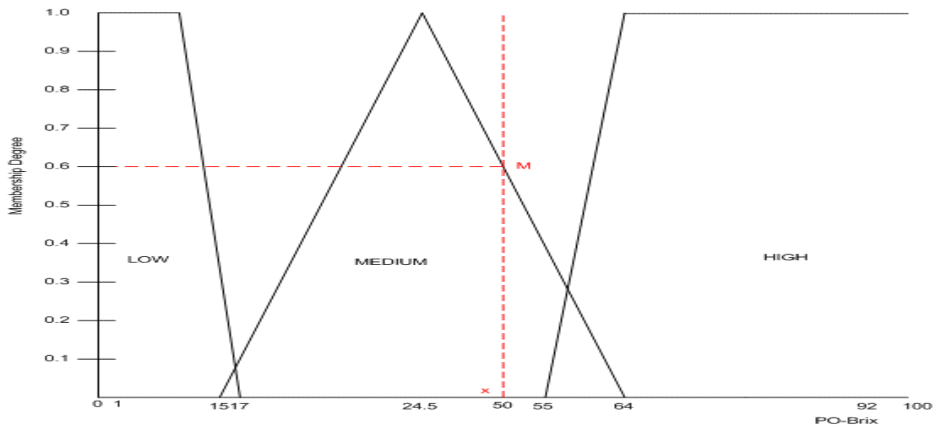


Figure 7 Membership degree of Brix

Membership degree of Brix is 0.6 MEDIUM.

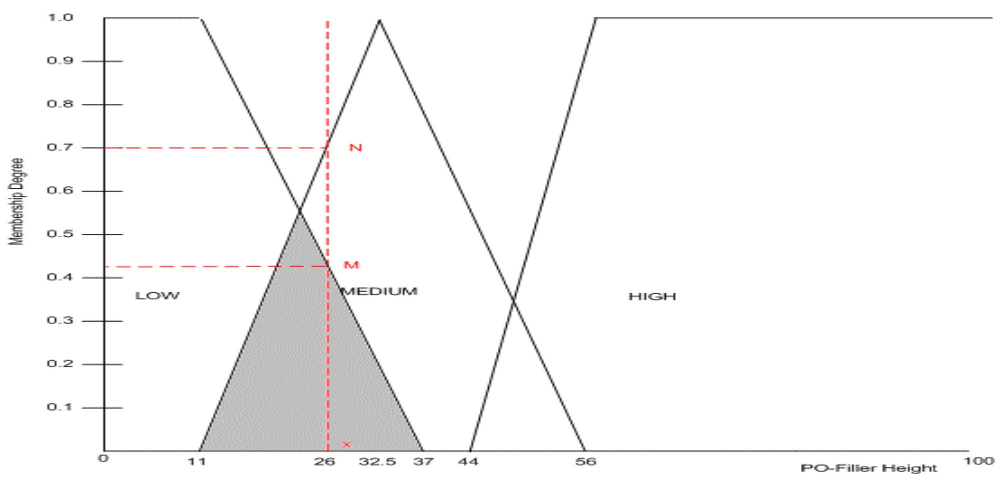


Figure 8 Membership degree of Filler Height

The membership degree of Filler Height is 0.42 LOW.

Using the IF-THEN rule to the above membership degrees, the following rule is match:

IF PO_1 is HIGH (0.29) and PO_2 is MEDIUM (0.60) and PO_3 is LOW (0.42)
THEN Apply SPC at Part-1

So the decision to apply SPC is on Part 1.

5.0 Conclusions and Future Research

In this paper we have implemented the fuzzy membership function to determine the optimal SPC allocation on a production line which consists of many parts. From this implementation, it is shown that the decision on the allocation of the SPC could be taken precisely and in more natural way compared to the decision which is taken based on the numerical value of the probability of out-of-control condition. It is our intention to further enhance the methodology as well as the fuzzy logic technique used in order to obtain better results.

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References

Bain, L.J & Engelhardt, M. 1992. Introduction to Probability and Mathematical Statistics 2nd. ed. Boston: PWS-KENT.

Bilgic, Taner & Turksen, Burhan. 1999. *Measurement of Membership Function: Theoretical and Experimental Work*. In D. Dubois and H. Prade (editors) Handbook of Fuzzy Systems, Vol. 1, Fundamentals of Fuzzy Sets, chapter 3, pages 195--202. Kluwer Academic Publishers.

Barr, D.B. & Zehna, P.W. 1983. Probability: Modeling Uncertainty. Reading, MA: Addison-Wesley.

Besterfield, D. H. 1998. Quality Control. 5th edition. New Jersey: Prentice Hall.

Dubois, D. & Prade, H. 1980. *Fuzzy Sets and Systems Theory and Application*. New York: Academic Press.

Eiklenborg, M., Ioanou, S., King II, G. & Vilcheck, M. 2002. Taguchi Methods for Achieving Quality: http://online.sfsu.edu (April 9, 2002).

El-Shal, S. M. & Morris, A.S. 1999. A Fuzzy Rule-Based Algorithms to Improve The Performance of SPC in Quality Systems, *Proceedings of IEEE International Conference on Systems, Man., and Cybernetics*. Tokyo, Japan: 284 –289.

Ertan, B. 1998. Analysis of Key Characteristic Methods and Enablers Used in Variation Risk Management. Masters Thesis. Department of Mechanical Engineering. Massachusetts Institute of Technology, Cambridge, MA.

Hajek, P. & Godo, L. 1995. Fuzzy Logic and Probability, *Proceeding of the Eleventh Annual Conference on Uncertainty in Artificial Intelligence (UAI-95)*, Montreal, Canada.

Jang, Y.J. 1999. Mathematical Model for Optimal Allocation Statistical Process Control. Masters Thesis. Department of Mechanical Engineering. Massachusetts Institute of Technology, Cambridge, MA.

Kaehler, S.D. 1998. Fuzzy Logic – An Introduction: www.seattlerobotics.org/encoder/mar98/fuz/flindex.html (October 20, 2003)

Kofke, D.A. 2004. Markov Processes. Lecture Note. Department of Chemical Engineering SUNY Buffalo http://www.cheme.buffalo.edu/courses/ce503/Lectures/Lecture8/sld001.htm (Januari 18, 2004)

Lucero, Y. & Nava, P. 1999. A Method for Membership Function Generation from Training Samples. Master Thesis. Department of Electrical and Computer Engineering. University of Texas at El Paso, TX.

Nababan, E.B, Hamdan, A.R, Hasan, M.K. & Mohamed, H. 2003. Transition Probability in Allocating SPC. *Proceeding on International Simposium on Information Technology (ITSim2003).* Kuala Lumpur. 337-346.

Negnevitsky, M. 2002. Artificial Intelligent: A Guide to Intelligent Systems. Essex: Addison Wesley.

Nembhard, H.B and Ming, S.K. 2001. Implementing SPC in a Simulation Model for Manufacturing Transition, *Journal of Quality Engineering, 14(1), 127-136.*

Thornton, A.C. 1999. A Mathematical Framework for The Key Characteristic Process, Working Paper, Massachusetts Institute of Technology, Cambridge, MA

Shi, Yigang and Sen, P.C. 2000. Effect of Different Slopes of Membership Functions on the Fuzzy Control of DC-DC Converters. *Proceedings of The Third International Power Electronics and Motion Control Conference (PIEMC 2000).* Beijing China: 1160 - 1165.

Zhao, Jin and Bose, B.K. 2002. Evaluation of Membership Function for Fuzzy Logic Controlled Induction Motor Drive. *Proceeding of the IEEE 28th Annual Conference of the Industrial Electronics Society (IECON'02).* Sevilla, Spain: 229-234.