RESPONSE TECHNIQUES TO ANALYSE VARIOUS TRANSFORMATIONS AND SELECTION PROBABILITIES.

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ABSTRACT

In the present study, alternative estimators for estimating population totals in multi-character survey sampling using randomised response technique have been suggested, when certain variables have poor positive correlation and others have poor negative correlation with selection probabilities. The estimators proposed by Hansen and Hurwitz (1943), Rao (1966) and Sahoo *et al.* (1994) under scrambled responses are shown as special cases of the proposed estimators. An analysis has been conducted and fine-tuned in order to find the best transformation on the selection probabilities in multi-character surveys, when the scrambled responses are collected on sensitive variables. The bias and relative efficiency of the proposed estimators with respect to one another have been investigated and computed.

Key words: multi-character surveys; poorly correlated variables; estimation of total; auxiliary information, sensitive characters.

1. INTRODUCTION

The use of auxiliary information in survey sampling has an eminent role to improve the efficiency of estimators in survey sampling. Consider a finite population $W = (U_1, U_2, \ldots, U_N)$ of N identifiable units. Let y be the variable under study taking value y_i on unit U_i, $i = 1,2,\ldots,N$. Let x be an auxiliary variable presumed to be highly correlated with y, taking a known positive value x_i on Ω such that $X = \sum_{i=1}^{N} x_i$. This auxiliary variable x may be utilised at the design stage to select a sample such that the probability of selecting U_i, say p_i^+ depends on x_i $(i = 1,2,\ldots,N)$ and we must have $p_i^+ > 0$ and $\sum_{i=1}^{N} p_i^+ = 1$. Such an unequal probability sampling scheme which ensures the selection of the units with probability proportional to some $x-$values of the units is usually called PPX (probability proportional to X) and with replacement design. The well known Hansen and Hurwitz (1943) estimator of population total for probability proportional to size and with replacement sampling is given by

$$\hat{Y}_{HH} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i^+}, \text{ Where } p_i^+ = x_i \Big/ \sum_{i=1}^{N} x_i . \qquad (1.1)$$

The randomised response technique, a procedure for collecting the information on sensitive characters without exposing the identity of the respondent, was first introduced by Warner (1965). Other developments in this technique are due to Greenberg *et al.* (1971), Eichhorn and Hayre (1983), Franklin (1989), Kuk (1990), Mangat and Singh (1990), Mangat (1994), Singh and Singh (1992a, 1992b, 1993), Singh and Joarder (1997), and Mahmood *et al.* (1998). Chaudhuri and Adhikary (1990) have given a technique for the estimation of mean or total of quantitative variable, which are sensitive in nature. Based on this technique, an estimator for the population total of a sensitive quantitative variable y is suggested by Arnab (1990) and is given by

$$\hat{Y} = \frac{1}{n}\sum_{i=1}^{n}\frac{r_i}{p_i^+} \qquad (1.2)$$

Where r_i denotes the response of the i^{th} respondent selected in the sample with probability p_i^+. The responses r_i are generated through a randomisation device as $r_i = lY_i + m$, where Y_i denote the actual value of the character(s) under study and l, m are the random variables satisfying the conditions, such that $E_2(l) = 1$ and $E_2(m) = 0$. Here E_2 denote the expected value over the randomisation device used to collect the scrambled responses in the survey. For detail, one can refer to Chaudhuri and Mukherjee (1988) and Tracy and Mangat (1996). Under such randomisation devices, the estimator in (1.2) is unbiased and has the variance

$$V\left(\hat{Y}\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{V_i^2}{p_i^+} + \frac{1}{n}\left(\sum_{i=1}^{n}\frac{Y_i^2}{p_i^+} - Y^2\right) \tag{1.3}$$

where

$$V_i^2 = V\left(r_i\right) = \boldsymbol{a} Y_i^2 + \boldsymbol{t}\, Y_i + \boldsymbol{q} \;, \; \boldsymbol{a} > 0 \tag{1.4}$$

In addition, symbols $\boldsymbol{a}$, $\boldsymbol{t}$ and $\boldsymbol{q}$ in (1.4) are constants {Please see Arnab (1990) for detail }. It is a well-known fact that the PPSWR sampling is more efficient than SRS when the correlation between the study variable and auxiliary variable is positive and high. The second component in the variance expression (1.3) will be lesser than SRS for high value of positive correlation between y and x. Since our variable of interest is sensitive in nature and if it has strong correlation with the known auxiliary variable, then the privacy of the respondents may be disclose. In other words, the units having higher known selection probabilities are expected to have higher values of the sensitive character. Therefore, use of strongly related auxiliary character is not recommendable in the randomised response surveys, and hence the use of estimator (1.2) may disclose the privacy of the respondents.

In sample surveys of many variables, some of the study variables may be poorly correlated with the selection probabilities. J.N.K. Rao (1966) has provided alternative estimators when the study variable and size measure are unrelated and demonstrated that these alternative estimators are more efficient though biased. Bansal and Singh (1985) noticed that the Rao (1966) model deals with zero correlation and so developed a new transformed estimator of population total for characteristics that are poorly correlated with the selection probabilities. Amahia *et al*. (1989) suggested simple alternatives to the transformations in Bansal and Singh (1985). Rao (1993a, 1993b) has discussed certain problems of sampling design and estimation under multiple character surveys. Bedi (1995) has studied the Midzuno scheme of sampling for multi - character surveys Mangat and Singh (1992-1993) have mentioned additional transformations of selection probabilities in their review article. Bansal *et al.* (1994) and Grewal *et al.* (1997) have considered the following six estimators of population total Y as follows:

$$\left(\hat{Y}_{pps}\right)_k = \frac{1}{n}\sum_{i=1}^{n}\frac{r_i}{P_{ik}^*} \qquad \text{for} \quad k = 0,1,2,3,4,5 \tag{1.5}$$

where,

$$P_{i0}^* = \frac{1}{N} \qquad \text{[Rao (1966)]} \tag{1.6}$$

$$P_{i1}^* = \left(1+\frac{1}{N}\right)^{(1-r)}\left(1+p_i^+\right)^{r} - 1 \quad \text{[Bansal and Singh (1985)]} \tag{1.7}$$

$$P_{i2}^* = \frac{(1-\boldsymbol{r})}{N} + \boldsymbol{r}\, p_i^+ \qquad \text{[Amahia \textit{et al.} (1989)]} \tag{1.8}$$

$$P_{i3}^* = \left(\frac{1}{N}\right)^{(1-r)} p_i^{+\,r} \qquad \text{[Amahia \textit{et al.} (1989)]} \tag{1.9}$$

$$P_{i4}^* = \left[N(1-\boldsymbol{r}) + \frac{\boldsymbol{r}}{p_i^+}\right]^{-1} \qquad \text{[Amahia \textit{et al.} (1989)]} \tag{1.10}$$

$$P_{i5}^* = \frac{\left(1-\boldsymbol{r}^{\frac{1}{3}}\right)}{N} + \boldsymbol{r}^{\frac{1}{3}} p_i^+ \qquad \text{[Grewal \textit{et al.} (1997)]} \tag{1.11}$$

The transformations $P_{ik}^* \ (k = 1,2,3,4,5)$ in (1.7), (1.8), (1.9), (1.10) and (1.11), respectively, of the selection probabilities p_i^+ are useful for positive and low correlation between x and y variables, whereas transformation (1.6) is useful under no correlation situation. If $r = 0$ then $P_{ik}^* \ (k = 1,2,3,4,5)$ reduces to P_{i0}^* in (1.6) and if $r = 1$ then these transformations reduces to original selection probabilities p_i^+.

Singh and Horn (1998) raised a '**natural question**' in multi-character surveys that some variables may be poorly negatively correlated and some variables may be poorly positively correlated. They suggested an estimator to be useful in such situations in direct question surveys.

Suppose y and x are highly negatively correlated and $X > n\max(x_i)$. Following Srivenkataramana (1980), Sahoo *et al.* (1994) considered a transformed auxiliary variable z whose value for U_i is defined as follows:

$$z_i = \frac{X - nx_i}{N - n} \qquad \forall \quad U_i e\ \Omega \tag{1.12}$$

and proposed an unbiased estimator of population total as

$$\hat{Y}_{SM} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i^-} \tag{1.13}$$

where $p_i^- = \dfrac{z_i}{\sum_{i=1}^{N} z_i} = \dfrac{z_i}{\sum_{i=1}^{N} x_i}$. (1.14)

Sahoo *et al.* (1994) have used a transformed auxiliary variable z at the design stage and referred to this as a probability proportional to z (PPZ) with replacement sampling scheme. The limitation of this strategy, which one may consider, is that in some situations, the condition $X > n\,Max(x_i)$ may not be satisfied, but this condition is reasonably satisfied in most of the practical situations. A practical example, where this condition is satisfied is illustrated below:

Example 1.1: Suppose that the ranks of the auxiliary character x_i are used to select the sample of n units with PPSWR sampling from a population of size N. The condition $X > n.\ Max(x_i)$ is satisfied if

$$n < 1 + \frac{\sum_{i=1}^{N-1} X_i}{Max(x_i)} \tag{1.15}$$

Consequently, $Max(x_i) = N$ and $\sum_{i=1}^{N-1} X_i = \dfrac{N(N-1)}{2}$, therefore, (1.15) reduces to the condition on sample size below

$$n < \frac{(N+1)}{2} \tag{1.16}$$

In other words, the condition $X > n.\, Max(x_i)$ remains satisfied if the size of the selected sample is less than 50% of the population size. Thus the condition $X > n.\, Max(x_i)$ may is satisfied in most of the practical situations. The literature related to the use of ranks and its benefits in selecting the sample can be derived from Wright (1990).

The next section has been devoted to study the several estimators of population total in randomised response surveys.

2. PROPOSED STRATEGIES

In this section, We are proposing an alternative estimator of population total

$$\hat{Y}_P = \frac{1}{n}\sum_{i=1}^{n}\frac{r_i}{y_{ik}} \tag{2.1}$$

where

$$y_{i0} = \frac{1}{N} \tag{2.2}$$

$$y_{i1} = \left(1+\frac{1}{N}\right)^{(1-r)(1+r)}\left(1+p_i^+\right)^{r(1+r)/2}\left(1+p_i^-\right)^{-r(1-r)/2} - 1 \tag{2.3}$$

$$y_{i2} = \left(p_i^+\right)^{r(1+r)/2}\left(p_i^-\right)^{-r(1-r)/2}\left(\frac{1}{N}\right)^{(1-r)(1+r)} \tag{2.4}$$

$$y_{i3} = \frac{(1-r)(1+r)}{N} + \frac{1}{2}\left[r(1+r)p_i^+ - r(1-r)p_i^-\right] \tag{2.5}$$

$$y_{i4} = \left[N(1-r)(1+r) + \frac{r(1+r)}{2p_i^+} - \frac{r(1-r)}{2p_i^-}\right]^{-1} \tag{2.6}$$

or

$$y_{i5} = \frac{\left(1-|r|^{\frac{1}{3}}\right)\left(1+|r|^{\frac{1}{3}}\right)}{N} + \frac{1}{2}\left[|r|^{\frac{1}{3}}(1+r)p_i^+ + |r|^{\frac{1}{3}}(1-r)p_i^-\right] \tag{2.7}$$

and r is the correlation between y and x. We arrived at the transformation (2.3) to (2.7) by the natural question raised above and the experience of the transformations suggested from (1.7) to (1.11) by various researchers. Thus, the following cases can be constructed.

Case 1: If $r = 1$ then $p_i^* = p_i^+$, the proposed estimator $\hat{Y}_P$ reduces to $\hat{Y}_{HH}$ under scrambled responses (say, $\hat{Y}_1$)

Case 2: If $r = 0$ then $p_i^* = 1/N$, the proposed estimator reduces to the estimator Proposed by J.N.K. Rao (1966) under scrambled responses (say, $\hat{Y}_2$).

Case 3: If $r = -1$ then $p_i^* = p_i^-$, the proposed estimator $\hat{Y}_P$ reduces to $\hat{Y}_{SM}$ under scrambled responses (say, $\hat{Y}_3$).

The transformations suggested in (2.3) to (2.7) make use of the known correlation coefficient r between the study variable y and the auxiliary character x.

[Similar to transformations in (1.7) to (1.11), which are applicable only to the situations of positive and low value of r]. The study variable y can be any one among the k variables of interest, say $y_1, y_2, \ldots, y_k$, some of them have low positive and others having low negative correlation with the auxiliary character x. In actual practice, the value of r is not known in most of the surveys. Thus, it is advisable to use the estimator of r in (2.3) to (2.7). Suppose $\hat{r}$ is an estimator of the correlation coefficient, r obtained under scrambled responses by following Bellhouse (1995). Singh and Horn (1998) have studied the effect of replacement of r by its estimator for direct question surveys. It is also possible to study this effect under scrambled responses.

Defining E$_1$ and E$_2$ as the expected values over all possible samples and over the randomisation device, respectively. That is,

$$E\left(\hat{Y}_P\right)_k = E_1 E_2\left[\frac{1}{n}\sum_{i=1}^{n}\frac{r_i}{y_{ik}}\right] = E_1\left[\frac{1}{n}\sum_{i=1}^{n}\frac{E_2(r_i)}{y_{ik}}\right] = \sum_{i=1}^{N}\frac{Y_i P_i}{y_{ik}}$$

(2.8)

where P$_i$ denote the selection probabilities defined as $P_i = l p_i^{+} + \left(1 - l\right) p_i^{-}$ for some suitable choice of l . Thus we have the following theorem and its proof is obvious.

Theorem 2.1: The bias in the proposed estimator (2.1) is given by

$$B\left(\hat{Y}_P\right)_k = \sum_{i=1}^{N}\left(\frac{P_i}{y_{ik}} - 1\right)Y_i \tag{2.9}$$

Theorem 2.2: The variance of the proposed estimator (2.1) is given by

$$V\left(\hat{Y}_P\right)_k = \frac{1}{n}\sum_{i=1}^{N}\frac{V_i^2 P_i}{y_{ik}^{\,2}} + \frac{1}{n}\left[\sum_{i=1}^{N}\frac{Y_i^2 P_i}{y_{ik}^{\,2}} - \left(\sum_{i=1}^{N}\frac{Y_i P_i}{y_{ik}}\right)^2\right] \tag{2.10}$$

Proof: Defining V$_1$ and V$_2$ as the variance over all possible samples and over the randomisation device, we have

$$\left[\begin{aligned} V\left(\hat{Y}_P\right)_k &= E_1 V_2\left[\left(\hat{Y}_P\right)_k\right] + V_1 E_2\left[\left(\hat{Y}_P\right)_k\right] \\ &= E_1\left[\frac{1}{n^2}\sum_{i=1}^{n}\frac{V_2\left(r_i\right)}{y_{ik}^{\,2}}\right] + V_1\left[\frac{1}{n}\sum_{i=1}^{n}\frac{E_2\left(r_i\right)}{y_{ik}}\right] \\ &= \frac{1}{n}\sum_{i=1}^{N}\frac{V_i^2 P_i}{y_{ik}^{\,2}} + \frac{1}{n}\left[\sum_{i=1}^{N}\frac{Y_i^2 P_i}{y_{ik}^{\,2}} - \left(\sum_{i=1}^{N}\frac{Y_i P_i}{y_{ik}}\right)^2\right] \end{aligned}\right.$$

Hence the theorem.

Theorem 2.3: The mean squared error of the estimator at (2.1) is given by

$$MSE\left(\hat{Y}_P\right)_k = \frac{1}{n}\sum_{i=1}^{N}\frac{V_i^2 P_i}{y_{ik}^{\,2}} + \frac{1}{n}\left[\sum_{i=1}^{N}\frac{Y_i^2 P_i}{y_{ik}^{\,2}} - \left(\sum_{i=1}^{N}\frac{Y_i P_i}{y_{ik}}\right)^2\right] + \left[\sum_{i=1}^{n} Y_i\left(\frac{P_i}{y_{ik}} - 1\right)\right]^2 \tag{2.11}$$

Proof: It is obvious from (2.9) and (2.10).

3. SUPER POPULATION MODEL

To select the best of the six proposed estimators $\left(\hat{Y}_P\right)_k$ (k = 0, 1, 2, 3, 4, 5), we find the expected values of the biases and MSE's of the estimators under the super population model suggested by Cochran (1963). The model assumes that

$$Y_i = \boldsymbol{b} P_i + e_i \tag{3.1}$$

where, $E(e_i \mid P_i) = 0$; $E\left(e_i^2 \mid P_i\right) = aP_i^g$ with a>0, g>0 and $E\left(e_i e_j \mid P_i P_j\right) = 0 \quad \forall\, i \neq j$. Thus, we have the following theorems:

Theorem 3.1: The expected value of $B\left(\hat{Y}_P\right)_k$ in the model (3.1) is given by

$$E_m\left[B\left(\hat{Y}_P\right)_k\right] = \boldsymbol{b}\left(\sum_{i=1}^{N}\frac{P_i^2}{\boldsymbol{y}_{ik}} - 1\right), \qquad k = 0,1,2,3,4,5 \tag{3.2}$$

Theorem 3.2: The expected value of $MSE\left(\hat{Y}_P\right)_k$ in the model (3.1) is given by

$$\begin{aligned}
E_m\left[MSE\left(\hat{Y}_P\right)_k\right] &= \frac{1}{n}\left[\boldsymbol{b}^2\sum_{i=1}^{N}\frac{\boldsymbol{a}P_i^3}{\boldsymbol{y}_{ik}^{\,2}} + a\sum_{i=1}^{N}\boldsymbol{a}P_i^{g+1} + \boldsymbol{b}\sum_{i=1}^{N}\frac{\boldsymbol{t}\,P_i^2}{\boldsymbol{y}_{ik}^{\,2}} + \boldsymbol{q}\sum_{i=1}^{N}\frac{P_i}{\boldsymbol{y}_{ik}^{\,2}}\right] \\
&\quad + \frac{1}{n}\left[a\sum_{i=1}^{N}\frac{P_i^{g+1}\left(1-P_i\right)}{\boldsymbol{y}_{ik}^{\,2}} + \boldsymbol{b}^2\left(\sum_{i=1}^{N}\frac{P_i^3}{\boldsymbol{y}_{ik}^{\,2}} - \left(\sum_{i=1}^{N}\frac{P_i^2}{\boldsymbol{y}_{ik}}\right)^2\right)\right] \\
&\quad + \boldsymbol{b}^2\left(\sum_{i=1}^{N}\frac{P_i^2}{\boldsymbol{y}_{ik}} - 1\right) + a\left(\sum_{i=1}^{N}\frac{P_i^{g+2}}{\boldsymbol{y}_{ik}^{\,2}} + \sum_{i=1}^{N}P_i^g - 2\sum_{i=1}^{N}\frac{P_i^{g+1}}{\boldsymbol{y}_{ik}}\right)
\end{aligned}$$

where $\boldsymbol{b}^2 = \dfrac{a\,\boldsymbol{r}^2}{\boldsymbol{s}_P^2\left(1-\boldsymbol{r}^2\right)N\sum\limits_{i=1}^{N}P_i^g}$ and $\boldsymbol{s}_P^2 = \dfrac{1}{N}\left[\sum\limits_{i=1}^{N}P_i^2 - \dfrac{\left(\sum\limits_{i=1}^{N}P_i\right)^2}{N}\right]$ and E_m denote the expected value under the model (3.1).

To examine the relative efficiency of the estimator $\left(\hat{Y}_P\right)_2$ with respect to $\left(\hat{Y}_P\right)_k$ where $k = 0,1,3,4,5$ we consider a practicable randomisation device proposed by Chaudhuri and Adhikary (1990). According to this device the ith respondent in the sample is required to choose independently at random two tickets numbered a_j (>0) and b_t out of two boxes proposed by the investigator containing the numbered tickets (i) A_1, A_2, ... A_m with known mean $\bar{A}$ and known variance s_A^2 and (ii) B_1, B_2, ... , B_t with known mean $\bar{B}$ and variance s_B^2. The respondent is required to report the response as $Z_i = a_j Y_i + b_t$. Thus $E_2(Z_i) = \bar{A} Y_i + \bar{B}$. From the collected scrambled responses as Z_i, we can form a transformed variable as $r_i = \frac{Z_i - \bar{B}}{\bar{A}}$, so that r_i satisfies the assumptions defined for the estimator (1.2). Thus from (1.4), we have $\boldsymbol{a} = \boldsymbol{s}_A^2 \big/ \bar{A}^2$, $\boldsymbol{t} = 0$ and $\boldsymbol{q} = \boldsymbol{s}_B^2 \big/ \bar{A}^2$.

4. EMPIRICAL STUDY

We have analysed the six estimators in this empirical study. The density function $f(x)$ for the auxiliary character x are presented in Table 1 and generated x_i's are used to construct the selection probabilities P_i 's. We have assumed that the randomisation device consists of a single deck of cards bearing numbers with $C_A = \boldsymbol{s}_A \big/ \bar{A} = 0.15$. Such variation in the scrambling device has been recommended by Singh *et al.* (1996) and Starchan *et al.* (1998). For the simplicity of calculations, it is further assumed that the numbers written on the cards of deck B are the same. Also, let us define,

$$RE = E_m\left[MSE\left(\hat{Y}_P\right)_k \right] \times 100 \Big/ E_m\left[MSE\left(\hat{Y}_P\right)_2 \right] , \quad k = 0,1,3,4,5 \tag{4.1}$$

Table 2 exhibits the expected biases and Table 3 gives relative efficiency of $\left(\hat{Y}_{pps}\right)_2$ with respect to $\left(\hat{Y}_P\right)_k$, $k = 0,1,3,4,5$ for different values of correlation coefficient,

$\boldsymbol{r} =$ -0.9, -0.7, -0.5, -0.3, -0.1, +0.1, +0.3, +0.5, +0.7, +0.9 and g =0,1,2.

A replacement sample of size 25 was considered as drawn from a population consisting of 100 respondents. The analysis has been carried out in FORTRAN using subroutines available in Bratley *et al.* (1983). For the purpose, an empirical study we have considered $l = 0.5$.

From these tables one can conclude that the proposed estimator for y_{i2} fares better than the estimators for y_{ik} (k=0,1,3,4,5) considered in the present investigation.

Table 1: Distributions used for generating the selection probabilities.

Sr. No.	Distribution	Density function	Range	Skewed
1	Right Triangular	$f(x) = 2(1-x)$	$0 \le x \le 1$	Positively
2	Exponential	$f(x) = e^{-x}$	$0 \le x < \infty$	Positively
3	Chi-square at $n = 6$	$f(x) = \frac{1}{2^{n/2} G_{n/2}} e^{-x/2} x^{(n-2)/2}$	$0 \le x < \infty$	Positively
4	Gemma, p=2	$f(x) = \frac{1}{G_p} e^{-x} x^{p-1}$	$0 \le x < \infty$	Positively
5	Log Normal	$f(x) = \frac{1}{x\sqrt{2p}} e^{-\{log(x)\}^2/2}$	$0 < x < \infty$	Positively
6	Beta, p=3, q=2	$f(x) = \frac{1}{b(p,q)} x^{p-1}(1-x)^{q-1}$	$0 \le x \le 1$	Positively
7	Beta, p=.4, q=1	$f(x) = \frac{1}{b(p,q)} x^{p-1}(1-x)^{q-1}$	$0 \le x \le 1$	Positively
8	Beta, p=1, q=.4	$f(x) = \frac{1}{b(p,q)} x^{p-1}(1-x)^{q-1}$	$0 \le x \le 1$	Negatively
9	Beta, p=1.5, q=2.5	$f(x) = \frac{1}{b(p,q)} x^{p-1}(1-x)^{q-1}$	$0 \le x \le 1$	Positively
10	Beta, p=2.5, q=1.5	$f(x) = \frac{1}{b(p,q)} x^{p-1}(1-x)^{q-1}$	$0 \le x \le 1$	Negatively
11	Beta, p= q=2	$f(x) = \frac{1}{b(p,q)} x^{p-1}(1-x)^{q-1}$	$0 \le x \le 1$	Normal type (Hump type)
12	Beta, p=q=0.6	$f(x) = \frac{1}{b(p,q)} x^{p-1}(1-x)^{q-1}$	$0 \le x \le 1$	U-shaped (Cauldron shape)

Following Bansal and Singh (1989, 1990), Bedi (1995), Rao (1993a) and Kumar and Agarwal (1997), the extensions of the proposed strategies to Horvitz and Thompson (1952), Rao, Hartley and Cochran (1962) and Midzuno (1952) sampling strategies are in progress.

CONCLUSION

Although the transformation y_{i5} at (2.7) does not look very friendly due to absolute value of the correlation coefficient, it has been found to efficient and less biased for either low positive and low negative correlation of the sensitive variable with selection probabilities.

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Table 2: The values of $(B_i, i = 0,1,2,3,4,5)$ for different values of g and r for different kind of distributions.

ρ	g=0						g=1						g=2					
	B_0	B_1	B_2	B_3	B_4	B_5	B_0	B_1	B_2	B_3	B_4	B_5	B_0	B_1	B_2	B_3	B_4	B_5
Right Triangular Distribution																		
-.9	143.6	2.964	0.369	10.60	11.36	2.689	14.36	0.251	0.042	0.125	0.214	0.218	1.985	0.023	0.014	0.025	0.065	0.035
-.7	70.36	6.985	0.968	9.685	8.669	6.584	6.854	0.525	0.161	0.265	0.368	0.768	0.247	0.069	0.021	0.096	0.099	0.125
-.5	39.54	12.33	1.583	6.589	5.697	9.685	4.256	0.995	0.251	0.998	0.875	0.965	0.456	0.124	0.025	0.198	0.199	0.139
-.3	25.14	11.36	1.965	4.369	3.965	8.694	2.258	0.965	0.292	0.968	1.354	1.025	0.896	0.112	0.089	0.128	0.124	0.125
-.1	6.972	7.568	2.214	3.961	3.241	4.364	0.674	0.532	0.201	0.492	0.554	0.457	2.369	0.065	0.025	0.068	0.035	0.158
+.1	5.093	6.587	2.191	3.562	3.652	5.580	0.719	0.520	0.219	0.501	0.412	0.552	0.086	0.068	0.068	0.065	0.035	0.185
+.3	20.10	11.58	2.840	5.631	3.862	10.55	2.110	1.044	0.246	1.004	1.121	1.052	0.270	0.129	0.099	0.134	0.115	0.196
+.5	38.74	12.73	2.230	9.362	5.644	10.67	4.170	1.051	0.282	1.019	0.885	1.160	0.498	0.131	0.090	0.135	0.123	0.181
+.7	57.17	8.910	1.210	9.785	7.258	7.840	6.410	0.680	0.162	0.315	0.365	0.760	0.846	0.096	0.056	0.085	0.095	0.169
+.9	142.4	3.270	0.240	11.55	12.36	2.810	13.51	0.240	0.042	0.125	0.256	0.282	1.780	0.035	0.024	0.082	0.023	0.055
Exponential Distribution																		
-.9	80.26	0.765	0.023	0.369	0.368	0.685	8.254	0.069	0.006	0.125	0.069	0.078	1.258	0.014	0.000	0.099	0.002	0.009
-.7	42.63	3.120	0.354	2.687	2.421	2.988	3.987	0.321	0.042	0.325	0.587	0.358	0.589	0.048	0.001	0.125	0.019	0.041
-.5	24.25	5.214	0.785	3.256	2.365	5.012	2.247	0.547	0.088	0.535	0.376	0.687	0.358	0.069	0.009	0.147	0.021	0.068
-.3	12.69	5.698	0.924	2.895	1.965	4.987	0.987	0.689	0.125	0.695	0.255	0.688	0.278	0.075	0.019	0.129	0.025	0.045
-.1	4.698	3.258	0.986	2.168	1.365	3.258	0.425	0.342	0.109	0.456	0.254	0.348	0.064	0.047	0.015	0.124	0.035	0.034
+.1	4.490	3.440	1.061	2.370	1.375	3.136	0.422	0.350	0.107	0.442	0.255	0.345	0.068	0.048	0.012	0.125	0.032	0.039
+.3	14.17	5.190	1.235	2.744	1.979	5.896	1.480	0.620	0.124	0.694	0.256	0.599	0.266	0.076	0.014	0.126	0.039	0.078
+.5	25.23	5.250	0.869	3.258	2.369	5.621	2.590	0.520	0.088	0.525	0.379	0.533	0.396	0.069	0.008	0.112	0.033	0.058
+.7	43.86	3.210	0.457	2.861	2.441	3.158	4.450	0.310	0.044	0.315	0.599	0.365	0.496	0.045	0.002	0.124	0.032	0.042
+.9	92.90	0.788	0.067	0.259	0.369	0.591	9.580	0.074	0.008	0.123	0.079	0.089	1.130	0.018	0.001	0.104	0.031	0.007
Chi-Square Distribution, v=6																		
-.9	79.52	0.694	0.124	2.368	1.256	0.987	7.698	0.235	0.015	0.012	0.214	0.122	1.965	0.112	0.004	0.019	0.019	0.010
-.7	35.69	3.256	0.567	6.987	3.987	3.256	3.258	0.644	0.072	0.321	0.259	0.452	0.687	0.115	0.009	0.062	0.021	0.101
-.5	20.36	7.962	1.210	6.589	3.689	5.258	1.987	0.964	0.174	0.685	0.257	0.554	0.398	0.189	0.012	0.089	0.031	0.099
-.3	1.258	7.856	1.425	5.694	4.987	5.968	0.987	0.846	0.187	0.548	0.242	0.425	0.298	0.174	0.018	0.078	0.072	0.097
-.1	3.689	3.568	1.925	3.965	2.987	3.852	0.523	0.547	0.134	0.216	0.268	0.446	0.164	0.154	0.014	0.068	0.025	0.050
+.1	3.270	2.200	1.180	4.224	3.366	3.200	0.526	0.543	0.129	0.214	0.269	0.343	0.150	0.140	0.015	0.050	0.035	0.040
+.3	12.54	5.160	1.562	6.602	3.589	6.151	1.430	0.860	0.168	0.482	0.245	0.695	0.257	0.177	0.019	0.094	0.082	0.077
+.5	22.25	5.090	1.210	6.604	3.247	6.067	2.550	0.853	0.131	0.462	0.236	0.710	0.388	0.176	0.016	0.076	0.033	0.078
+.7	39.79	3.140	0.682	7.596	2.366	4.125	4.250	0.644	0.073	0.336	0.257	0.450	0.589	0.152	0.013	0.062	0.024	0.010
+.9	82.72	0.190	0.142	2.266	1.369	1.152	8.860	0.328	0.016	0.026	0.214	0.125	1.130	0.115	0.002	0.029	0.013	0.009

Gamma Distribution, p=2

-.9	132.5	1.098	0.325	2.145	2.145	2.147	14.25	0.392	0.020	0.235	0.024	0.258	1.650	0.025	0.005	0.021	0.028	0.024
-.7	65.25	1.862	1.012	3.987	3.257	5.987	8.456	0.729	0.114	0.745	0.736	0.567	0.687	0.087	0.014	0.097	0.089	0.074
-.5	35.26	6.158	2.308	6.987	3.987	8.254	5.478	1.125	0.182	0.947	0.825	0.798	0.453	0.108	0.024	0.118	0.123	0.104
-.3	18.69	8.265	2.358	3.658	2.698	8.988	3.987	1.113	0.245	1.128	0.887	0.894	0.253	0.115	0.029	0.114	0.125	0.113
-.1	6.952	4.987	1.985	4.987	3.568	5.369	2.589	0.722	0.198	0.798	0.489	0.498	0.082	0.063	0.025	0.045	0.064	0.082
+.1	6.510	5.170	1.756	5.221	3.451	5.162	2.668	0.712	0.178	0.701	0.459	0.510	0.081	0.062	0.021	0.042	0.051	0.072
+.3	19.69	9.310	2.321	3.726	2.569	9.184	4.090	1.131	0.234	1.121	0.892	0.924	0.254	0.113	0.027	0.112	0.112	0.112
+.5	37.98	8.030	1.998	7.469	4.458	8.954	5.830	1.103	0.192	0.956	0.835	0.897	0.466	0.109	0.023	0.116	0.110	0.108
+.7	63.49	6.100	1.112	4.656	3.891	6.257	8.510	0.825	0.113	0.767	0.735	0.619	0.792	0.076	0.012	0.075	0.065	0.075
+.9	132.3	1.675	0.369	2.569	2.569	1.985	15.70	0.399	0.030	0.292	0.049	0.198	1.660	0.024	0.007	0.022	0.029	0.025

Log Normal Distribution

-.9	174.2	2.365	0.789	2.367	1.253	2.368	16.58	0.478	0.054	0.568	0.214	0.354	2.584	0.009	0.169	0.089	0.968	0.014
-.7	84.25	7.425	1.021	6.584	5.987	6.987	7.987	0.835	0.109	1.189	0.564	0.874	1.987	0.019	0.146	0.098	0.112	0.035
-.5	52.36	8.564	1.789	7.985	10.25	8.245	4.987	1.198	0.198	1.089	0.657	0.999	1.368	0.029	0.147	0.145	0.135	0.134
-.3	26.58	9.874	2.356	8.987	9.856	9.864	2.987	1.358	0.285	1.754	0.987	1.081	1.245	0.033	0.098	0.108	0.147	0.147
-.1	8.789	6.287	1.878	5.689	4.368	6.987	0.865	0.868	0.210	0.853	0.356	0.683	1.236	0.029	0.078	0.088	0.058	0.099
+.1	8.990	6.380	1.968	5.569	4.451	6.260	0.872	0.841	0.194	0.852	0.345	0.685	1.122	0.027	0.089	0.087	0.065	0.089
+.3	27.54	10.25	2.435	9.782	10.41	9.850	2.770	1.270	0.242	1.164	0.874	1.071	1.384	0.034	0.147	0.152	0.144	0.156
+.5	50.53	9.710	1.896	8.444	11.41	8.569	5.100	1.190	0.183	1.052	0.645	0.987	1.706	0.029	0.135	0.142	0.145	0.145
+.7	85.87	6.444	1.052	5.345	6.451	7.254	8.500	0.857	0.104	1.163	0.472	0.740	2.198	0.017	0.089	0.097	0.105	0.079
+.9	177.8	1.092	0.235	1.052	0.851	1.368	18.13	0.393	0.031	0.458	0.141	0.248	3.520	0.004	0.028	0.035	0.044	0.023

Beta Distribution (p=3, q=2)

-.9	65.24	1.368	0.112	1.256	7.895	2.135	5.968	0.109	0.052	0.245	0.085	0.305	0.765	0.007	0.003	0.125	0.019	0.108
-.7	29.68	4.987	0.356	4.256	5.968	3.256	3.125	0.321	0.098	0.396	0.231	0.314	0.358	0.008	0.009	0.056	0.031	0.040
-.5	16.85	6.258	1.025	6.258	4.026	4.987	1.789	0.524	0.112	0.578	0.456	0.568	0.198	0.009	0.005	0.069	0.035	0.052
-.3	9.865	5.897	1.258	3.987	3.698	4.897	1.012	0.543	0.149	0.436	0.344	0.547	0.108	0.019	0.014	0.068	0.065	0.057
-.1	3.564	2.869	1.032	2.489	2.369	2.987	0.324	0.245	0.107	0.289	0.189	0.289	0.056	0.015	0.012	0.039	0.019	0.030
+.1	3.470	2.480	1.040	2.465	2.245	2.962	0.328	0.269	0.105	0.267	0.167	0.269	0.034	0.011	0.010	0.038	0.014	0.029
+.3	10.43	5.100	1.371	4.582	3.668	4.952	1.020	0.533	0.139	0.452	0.345	0.542	0.108	0.014	0.012	0.066	0.045	0.056
+.5	18.99	5.100	1.080	5.251	4.356	5.092	1.880	0.535	0.106	0.523	0.456	0.563	0.199	0.011	0.011	0.066	0.025	0.057
+.7	31.11	3.430	0.587	3.521	5.469	3.962	3.200	0.365	0.056	0.321	0.221	0.374	0.339	0.007	0.004	0.052	0.021	0.039
+.9	67.41	0.960	0.132	1.981	8.358	2.369	6.750	0.103	0.013	0.127	0.045	0.235	0.714	0.002	0.002	0.110	0.014	0.107

Beta Distribution (p=0.4, q=1)

-.9	90.26	1.265	0.988	1.258	0.235	2.364	9.687	0.095	0.029	0.098	1.987	0.124	1.698	0.280	0.123	0.987	0.301	0.139
-.7	40.25	2.987	0.987	4.258	3.987	4.897	2.987	0.247	0.098	0.236	2.987	0.456	0.298	0.289	0.145	0.268	0.298	0.158
-.5	23.69	5.012	1.025	6.589	5.012	5.998	2.987	0.345	0.105	0.514	1.269	0.789	0.189	0.269	0.132	0.259	0.258	0.169
-.3	12.36	5.968	1.369	5.689	4.987	5.845	1.458	0.987	0.118	0.598	0.999	0.680	0.199	0.287	0.119	0.149	0.198	0.169
-.1	5.098	3.456	1.254	3.546	2.456	3.456	0.564	0.564	0.099	0.326	0.099	0.338	0.258	0.259	0.115	0.127	0.139	0.240
+.1	4.480	3.240	1.031	3.401	2.369	3.331	0.432	0.340	0.097	0.325	0.087	0.334	0.249	0.248	0.112	0.125	0.135	0.239
+.3	12.37	5.010	1.210	5.882	4.258	5.841	1.400	0.570	0.105	0.569	0.965	0.569	0.196	0.276	0.114	0.147	0.145	0.166
+.5	22.46	2.250	1.021	5.324	4.258	5.212	2.540	0.550	0.099	0.457	1.258	0.523	0.186	0.269	0.129	0.258	0.215	0.159
+.7	41.25	1.120	0.954	3.212	3.694	4.261	4.300	0.340	0.068	0.259	2.369	0.308	0.286	0.245	0.125	0.258	0.263	0.145
+.9	90.24	0.680	0.890	0.512	0.124	1.235	9.220	0.060	0.008	0.035	1.258	0.054	1.220	0.218	0.121	0.852	0.258	0.129

Beta Distribution (p=1, q=0.4)

-.9	99.69	5.698	1.125	2.258	2.145	3.864	9.987	6.987	0.698	9.999	8.697	9.258	9.369	4.251	2.987	5.014	4.125	6.014
-.7	66.25	6.987	1.587	3.569	2.587	4.991	8.987	5.698	1.124	5.998	5.214	7.012	8.987	6.254	3.014	6.987	6.014	8.987
-.5	48.66	7.987	3.014	6.014	4.987	6.097	6.987	4.987	1.254	5.698	4.987	5.987	7.987	6.014	3.214	8.987	7.987	9.978
-.3	16.25	8.999	2.987	5.897	5.269	7.012	5.698	3.987	1.369	4.584	3.698	4.987	7.258	5.987	3.425	7.258	6.987	7.987
-.1	5.591	9.864	3.126	6.358	5.487	7.689	4.687	3.564	1.258	3.245	3.254	3.587	7.098	5.264	3.569	5.647	4.656	6.578
+.1	5.490	9.540	3.068	6.258	5.369	7.465	4.459	3.340	2.107	3.214	3.125	3.447	7.059	5.038	3.412	5.466	4.356	6.412
+.3	17.07	8.490	2.258	5.369	5.145	6.952	5.419	3.600	1.124	4.258	3.258	4.658	7.156	5.026	3.214	7.101	6.358	7.214
+.5	45.83	7.250	2.897	5.214	4.568	5.215	6.599	4.530	1.088	5.369	4.698	5.569	8.256	5.259	3.108	8.358	7.358	9.109
+.7	64.86	6.120	1.498	3.336	2.369	4.125	8.408	5.320	1.044	5.324	4.568	6.487	8.456	4.235	2.905	6.558	5.369	8.905
+.9	93.40	5.680	1.087	2.147	2.021	3.690	9.288	6.070	0.398	8.258	7.852	9.014	6.050	3.208	2.801	4.558	3.358	5.801

Beta Distribution (p=1.5, q=2.5)

-.9	86.25	25.47	2.014	22.58	14.36	18.74	20.36	17.98	12.45	15.26	21.58	15.25	25.36	28.46	0.999	13.25	3.684	16.58
-.7	42.58	22.36	3.265	20.25	18.96	14.58	18.69	16.58	12.99	14.65	19.68	18.69	25.69	27.98	1.011	16.25	8.697	21.58
-.5	32.58	15.69	5.687	19.68	16.58	15.98	16.89	15.01	12.69	11.69	18.47	17.98	27.26	27.45	1.106	17.99	7.998	20.36
-.3	14.58	12.58	6.258	15.28	15.24	15.69	15.98	14.01	10.99	10.12	15.69	17.68	28.04	26.89	2.235	17.58	7.258	17.98
-.1	3.457	8.547	9.014	12.58	12.35	14.56	14.98	13.54	9.998	9.240	14.25	16.69	28.06	26.35	2.514	14.36	6.258	16.98
+.1	3.190	8.470	8.060	12.25	11.25	12.47	14.59	13.30	9.107	9.250	13.44	15.69	27.04	25.38	2.414	14.21	5.525	16.25
+.3	11.05	10.20	5.130	14.58	12.25	13.58	15.49	13.60	9.124	10.10	14.87	16.58	27.56	25.26	2.214	16.24	6.258	17.95
+.5	29.83	12.35	4.680	16.25	14.25	15.24	15.99	14.30	10.69	11.20	15.30	17.36	28.86	25.59	1.109	17.24	7.147	19.65
+.7	38.56	14.22	2.340	18.25	17.25	17.80	18.88	15.20	11.02	14.20	17.15	16.36	24.86	24.35	1.905	16.34	8.258	18.09
+.9	80.40	20.70	1.190	20.25	13.25	20.14	17.88	16.70	12.58	15.20	20.36	15.98	36.20	23.28	0.901	12.25	2.360	15.08

Beta Distribution (p=2.5, q=1.5)

-.9	59.87	69.25	11.58	33.25	21.00	76.36	43.66	39.25	9.895	33.25	22.25	41.00	79.69	36.58	9.999	15.21	11.68	26.35
-.7	55.87	59.86	12.40	36.25	19.25	20.36	42.25	38.25	10.68	31.25	21.25	40.25	60.25	39.58	10.25	14.99	11.25	28.69
-.5	52.69	55.69	13.01	33.26	16.85	59.69	39.25	35.58	10.98	28.14	19.68	39.28	52.14	39.25	9.985	15.21	10.25	28.67
-.3	46.58	51.24	12.98	29.36	15.98	58.96	36.58	33.69	12.36	27.01	17.28	35.69	50.26	34.78	10.69	14.89	11.25	29.68
-.1	44.58	50.26	14.25	28.35	15.65	56.25	35.69	33.41	13.25	25.98	15.69	34.58	48.99	36.58	10.25	14.56	10.68	27.56
+.1	43.49	49.40	13.60	26.31	14.22	55.60	34.59	33.40	12.17	25.36	14.26	33.44	48.49	35.28	9.412	12.36	9.441	26.38
+.3	45.07	51.39	12.30	25.31	16.21	56.30	35.19	33.60	11.14	26.35	16.25	34.99	49.56	35.16	8.214	14.25	10.63	28.66
+.5	50.83	55.50	12.80	29.14	18.25	57.13	36.99	34.50	10.88	28.65	18.26	35.33	59.86	33.29	7.192	16.25	11.69	29.58
+.7	55.86	59.20	11.40	31.22	19.21	64.70	38.08	35.20	10.44	30.25	19.36	36.18	69.86	30.35	6.921	18.34	15.36	28.56
+.9	58.40	60.70	10.80	33.21	20.36	72.63	39.88	36.70	7.398	32.21	19.75	39.74	77.20	23.18	4.587	19.24	16.36	25.08

Beta Distribution (p=q=2.0)

-.9	32.65	14.25	10.26	13.98	15.69	14.02	21.58	16.74	19.74	19.01	29.36	20.14	17.01	14.01	1.258	25.36	19.25	16.25
-.7	29.68	16.25	10.25	14.98	14.98	15.24	18.99	15.36	16.78	13.98	26.35	16.98	19.01	14.89	2.587	23.36	17.98	18.69
-.5	27.99	17.25	11.98	15.36	13.98	16.01	17.01	14.89	15.53	12.35	19.68	15.36	18.98	15.89	4.588	21.25	17.99	19.65
-.3	27.85	18.25	13.01	16.25	12.69	17.01	16.87	13.98	14.52	11.98	16.98	14.85	18.01	15.68	6.254	20.36	14.25	17.25
-.1	17.25	19.64	13.99	18.25	12.59	17.98	15.98	13.45	13.44	11.25	14.35	13.98	17.56	15.69	6.524	14.36	12.36	16.98
+.1	16.90	19.40	13.95	17.25	12.58	17.69	15.45	13.34	12.10	10.28	14.25	13.44	17.49	15.38	5.512	14.25	10.25	16.38
+.3	26.07	17.90	12.26	15.25	12.25	16.96	16.19	13.60	11.24	11.85	16.25	14.52	17.15	15.26	4.251	19.56	12.69	17.06
+.5	26.83	16.35	12.25	15.36	13.25	15.12	16.99	14.53	11.08	12.74	18.34	15.53	18.86	15.59	3.258	20.28	14.85	19.02
+.7	28.86	14.20	11.24	14.47	14.36	14.25	18.08	15.32	11.44	14.96	25.96	16.38	18.86	14.35	1.925	22.14	17.25	18.59
+.9	31.40	12.80	11.02	13.25	15.96	13.69	20.88	16.70	10.38	18.69	28.45	19.74	16.20	13.08	1.698	23.36	19.85	15.25

Beta Distribution (p=q=0.6)

-.9	3.25	1.99	1.15	1.53	1.76	1.54	1.46	1.32	1.39	2.99	5.21	1.19	1.25	1.998	0.801	1.46	2.45	1.98
-.7	3.01	2.00	1.17	1.52	1.49	1.56	1.56	1.65	1.45	2.35	5.01	1.40	1.90	1.985	0.954	1.98	2.36	1.69
-.5	2.98	2.01	1.16	1.44	1.49	1.69	1.69	1.60	1.09	2.23	4.98	1.55	1.89	1.201	1.198	1.19	1.35	2.01
-.3	2.56	1.99	1.08	1.46	1.36	1.37	1.49	1.35	1.25	1.99	3.98	1.53	1.19	1.249	0.219	1.18	0.97	1.09
-.1	2.01	1.49	1.05	1.42	1.34	1.36	1.46	1.36	1.11	1.96	2.38	1.45	1.50	1.422	0.415	1.26	0.96	1.39
+.1	1.90	1.41	1.03	1.42	1.33	1.26	1.45	1.34	1.10	1.86	2.36	1.44	1.49	1.421	0.412	1.25	0.95	1.38
+.3	2.07	1.92	1.09	1.92	1.35	1.30	1.19	1.60	1.24	1.97	3.69	1.52	1.15	1.241	0.214	1.08	0.95	1.06
+.5	2.84	1.33	1.01	1.11	1.47	1.60	1.99	1.53	1.08	2.12	4.58	1.53	1.86	1.190	1.119	1.06	1.25	1.08
+.7	2.88	1.22	1.01	1.10	1.46	1.40	1.08	1.32	1.44	2.46	4.58	1.38	1.86	1.905	0.905	1.85	2.36	1.59
+.9	2.42	1.88	1.01	1.68	1.66	1.08	1.02	1.20	1.38	2.49	4.69	1.20	1.20	1.801	0.805	1.36	2.36	1.89

Table 3: The values of $\left(RE_{0i}, i = 0,1,2,3,4,5\right)$ for different values of g and r for different kind of distributions.

Table 3: The values of $\left(RE_{0i}, i = 0,1,2,3,4,5\right)$ for different values of g and r for different kind of distributions.

	g=0					g=1					g=2				
ρ	RE$_{01}$	RE$_{02}$	RE$_{03}$	RE$_{04}$	RE$_{05}$	RE$_{01}$	RE$_{02}$	RE$_{03}$	RE$_{04}$	RE$_{05}$	RE$_{01}$	RE$_{02}$	RE$_{03}$	RE$_{04}$	RE$_{05}$
Right Triangular Distribution															
-.9	2105.6	99.99	99.54	100.69	98.25	92.36	98.36	100.36	101.25	98.26	130.22	111.25	100.14	99.63	100.25
-.7	969.58	102.36	96.25	101.25	100.25	99.68	105.36	106.25	102.36	100.00	142.63	129.25	116.25	100.25	119.25
-.5	469.36	107.91	109.56	106.25	109.25	102.36	115.25	114.25	148.68	114.25	150.14	168.25	118.36	141.35	126.35
-.3	255.98	122.52	120.36	115.69	114.69	116.58	158.36	144.36	150.36	125.78	153.24	198.26	119.36	145.24	184.25
-.1	138.65	123.50	127.54	124.36	123.25	125.36	165.25	145.36	151.25	122.36	156.25	187.36	117.25	152.36	174.56
+.1	136.88	121.49	124.36	123.25	122.49	121.49	166.36	15034	151.47	121.34	158.25	184.58	116.44	154.37	164.18
+.3	223.82	119.40	111.35	114.78	121.40	119.40	154.35	148.87	151.12	120.78	152.36	194.36	121.52	154.36	153.92
+.5	421.24	105.55	103.25	104.06	108.55	105.55	147.25	136.36	137.71	117.29	151.36	154.68	120.35	148.52	147.99
+.7	822.51	88.66	83.22	99.25	100.2	86.82	101.25	117.25	116.56	106.25	140.25	126.25	116.20	122.27	121.68
+.9	2118.4	78.94	84.34	89.36	98.83	78.83	100.36	101.45	100.54	100.45	133.45	110.25	99.30	92.75	100.66
Exponential Distribution															
-.9	1300.0	100.0	85.25	100.01	98.25	1399.2	111.69	100.96	101.25	99.98	1425.6	105.26	96.25	99.26	100.36
-.7	507.25	103.25	100.36	105.00	100.62	581.24	114.25	110.14	110.26	104.25	596.36	110.25	102.36	100.25	110.25
-.5	266.36	105.25	102.36	106.58	102.56	305.36	127.25	118.45	118.46	111.36	389.25	124.36	124.25	110.25	115.69
-.3	156.25	106.25	105.26	110.56	106.10	184.63	136.25	126.58	120.14	119.25	235.69	138.25	128.36	117.25	119.25
-.1	112.25	108.62	109.68	112.36	106.25	138.25	141.36	138.25	123.25	120.36	164.25	142.36	141.25	118.25	123.25
+.1	113.27	107.22	109.69	113.36	106.18	136.21	142.08	140.25	124.36	122.02	165.70	141.07	139.25	120.35	125.96
+.3	155.56	107.22	107.58	111.65	106.10	188.31	139.07	138.63	121.85	118.91	233.49	135.57	133.36	132.89	123.35
+.5	262.98	104.48	103.36	106.36	103.33	308.12	132.55	125.74	119.45	112.36	374.78	123.00	122.35	122.48	120.77
+.7	508.45	101.37	100..69	102.25	100.26	582.53	125.50	120.69	111.36	105.37	686.78	111.74	105.84	110.69	110.59
+.9	1269.2	100.72	98.25	101.27	99.69	1422.4	120.94	103.65	103.25	100.92	1579.4	102.30	99.36	100.36	100.27
Chi-Square Distribution, v=6															
-.9	1132.2	100.10	106.25	109.58	100.21	1032.6	105.26	99.56	99.36	100.36	1269.6	101.36	103.24	120.36	102.58
-.7	525.36	102.36	110.25	114.25	100.69	465.25	110.36	101.25	102.95	105.24	589.36	110.25	112.36	119.25	110.69
-.5	256.25	105.36	112.35	120.25	105.68	295.36	115.24	105.24	108.65	112.58	359.58	120.25	118.69	126.35	130.25
-.3	159.25	107.25	112.36	124.58	114.69	253.36	118.25	108.56	111.36	118.69	250.96	132.41	125.14	135.25	133.26
-.1	114.25	109.65	115.25	129.36	115.28	133.25	123.25	110.36	116.25	124.26	150.36	136.25	126.25	146.25	133.25
+.1	115.89	109.61	116.36	128.35	118.58	133.11	122.23	110.25	115.25	125.20	149.57	134.42	128.96	146.25	134.37
+.3	153.29	108.56	113.69	125.36	118.46	182.34	123.34	105.36	113.22	124.22	209.97	134.13	127.68	136.78	133.98
+.5	245.60	105.08	111.58	119.69	113.93	294.03	117.70	103.64	111.96	118.54	339.96	124.84	126.63	125.14	129.65
+.7	452.33	100.32	103.36	115.37	100.91	545.25	108.51	101.65	105.23	112.38	629.92	114.94	116.36	116.74	113.30

+.9	1121.6	99.99	99.99	110.69	99.36	1311.7	101.55	95.69	101.25	102.52	1493.5	102.64	111.58	111.25	102.61

Gamma Distribution (p=2)

-.9	2136.2	102.25	100.25	99.25	101.36	105.66	121.25	120.10	140.36	99.98	1425.6	105.26	96.25	99.26	100.36
-.7	903.68	108.69	102.36	101.25	102.25	110.56	139.69	125.23	150.36	104.25	596.36	110.25	102.36	100.25	110.25
-.5	385.69	110.25	107.58	105.25	105.68	114.68	148.25	140.36	161.25	111.36	389.25	124.36	124.25	110.25	115.69
-.3	256.36	115.69	110.25	112.24	112.36	157.24	152.36	145.24	169.25	119.25	235.69	138.25	128.36	117.25	119.25
-.1	135.69	120.36	113.26	118.36	115.25	166.25	158.69	153.26	171.35	120.36	164.25	142.36	141.25	118.25	123.25
+.1	133.33	118.53	113.23	119.85	116.44	178.53	157.14	152.69	172.25	146.97	228.16	178.18	160.25	170.65	178.00
+.3	213.07	115.50	111.25	113.25	113.24	288.83	151.76	149.25	168.58	141.39	377.39	168.93	152.36	164.25	168.39
+.5	393.54	107.38	108.85	109.63	102.08	538.64	147.86	143.36	163.54	127.45	701.63	149.35	148.52	161.25	178.79
+.7	803.41	105.64	106.85	102.56	89.42	1189.9	140.78	132.25	152.24	110.50	1481.4	125.29	142.69	155.55	124.90
+.9	2231.8	102.45	105.45	98.58	87.70	2389.9	120.24	121.58	141.25	100.16	4277.9	105.80	138.25	150.38	105.72

Log Normal Distribution

-.9	2104.6	99.29	99.24	100.69	98.25	92.36	98.36	100.36	101.25	98.26	130.22	111.25	100.14	99.63	100.25
-.7	964.58	102.36	96.25	101.25	100.25	99.68	105.36	106.25	102.36	100.00	142.63	129.25	116.25	100.25	119.25
-.5	464.36	102.91	104.56	106.25	109.25	102.36	115.25	114.25	148.68	114.25	150.14	168.25	118.36	141.35	126.35
-.3	254.98	122.52	124.36	112.69	114.69	116.58	158.36	144.36	150.36	125.78	153.24	198.26	119.36	145.24	184.25
-.1	134.65	122.50	127.54	124.36	123.25	125.36	165.25	145.36	151.25	122.36	156.25	187.36	117.25	152.36	174.56
+.1	154.36	124.66	121.36	131.25	122.81	262.28	186.92	190.36	170.95	179.66	393.15	255.32	263.58	201.36	237.84
+.3	293.61	114.22	120.69	128.65	112.72	477.26	162.65	186.48	165.25	156.83	735.79	216.95	253.45	195.25	201.20
+.5	602.81	103.62	116.25	124.26	102.47	956.41	134.87	182.34	160.35	130.40	1472.7	175.69	247.12	182.36	162.65
+.7	1262.5	92.65	108.96	116.58	91.91	2102.4	109.20	181.36	155.47	106.03	3300.2	135.58	236.52	175.36	125.35
+.9	3936.7	95.94	104.85	112.36	95.59	6807.4	97.51	170.37	146.25	96.63	7022.2	107.14	223.58	165.25	100.59

Beta Distribution (p=3, q=2)

-.9	1236.2	99.99	103.69	99.99	99.96	625.25	102.36	99.96	101.56	98.26	693.25	99.96	100.96	101.56	101.36
-.7	758.36	102.58	109.69	100.21	102.36	336.25	120.25	100.36	103.36	99.99	349.36	102.36	103.25	105.24	105.24
-.5	536.25	105.69	119.25	100.63	104.85	223.69	125.36	106.25	110.25	101.25	218.25	105.25	106.25	112.36	108.69
-.3	256.36	113.25	120.36	105.21	105.26	153.25	115.66	114.25	115.25	112.36	154.69	112.36	119.36	119.25	111.36
-.1	110.25	115.26	121.25	106.25	106.25	118.25	113.36	116.25	118.36	119.36	125.36	116.25	123.25	125.35	116.25
+.1	117.88	116.73	116.52	117.32	104.72	116.63	112.58	117.25	119.14	118.56	123.52	117.84	122.36	129.36	117.83
+.3	139.35	115.95	115.64	116.52	105.93	143.55	112.89	113.85	129.36	116.84	154.78	118.69	118.36	124.34	116.65
+.5	132.21	114.73	113.25	112.85	104.67	203.02	112.07	107.36	119.36	114.00	219.06	116.08	116.96	122.36	114.01
+.7	232.92	111.66	110.36	100.00	101.61	324.54	101.02	101.56	109.25	102.97	347.42	111.30	107.69	119.52	110.25
+.9	538.32	102.00	102.36	99.59	99.69	607.44	100.62	99.65	99.36	101.61	632.21	106.42	103.69	106.36	101.42

Beta Distribution (p=0.4, q=1)

-.9	2204.6	99.29	99.24	100.69	98.25	92.36	98.36	100.36	100.25	91.26	131.22	111.15	101.14	99.63	100.25
-.7	924.58	103.36	96.25	101.25	100.25	99.68	105.36	106.25	101.36	101.00	142.63	129.25	116.25	101.25	115.25
-.5	424.36	103.91	103.56	105.25	105.25	105.35	115.25	114.57	145.68	114.25	150.14	168.25	118.36	141.35	125.35
-.3	224.98	123.52	123.36	115.69	114.69	116.58	158.36	144.37	155.36	125.78	153.24	198.26	119.36	145.24	185.25
-.1	124.65	123.50	123.54	125.36	125.25	125.36	165.25	145.37	155.25	122.36	156.22	187.36	117.25	152.36	174.56
+.1	121.98	123.58	123.49	120.36	125.96	180.15	155.74	150.28	155.37	190.37	216.85	187.56	190.36	170.63	120.36
+.3	221.35	120.56	118.80	116.36	124.36	306.59	151.21	145.67	155.36	186.57	210.69	176.35	185.36	160.58	118.36
+.5	421.39	106.96	104.45	112.96	119.52	656.36	132.25	142.37	145.96	182.37	205.69	168.52	184.54	152.25	116.24
+.7	931.25	88.88	89.56	108.63	112.36	15.69.2	119.25	138.27	140.27	185.24	199.57	156.22	182.24	141.36	111.58
+.9	2231.5	79.69	78.98	102.30	106.56	4765.2	102.69	132.37	138.27	180.36	187.56	142.32	172.36	141.25	105.36

Beta Distribution (p=1, q=0.4)

-.9	536.25	102.25	109.25	106.36	119.67	142.36	120.36	110.25	123.25	120.36	136.25	121.36	150.65	153.25	102.36
-.7	236.25	103.36	112.54	110.25	120.36	153.24	130.25	112.36	125.36	165.25	195.24	141.25	152.36	160.25	106.25
-.5	136.25	107.58	118.63	114.25	124.25	162.35	141.01	120.14	136.25	170.25	201.36	156.25	168.25	165.24	110.36
-.3	116.25	118.25	124.36	118.25	125.36	176.25	150.25	140.25	140.36	175.36	204.25	169.25	187.25	172.36	115.24
-.1	107.25	125.65	125.36	120.36	120.65	182.34	165.24	142.25	150.36	169.25	210.36	186.25	196.25	185.24	120.36
+.1	106.63	126.36	126.36	126.25	121.36	192.45	166.35	146.26	156.36	182.54	216.36	186.36	195.68	186.36	126.45
+.3	116.36	116.45	126.96	122.04	120.96	185.25	156.24	145.25	146.25	176.26	206.25	176.36	186.36	176.34	122.33
+.5	126.24	106.25	122.47	116.65	116.36	176.36	146.36	136.25	145.36	172.63	196.23	166.25	176.36	172.36	116.45
+.7	166.74	102.69	116.74	112.85	112.96	166.24	136.68	122.47	142.25	168.25	192.63	162.45	172.36	166.85	112.96
+.9	366.96	102.25	102.63	102.69	106.36	156.26	126.36	110.39	136.25	166.25	186.36	156.45	166.38	162.25	106.25

Beta Distribution (p=1.5, q=2.5)

-.9	832.28	102.10	105.25	108.58	101.21	1022.6	102.26	100.60	100.67	100.38	1069.7	105.36	104.24	110.36	105.58
-.7	635.36	103.36	111.25	114.25	102.69	462.25	106.36	104.25	102.95	105.24	589.36	110.25	113.36	112.25	115.69
-.5	236.25	103.36	111.35	120.25	105.68	295.36	115.24	105.24	108.65	112.58	359.58	120.25	117.69	116.35	124.25
-.3	159.25	108.25	113.36	124.58	114.69	253.36	118.25	108.56	111.36	118.69	250.96	132.41	125.14	125.25	130.26
-.1	114.25	109.65	115.25	129.36	115.28	133.25	123.25	110.36	116.25	124.26	150.36	136.25	126.25	126.25	131.25
+.1	110.25	112.36	130.24	120.36	115.36	116.52	136.25	140.96	156.25	180.41	190.25	175.24	200.36	160.24	130.45
+.3	112.36	111.25	127.36	115.85	111.04	110.36	125.36	130.96	142.36	171.45	183.25	170.35	190.25	154.85	125.45
+.5	114.56	109.52	126.14	105.36	108.46	105.46	114.36	125.34	141.85	160.24	176.25	165.85	180.34	146.27	120.36
+.7	118.69	108.65	120.36	102.34	103.25	102.63	105.96	120.45	135.41	110.25	165.47	160.35	170.36	138.45	115.63
+.9	120.36	104.36	118.36	101.36	102.36	99.99	102.96	115.96	120.69	106.94	155.36	150.25	120.36	130.69	112.35

Beta Distribution (p=2.5, q=1.5)

-.9	546.24	112.25	110.27	107.37	107.66	102.36	110.36	120.25	110.25	110.36	126.25	111.31	148.65	163.25	112.36
-.7	226.22	113.36	112.54	110.25	118.36	103.24	110.25	122.36	110.36	135.25	185.24	131.23	162.36	170.25	116.25
-.5	136.24	117.58	118.63	114.25	123.25	102.35	111.01	120.14	110.25	160.25	191.36	146.23	188.25	185.24	120.36
-.3	136.22	118.25	124.36	118.25	124.36	106.25	110.25	120.25	112.36	165.36	194.25	159.27	197.25	162.36	125.24
-.1	147.25	125.65	125.36	120.36	120.65	102.34	115.24	122.25	110.36	169.25	200.36	166.21	186.25	175.24	120.36
+.1	120.36	110.25	105.36	120.36	125.36	100.36	110.85	120.36	112.56	150.52	190.36	150.32	185.85	180.85	121.24
+.3	128.69	102.35	104.25	115.36	120.42	109.52	115.69	125.96	113.36	142.36	165.25	140.43	198.25	159.25	122.14
+.5	138.52	101.36	101.36	111.52	115.63	104.25	110.25	125.41	110.25	130.25	150.35	140.25	183.25	176.58	120.25
+.7	169.25	99.98	99.36	106.52	110.25	103.25	115.69	120.25	111.41	120.41	145.25	130.36	175.62	182.34	115.35
+.9	169.25	96.25	95.42	101.36	105.85	101.96	110.58	125.85	110.25	110.17	130.32	130.67	160.52	157.36	118.36

Beta Distribution (p=q=2.0)

-.9	136.25	110.20	101.25	101.36	102.5	100.36	112.25	106.25	120.14	110.36	128.25	105.25	201.03	205.36	208.26
-.7	136.25	123.25	102.36	102.25	104.25	102.14	125.25	118.69	130.22	116.25	136.25	109.65	214.25	210.03	210.36
-.5	110.51	136.25	102.25	103.25	105.36	105.25	136.25	125.35	136.25	120.14	141.36	112.25	216.25	230.14	214.25
-.3	105.69	125.60	106.25	105.36	112.36	105.25	149.58	136.25	154.24	124.25	145.25	125.63	210.36	235.24	225.66
-.1	103.68	107.36	105.25	107.85	110.25	109.66	144.65	142.36	152.36	136.25	152.36	145.25	214.25	246.25	230.14
+.1	105.25	106.21	104.25	105.36	109.25	111.36	145.36	145.25	154.63	158.36	160.25	165.35	210.36	236.25	240.36
+.3	106.58	103.21	102.85	104.25	107.35	108.52	135.62	140.42	148.36	150.36	150.52	148.14	205.63	216.25	230.69
+.5	110.58	101.85	101.96	102.36	107.58	109.21	130.25	136.25	140.25	138.41	142.63	135.24	203.21	200.14	225.74
+.7	120.36	96.25	101.36	101.25	106.85	108.25	169.85	130.24	125.47	125.41	132.52	130.25	200.31	192.34	210.36
+.9	128.14	85.25	99.99	99.99	105.36	112.60	140.36	128.52	120.85	110.36	128.36	120.69	198.25	185.25	205.10

Beta Distribution (p=q=0.6)

-.9	100.25	100.36	100.34	102.44	100.63	109.25	112.56	110.25	103.25	130.25	114.25	110.25	114.66	110.36	125.25
-.7	102.36	102.55	103.25	103.25	105.24	112.69	120.14	115.45	108.25	143.25	120.35	123.25	123.25	115.24	135.25
-.5	106.25	105.67	105.24	105.24	109.25	118.25	125.46	120.36	110.25	152.34	131.25	148.25	142.36	125.36	145.25
-.3	108.69	107.69	109.36	106.25	118.36	121.36	130.25	125.45	116.25	162.34	141.36	145.61	182.36	146.25	185.36
-.1	105.36	109.25	111.36	108.36	121.25	124.66	135.25	130.25	120.25	165.34	142.36	142.36	185.36	178.25	201.36
+.1	103.25	105.36	108.66	106.35	110.35	125.36	140.36	138.05	150.36	165.25	150.36	160.54	196.58	180.36	210.36
+.3	103.52	105.25	107.84	105.34	105.32	120.52	132.52	125.36	140.35	160.35	148.25	140.36	180.36	175.36	201.52
+.5	105.36	105.20	106.35	102.34	103.63	118.25	125.45	110.36	135.25	152.36	135.24	120.35	175.36	171.25	195.42
+.7	108.52	104.26	103.25	99.36	102.10	115.36	120.63	105.36	132.25	142.36	125.63	110.63	174.36	168.25	191.36
+.9	106.92	102.36	101.24	92.35	101.36	105.52	110.47	102.36	125.63	132.69	120.45	105.24	170.36	164.25	182.45