Applying Bayesian Inference for Signal Extraction Model of Time Series

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Abstract

This paper explores a way to apply Bayesian Inference to Signal Extraction model of time series. The basic idea is to treat hyperparameters in stochastic time series models of the components as a random vector and compute a posterior density function using Bayesian inference. Then Importance Sampling as a variance reduction technique is applied to integrate out the hyperparameters. A real time series data are used to explain how to apply this concept.

Key Words: Hyperparameters, Bayesian Inference, Importance Sampling, Signal Extraction, Time Series Analysis.

1.INTRODUCTION

Many observable economic time series Z_t can be written using a signal extraction specification:

(1) $$Z_t = S_t + N_t \qquad t = 1, 2, ..., T,$$

where S_t is an unobserved signal component and N_t an unobserved noise component. The signal and noise components are assumed to be ARIMA processes, respectively

(2) $$\phi_s(B)\delta_s(B)S_t = \theta_s(B)a_t ,$$

and

(3) $$\phi_n(B)\delta_n(B)N_t = \theta_n(B)b_t .$$

In (2), $\phi_s(B)$, $\delta_s(B)$, and $\theta_s(B)$ are polynomials of degree ps, ds and qs, respectively, in the backshift operator B (defined by $BS_t = S_{t-1}$), namely

$$\phi_s(B) = 1 - \phi_{1s}B - \cdots - \phi_{ps}B^{ps} ,$$

$$\delta_s(B) = 1 - \delta_{1s}B - \cdots - \delta_{ds}B^{ds} ,$$

and

$$\theta_s(B) = 1 - \theta_{1s}B - \cdots - \theta_{qs}B^{qs} .$$

In (3), $\phi_n(B)$, $\delta_n(B)$, and $\theta_n(B)$ are similar polynomials of degree pn, dn and qn, respectively, in the backshift operator B. It is assumed that a_t and b_t are uncorrelated white noise processes with variances σ_a^2 and σ_b^2 respectively, and that $\delta_s(B)$ and $\delta_n(B)$ have no common zeros. In some instances it may not be appropriate to represent the time series Z_t as in (1), but the model may well be appropriate for some nonlinear transformations (for example, logarithm transformation) of the original time series.

Signal extraction model (1) is used in many literatures to analyze economic and business time series. Scott and Smith (1974) suggest the model (1) to analyze time series created by repeated sample surveys. More details of applying to repeated sample surveys can be found in Wolter (1985), Bell and Hillmer (1990; 1991). Box et al. (1978) use the model (1) for seasonal adjustment of time series. More details can be found in Burman (1980), Hillmer and Tiao (1982) and Bell and Hillmer (1984). Harvey and Todd (1983) use the model (1) for structural time series approach. More details can be found in Harvey (1989; 1993) and Boone and Hall (1999). More comprehensive literature review for signal extraction can be found in Dagum et al. (1998) and Pena et al. (2001).

In theory, given the observed time series $\mathbf{Z} = (Z_1, Z_2, ..., Z_T)'$, a minimum mean square estimator (MMSE) of S_t and its mean square error (MSE) at some point in the sample can be computed: Conditional on the full set of observations $\mathbf{Z}$, the MMSE of the unobserved component S_t, $\tilde{S}_t$, is

$$\tilde{S}_t = E(S_t|\mathbf{Z},\boldsymbol{\Psi}) \tag{4}$$

with mean square error

(5) $$\mathrm{MSE}(\tilde{S}_t) = \mathrm{Var}(S_t|\mathbf{Z},\boldsymbol{\Psi})$$

(Harvey; 1993) where $\boldsymbol{\Psi}$ is a vector of hyperparameters in $\phi_s(B)$, $\theta_s(B)$, $\phi_n(B)$, and $\theta_n(B)$, coupled with σ_a^2 and σ_b^2, namely

(6) $$\boldsymbol{\Psi} = (\phi_{1s}, ..., \phi_{ps}, \theta_{1s}, ..., \theta_{qs}, \phi_{1n}, ..., \phi_{pn}, \theta_{1n}, ..., \theta_{qn}, \sigma_a^2, \sigma_b^2).$$

Computation of (4) and (5) is based on two assumptions: S_t and N_t are known stochastic processes, and S_t and N_t are uncorrelated. With these assumptions, (4) and (5) can be computed using two equivalent approaches. One is the *classical approach* (Whittle, 1963; 1983) based on the Kolmogorov and Wiener theory (Wiener, 1949). This renders a linear filter (or a weight function) which uses a set of observable data **Z** as an input to estimate S_t and mean square error. The second approach, the *state space approach* (Pagan, 1975), is based upon converting the signal extraction model (1) to state space form and using the Kalman filter/smoother to estimate S_t and mean square error.

In practice, however, the models for S_t and N_t are often unknown. In this case the forms of the models for S_t and N_t must first be identified and then the hyperparameters of the models, $\boldsymbol{\Psi}$, must be estimated. Once the models are identified, a commonly used approach to estimate the hyperparameters **Ψ** is to get the maximum likelihood estimates (MLEs) by maximizing the likelihood function. Therefore, in practice, given the models and MLEs of hyperparameters, estimates of signal S_t and its MSE can be computed.

In this paper an alternative way to handle the hyperparameters **Ψ** in signal extraction model of time series is studied. The hyperparameters **Ψ** is treated as a random vector, and a posterior distribution function for the hyperparameters **Ψ** is obtained using Bayesian inference. Then estimator of S_t and its mean square error (MSE) at some point in the sample are computed using the posterior distribution function and Importance Sampling.

To show this idea, a real time series data, U.S. Teen Unemployment from January 1972 through December 1983, published by U.S. Census Bureau are used.

The organization of the paper is as follows. Section 2 explains Bayesian Inference and Importance Sampling method in the context of signal extraction. Section 3 shows the application and results.

2. BAYESIAN INFERENCE AND IMPORTANCE SAMPLING

2.1. Bayesian Inference

From the Bayesian inference, a vector of unknown hyperparameters **Ψ** is a random vector with a known prior probability density function (p.d.f.) P(**Ψ**). Given observable data **Z**, a posterior p.d.f. of the vector **Ψ**, P(**Ψ**|**Z**), is obtained up to a constant using Baye's theorem, namely

$$P(\mathbf{\Psi}|\mathbf{Z}) \;\propto\; P(\mathbf{\Psi})L((\mathbf{\Psi}|\mathbf{Z})$$

where L(**Ψ**|**Z**) is a likelihood function and $\propto$ means 'proportional to'. A prior p.d.f. is specified according to prior knowledge about the hyperparameters **Ψ**. To express relative ignorance about values of the hyperparameters, a diffuse prior p.d.f. will be specified in this paper. This choice of the prior is appropriate since this paper will deal with situations where the sample size, T, is large, so that the likelihood function dominates the prior p.d.f.. The likelihood function L(**Ψ**|**Z**) is computed by using the Kalman filter, so the posterior p.d.f. can be readily obtained.

Using the posterior p.d.f. P(**Ψ**|**Z**), the Bayesian inference leads to a way of estimating $E(S_t|\mathbf{Z})$ and $Var(S_t|\mathbf{Z})$ unconditional on the vector **Ψ**, from the following relations (Bell & Otto, 1993):

(7) $$E(S_t|\mathbf{Z}) = E_{P(\Psi|\mathbf{Z})}[E(S_t \mid \mathbf{Z}, \mathbf{\Psi})]$$

$$= \int E(S_t \mid \mathbf{Z}, \mathbf{\Psi})P(\mathbf{\Psi} \mid \mathbf{Z})d\mathbf{\Psi}$$

and

(8) $Var(S_t|\mathbf{Z})$

$$= E_{P(\Psi|\mathbf{Z})}[Var(S_t \mid \mathbf{Z}, \Psi)] + E_{P(\Psi|\mathbf{Z})}[\{E(S_t \mid \mathbf{Z}, \Psi) - E(S_t \mid \mathbf{Z}\}^2]$$

$$= \int [\mathrm{Var}(S_t \mid \mathbf{Z}, \boldsymbol{\Psi}) + \{E(S_t \mid \mathbf{Z}, \boldsymbol{\Psi}) - E(S_t \mid \mathbf{Z})\}^2] P(\boldsymbol{\Psi} \mid \mathbf{Z}) d\boldsymbol{\Psi} .$$

It is noted that the integrals in (7) and (8) are with respect to posterior p.d.f. P(**Ψ**|**Z**). With the exception of a few special cases where analytical integration is possible, computations of (7) and (8) pose technical problems. Since computing costs have become inexpensive, an appealing approach to compute (7) and (8) is based on Importance Sampling.

2.2. Importance Sampling

As a variance reduction technique (Hammersley & Handscomb, 1964), Importance Sampling provides a systematic approach that can be applied readily in any situation where the dimension of the integrand is high.

Importance Sampling makes use of a p.d.f. I(**Ψ**), called the importance sampling distribution function. The idea of Importance Sampling is that a set of random variates of **Ψ** that is in the region that is most important for the value of integration is generated from I(**Ψ**). The generated random variates are used to evaluate the integration and this approach can reduce the variance of the estimate of the integration compared to using random variates spread out evenly over the space of **Ψ**.

In general, Geweke (1991) suggests the importance function, I(**Ψ**), needs to have four properties: First, the support of I(**Ψ**) covers that of the posterior distribution of **Ψ** given **Z**, in other words, I(**Ψ**) > 0 over the space of the integrand. Second, it should have convenient Monte Carlo properties, that is, a set of synthetic, independent and identical variates of **Ψ** can be generated readily. Third, it does not have a sharper tail than that of the posterior. Fourth, it must have a shape similar to that of the posterior.

Importance Sampling was discussed by Hammersley and Handscomb (1964) and it was introduced by Kloek and Van Dijk (1978) to solve econometric problems. Since then, several econometric and statistics problems have been solved using Importance Sampling. See, for example, Van Dijk and Kloek (1980,1983), Geweke (1989,1991), Huzurbazar & Butler (1998), and Salmeron et al. (2000).

Importance Sampling for signal extraction is based on the following idea: Let I(**Ψ**) be any distribution from which a set of synthetic, independent and identical variates is readily generated. From the relations (7),

$$E(S_t|\mathbf{Z}) = \frac{\int E(S_t \mid \mathbf{Z}, \mathbf{\Psi})P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z})d\mathbf{\Psi}}{\int P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z})d\mathbf{\Psi}}$$

and since I(**Ψ**) > 0 over the space in which the integrand is defined, it follows that

$$\text{(9)} \qquad E(S_t|\mathbf{Z}) = \frac{\int [E(S_t \mid \mathbf{Z}, \mathbf{\Psi})P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z}) / I(\mathbf{\Psi})]I(\mathbf{\Psi})d\mathbf{\Psi}}{\int [P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z}) / I(\mathbf{\Psi})]I(\mathbf{\Psi})d\mathbf{\Psi}}.$$

From the (8),

$$\text{Var}(S_t|\mathbf{Z}) = \frac{\int[(\text{Var}(S_t \mid \mathbf{Z},\mathbf{\Psi}) + \{E(S_t \mid \mathbf{Z},\mathbf{\Psi}) - E(S_t \mid \mathbf{Z})\}^2]P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z})d\mathbf{\Psi}}{\int P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z})d\mathbf{\Psi}}$$

and since $I(\mathbf{\Psi}) > 0$,

(10) $\text{Var}(S_t|\mathbf{Z})$

$$= \frac{\int[(\text{Var}(S_t \mid \mathbf{Z},\mathbf{\Psi}) + \{E(S_t \mid \mathbf{Z},\mathbf{\Psi}) - E(S_t \mid \mathbf{Z})\}^2][P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z}) / I(\mathbf{\Psi})]I(\mathbf{\Psi})d\mathbf{\Psi}}{\int[P(\mathbf{\Psi})L(\mathbf{\Psi} \mid \mathbf{Z}) / I(\mathbf{\Psi})]I(\mathbf{\Psi})d\mathbf{\Psi}}.$$

Expressions (9) and (10) can be simplified by defining the weight function $w(\mathbf{\Psi}) = P(\mathbf{\Psi})L(\mathbf{\Psi}|\mathbf{Z})/I(\mathbf{\Psi})$. Then (9) and (10) can be written as, respectively,

$$E(S_t|\mathbf{Z}) = \frac{E_{I(\mathbf{\Psi})}[E(S_t \mid \mathbf{Z},\mathbf{\Psi})w(\mathbf{\Psi})]}{E_{I(\mathbf{\Psi})}[w(\mathbf{\Psi})]}$$

and

$$\text{Var}(S_t|\mathbf{Z}) = \frac{E_{I(\mathbf{\Psi})}[\{\text{Var}(S_t \mid \mathbf{Z},\mathbf{\Psi}) + (E(S_t \mid \mathbf{Z},\mathbf{\Psi}) - E(S_t \mid \mathbf{Z}))^2\}w(\mathbf{\Psi})]}{E_{I(\mathbf{\Psi})}[w(\mathbf{\Psi})]}.$$

Let $\boldsymbol{\psi}_1$, $\boldsymbol{\psi}_2$, ..., $\boldsymbol{\psi}_M$ be a set of synthetic, independent and identical variates from $I(\mathbf{\Psi})$. Then the importance sampling estimates of integrals in (7) and (8) are, respectively

(11) $$\widetilde{E}_I(S_t|\mathbf{Z}) = \frac{\sum_{i=1}^{M} E(S_t \mid \mathbf{Z}, \boldsymbol{\psi}_i) w(\boldsymbol{\psi}_i)}{\sum_{i=1}^{M} w(\boldsymbol{\psi}_i)}$$

and

(12) $\widetilde{\mathrm{Var}}_I(S_t|\mathbf{Z})$

$$= \frac{\sum_{i=1}^{M} [\mathrm{Var}(S_t \mid \mathbf{Z}, \boldsymbol{\psi}_i) + \{E(S_t \mid \mathbf{Z}, \boldsymbol{\psi}_i) - \widetilde{E}_I(S_t \mid \mathbf{Z}, \boldsymbol{\psi}_i)\}^2] w(\boldsymbol{\psi}_i)}{\sum_{i=1}^{M} w(\boldsymbol{\psi}_i)}.$$

Geweke(1989) shows that if the support of $I(\boldsymbol{\Psi})$ includes that of the posterior p.d.f. $P(\boldsymbol{\Psi}|\mathbf{Z})$ (or that of $P(\boldsymbol{\Psi})L(\boldsymbol{\Psi}|\mathbf{Z})$), and a set of synthetic, independent and identical variates of $\boldsymbol{\Psi}$ is generated from $I(\boldsymbol{\Psi})$, then the estimates (11) and (12) converge almost surely to $E(S_t|\mathbf{Z})$ and $\mathrm{Var}(S_t|\mathbf{Z})$, respectively.

2.3. Software Available

It is noted that in order to compute (11) and (12), it is necessary to compute not only $E(S_t|\mathbf{Z},\boldsymbol{\psi}_i)$ and $\mathrm{Var}(S_t|\mathbf{Z},\boldsymbol{\psi}_i)$, but $w(\boldsymbol{\psi}_i) = P(\boldsymbol{\psi}_i)L(\boldsymbol{\psi}_i|\mathbf{Z})/I(\boldsymbol{\psi}_i)$. Given $\boldsymbol{\psi}_i$, $E(S_t|\mathbf{Z},\boldsymbol{\psi}_i)$,

$Var(S_t|\mathbf{Z},\psi_i)$ and $L(\psi_i|\mathbf{Z})$ can be computed from the Census Bureau program, REGCMPNT and $P(\psi_i)$ and $I(\psi_i)$ can also be easily evaluated.

The software REGCMPNT has been developed by the Time Series Staff, Statistical Research Division in the U.S. Bureau of the Census. REGCMPNT performs parameter estimation and computes signal extraction estimates for time series models with regression terms in which the component models are of the ARIMA form. REGCMPNT is written in the FORTRAN programming language and is executable in UNIX, PC and VAX operating systems. REGCMPNT can be downloaded via anonymous ftp from the Census Bureau's anonymous ftp site on ftp.census.gov.

3. APPLICATION TO REAL DATA

Data used are U.S. Teen Unemployment for the period of January 1972 to December 1983 (See Table 1) published by U.S. Census Bureau. Since the data are collected by repeated sample surveys, S_t in signal extraction model (1) represents the population value, Z_t an estimate of the value derived from standard sample surveys, and N_t sampling error for time t.

In this application, a stochastic model of N_t is ARMA(1,1), namely $(1- \phi_1 B)N_t = (1- \theta_1 B)b_t$ and b_t is white noise processes with variances σ_b^2. This specification is based on the sampling error variances and autocorrelations. In repeated sample surveys, structures of sampling error variances and autocorrelations are usually known by design of surveys. From the design the values of parameters ϕ_1, θ_1 and σ_b^2 are known as 0.6, 0.3

and 0.876712, respectively and thus they are not components of unknown parameter **Ψ** in this application.

The stochastic model of S_t are ARIMA(0,1,1)(0,1,1)$_{12}$ model (called airline model, Box and Jenkins (1976, 1994)), namely $(1-B)(1-B^{12})S_t = (1-\theta B)(1-\Theta B^{12})a_t$, and a_t is white noise process with variances σ_a^2. This model has been widely used for representing many different seasonal economic time series. In this application, the natural logarithm transformation of σ_a^2, $\ln\sigma_a^2$, is used to make the scales of the parameters consistent. This makes the specification of the prior density P(**Ψ**) and the function I(**Ψ**) simpler. Therefore, for this application the parameter vector **Ψ** is $(\theta, \Theta, \ln\sigma_a^2)$.

In this application, the noninformative prior $P(\theta, \Theta, \ln\sigma_a^2)$ truncated to $|\theta| \le 1$ and $|\Theta| \le 1$ (for stationary and invertibility conditions (Box and Jenkins, 1976, 1994)), is used. Therefore, the posterior p.d.f., $P(\theta, \Theta, \ln\sigma_a^2 \mid \mathbf{Z})$, of this application is proportional to the likelihood function, $L(\theta, \Theta, \ln\sigma_a^2 \mid \mathbf{Z})$.

A multivariate t ($\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, ρ) (Johnson & Kotz, 1972) distribution will be used for the importance function I(**Ψ**). The value of a parameter $\boldsymbol{\mu}$ is the mode of the likelihood function and the value of $\boldsymbol{\Sigma}$ is the negative of the inverse of the Hessian matrix evaluated at the mode. One degree of freedom will be used, namely ρ=1.

This choice of importance function satisfies the requirements stated in section 2.2: First, by the way to specify the values of parameters of t distribution, it is noted that I(**Ψ**)

> 0 over the space of the integrand. Second, a synthetic, independent and identical variates of $\boldsymbol{\Psi}$ from the multivariate t ($\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, ρ) distribution can be readily generated. Third, since this is a Cauchy distribution (namely multivariate t distribution with 1 degree of freedom), it has heavy tails and therefore weights w($\boldsymbol{\Psi}$) should be bounded. Fourth, it has a shape similar to that of the posterior p.d.f. due to the fact that the component models of signal extraction model (1) are ARIMA models (DeGroot 1971; Broemeling & Shaarawy, 1986).

The values of parameters of the multivariate t ($\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, 1) are obtained by the software REGCMPNT of U.S. Census Bureau. The vector $\boldsymbol{\mu}$ and the matrix $\boldsymbol{\Sigma}$ of U.S. Teen Unemployment data are obtained as:

$$\boldsymbol{\mu} = \begin{bmatrix} 0.2711 \\ 0.6801 \\ 8.3648 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.02130 & 0.00605 & 0.02007 \\ 0.00605 & 0.01849 & 0.01001 \\ 0.02007 & 0.01001 & 0.10074 \end{bmatrix}.$$

M = 10,000 synthetic, independent and identical variates of $\boldsymbol{\Psi}$ are generated from the multivariate t ($\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$, 1) distribution and using (11) and (12), estimates of S_t (See Table 2) and its mean square error (MSE) (See Table 3) are obtained. From an extensive simulated study, Lee (1998) shows that using Bayesian approach with importance sampling provides estimates of the signal component S_t as good as or better, in terms of mean square errors, than a commonly used approach using MLE for the hyperparameters.

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Table 1:Teen Unemployment in US from January 1972 – December 1983

	Jan	Fab	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1972	1272	1361	1284	1140	959	1888	1631	1353	1266	1154	1237	1155
1973	1058	1209	1112	1117	996	1804	1582	1184	1219	1129	1257	1152
1974	1281	1271	1237	1040	1095	2067	1863	1306	1492	1412	1526	1473
1975	1744	1667	1689	1535	1569	2451	2194	1839	1688	1617	1598	1615
1976	1753	1670	1638	1568	1453	2268	2030	1810	1621	1587	1656	1576
1977	1699	1655	1667	1456	1419	2391	1981	1676	1631	1501	1564	1315
1978	1561	1592	1571	1380	1342	2065	1954	1567	1536	1454	1503	1470
1979	1539	1539	1473	1381	1362	2065	1832	1567	1538	1471	1461	1427
1980	1541	1547	1456	1314	1616	2307	2137	1750	1614	1616	1653	1472
1981	1730	1747	1673	1565	1627	2258	1971	1682	1675	1703	1841	1679
1982	1849	1918	1777	1778	1892	2415	2326	2015	1937	1924	2007	1883
1983	1863	1805	1818	1718	1742	2527	2179	1907	1700	1627	1584	1474

Table 2:Estimates of Signal Component Using Importance Sampling

	Jan	Fab	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1972	1272	1328	1261	1121	996	1880	1639	1313	1254	1153	1224	1127
1973	1125	1206	1138	1076	994	1823	1589	1213	1215	1138	1240	1151
1974	1270	1279	1239	1086	1106	2014	1806	1386	1462	1411	1520	1472
1975	1667	1664	1655	1530	1537	2388	2147	1790	1700	1618	1646	1595
1976	1723	1691	1647	1537	1477	2293	2055	1752	1648	1588	1650	1559
1977	1683	1664	1633	1466	1442	2292	1999	1665	1604	1503	1548	1380
1978	1558	1575	1535	1379	1361	2122	1927	1577	1527	1458	1510	1434
1979	1541	1539	1479	1366	1369	2106	1872	1569	1521	1463	1490	1415
1980	1546	1554	1489	1375	1535	2275	2069	1727	1633	1604	1650	1515
1981	1703	1721	1661	1554	1605	2298	2044	1737	1701	1702	1799	1683
1982	1841	1880	1806	1751	1830	2497	2308	1998	1923	1891	1948	1820
1983	1881	1850	1811	1704	1737	2464	2181	1859	1712	1636	1630	1504

Table 3:MSE of Estimates of Signal Component Using Importance Sampling

	Jan	Fab	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1972	2005	1930	1806	1650	1473	2248	2082	1849	1757	1649	1713	1653
1973	1540	1667	1566	1577	1460	2096	1965	1645	1666	1581	1687	1603
1974	1696	1681	1662	1483	1552	2240	2169	1791	1917	1825	1909	1900
1975	2057	2045	2024	1944	1971	2484	2386	2152	2054	1989	1982	1981
1976	2061	2018	1986	1948	1875	2363	2265	2136	2000	1955	1994	1944
1977	2019	2001	1996	1853	1847	2440	2231	2023	1976	1880	1933	1754
1978	1915	1931	1916	1783	1770	2236	2198	1942	1904	1842	1873	1854
1979	1903	1899	1851	1788	1787	2236	2120	1944	1909	1857	1850	1831
1980	1911	1922	1856	1782	2010	2424	2330	2099	2006	1989	2009	1907
1981	2083	2094	2046	1977	2036	2422	2277	2086	2068	2083	2172	2090
1982	2218	2270	2200	2188	2277	2673	2558	2373	2321	2313	2354	2273
1983	2308	2272	2279	2210	2270	2745	2575	2379	2225	2194	2210	2304