Supplement on Covariances between model-based estimated totals, August 2012**:**

The estimator for covariance between ratio-type model-based estimated totals, found on page 7, was designed to be a rough estimate made by adjusting the covariance due to the residuals by a factor reflecting the relative importance of residuals in overall covariance for each estimated total. That may sometimes be too crude. More exact work could be done, and for cases with one regressor in each model, please note the results provided by Prof. Poduri S. R. S. Rao, found at the following URL: www.amstat.org/sections/srms/proceedings/papers/1994_048.pdf.

This can be used for estimating standard errors for price-type estimators, and for year-to-date estimated totals.

For year-to-date estimates taken from the sum of monthly estimates from a sample, the variance estimate for the full year result would require 12!/(10!2!) = 66 pairs of estimated covariances between estimated totals.

For many uses for covariance between models, the number of regressors, and/or the regressors themselves vary. The numerator and denominator for price involve different regressors. If more than one regressor were required in either case, an exact result is not provided here.

Note, however, that there are really only two parts to the variance, as in Shmueli(2009), and as seen in Maddala(1992)**:** (1) part due to the residuals, and (2) part due to the model regression coefficients. When there is more than one regressor for either of the models for the two sets of data for which we are to estimate, we can put this into a one regressor format for each model, one for numerator here, and one for denominator, by using a linear combination of regressors as one regressor, in each model, as needed. Perhaps the best such linear combination would be an estimate of y.

Using the format $y_i = y_i^* + e_{0i} w_i^{-0.5}$, for each model, as needed, we have a simple way to calculate an estimate of the covariance between estimated totals, for random variables, using a one regressor form from Rao. This may be fairly accurate, and perhaps less complicated/messy, for covariance estimation purposes, for the equation on page 6.

References in addition to primary list:

Shmueli, G.(2009), "To Explain or to Predict?" Appendix A, Submitted to Statistical Science, Working Paper (RHS 06-099) Smith School of Business, University of Maryland

Särndal, C.-E., Swensson, B. and Wretman, J. (1992), *Model Assisted Survey Sampling*, Springer-Verlag.

Also, for more on the variance of a ratio, but under design-based conditions, see

Knaub, J.R., Jr. (1989), "Ratio Estimation and Approximate Optimum Stratification in Electric Power Surveys," Proceedings of the Section on Survey Research Methods, American Statistical Association, pp. 848-853. Appendix A.
http://www.amstat.org/sections/srms/proceedings/, and

Wolter, K.M. (2007), *Introduction to Variance Estimation*, 2nd ed., Springer.

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